

Chapter 1

Concept Application Exercises

Exercise 1.1

1. (a) $q = ne$

$$n = \frac{q}{e} = \frac{1}{1.6 \times 10^{-19}} = 6.25 \times 10^{18}$$
- (b) Repulsion. As a charged body can attract uncharged body.
- (c) No, as charge is quantized.
2. By induction.
3. (a) Yes. Two bodies are placed close to each other where one has much more charge than the other. Then due to induction, force of attraction becomes more than force of repulsion.
- (b) Yes, due to induction effect.
- (c) Yes, mass of negatively charged sphere will be more.
4. (a) Since the particle has positive charge, it has deficiency of electrons. So it has less number of electrons compared to the neutral state.
- (b) $q = ne$
 $10^{-12} = n \times 1.6 \times 10^{-19}$
 $n = 6.25 \times 10^6$
5. (a) $q = ne$
 $3.2 \times 10^{-8} = n \times 1.6 \times 10^{-19}$
 $n = 2 \times 10^{11}$
- (b) Charge on fur should be equal and opposite to that on ebonite, i.e., $+3.2 \times 10^{-8}$ C.
6. Protons cannot be transferred, as they reside in nucleus. To remove protons, a great amount of energy is required.
7. When the paper comes in contact with the rod, the paper also gets similarly charged. Hence, repulsion takes place.
8. Conductor will not get completely discharged, because the person is standing on the insulating stool. Some charge of the rod may transfer to the person to make the potential of the both same.
9. There should be other charged particles to feel the effect of a charge particle, otherwise the concept of electric charge would be meaningless.
10. Number of molecules in 2.0 mol of H_2 gas $= 2(6.02 \times 10^{23}) = 12.04 \times 10^{23}$
 As each H_2 molecule contains 2 electrons (and 2 protons), number of electrons (and protons) in 2.0 mol of H_2 gas is
 $n = 2(12.04 \times 10^{23}) = 24.08 \times 10^{23}$
 Thus, $q = ne = (24.08 \times 10^{23})(1.6 \times 10^{-19} \text{ C})$
 $= 0.358 \times 10^6 \text{ C} = 0.358 \text{ MC}$
11. (a) Since the polythene piece acquires a negative charge (q) on rubbing with wool, electrons are transferred from wool to polythene. If n is the number of electrons transferred, then
 $q = ne$ or $n = \frac{q}{e} = \frac{3 \times 10^{-7} \text{ C}}{1.6 \times 10^{-19} \text{ C}} = 1.875 \times 10^{12}$
- (b) Mass transferred from wool to polythene is
 $(1.875 \times 10^{12})(9.1 \times 10^{-31} \text{ kg}) = 1.7 \times 10^{-18} \text{ kg}$
- Of course, this mass is negatively small.
12. (a) This is due to the reason that the spheres are identical and they share the total charge equally.
- (b) The charge on other sphere will be $+Q$. Because of induction the charge on other sphere will be $+Q$.

13. Due to conservation of charge, it is clear from figure that

$$Q_1 = \frac{Q}{2}, Q_2 = -\frac{Q}{4}, \text{ and } Q_3 = -\frac{Q}{4}$$

$$\begin{array}{ccc|ccc} +Q & -Q & & +Q & +Q/2 & +Q/2 \\ \textcircled{1} & \textcircled{2} & & \textcircled{1}\textcircled{3} & \Rightarrow \textcircled{1} & \textcircled{3} \\ -Q & +Q/2 & & -Q/4 & -Q/4 & \\ \textcircled{2}\textcircled{3} & \Rightarrow & & \textcircled{2} & \textcircled{3} & \end{array}$$

14. The table tennis ball when slightly displaced, say toward the positive plate, gets attracted toward the positive plate due to induced negative charge on its near surface.

The ball touches the positive plate and itself gets positively charged by the process of conduction from the plate connected to high voltage generator. On getting positively charged, it is repelled by the positive plate and, therefore, the ball touches the other plate (earthed), which has negative charge due to induction. On touching this plate, the positive charge of the ball gets neutralized and in turn the ball shares negative charge of the earthed plate and is again repelled from this plate also, and this process is repeated again and again.

Here it should be understood that since the positive plate is connected to high voltage generator, its potential and hence its charge will always remain same. As soon as this plate gives some of its charge to ball, excess charge flows from generator to the plate, and an equal negative charge is always induced on the other plate.

15. To calculate charge, we will apply formula $Q = ne$; for this, we must have number of electrons. Here, number of electrons is $n = 0.01\%$ of 5×10^{21}

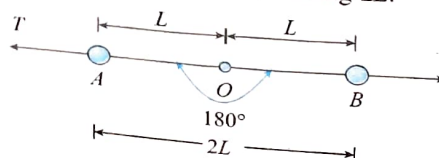
$$\text{i.e., } n = \frac{5 \times 10^{21} \times .01}{100} = 5 \times 10^{21} \times 10^{-4} = 5 \times 10^{17}$$

$$\text{So } Q = 5 \times 10^{17} \times 1.6 \times 10^{-19} = 8 \times 10^{-2} = 0.08 \text{ C}$$

Since electrons have been removed, charge will be positive i.e., $Q = +0.08 \text{ C}$

Exercise 1.2

1. (a) Right, because third particle should be placed near the charge smaller in magnitude and not between the charges.
- (b) The third particle should be negatively charged, only then net force on any charge due to other two charges can be zero.
- (c) Equilibrium is unstable, because if we displace any of the charged particles from its equilibrium position, it may not return to its initial position for all directions of displacement.
2. Let A and B be two charged balls suspended from hook, O as shown in figure.
- (a) In the absence of gravity, tension in the strings is due to only Coulomb's repulsive force. As such the two strings will be in the same straight line making an angle of 180° with each other, the separation between the balls being $2L$.



5.2 Electrostatics and Current Electricity

- (b) Tension in each string $T = \text{Coulomb's force between the charged balls, i.e.,}$

$$T = k_e \frac{Q \times Q}{(2L)^2} = k_e \frac{Q^2}{4L^2} \text{ (N)}$$

3. (a) Since the size of group is small, the group of n small charged particles must behave as single point charge, so that it can have a separation of 10 cm from the particle in question. Obviously, force on this particle due to the group of n particles is n times the force due to a single particle. Hence, force due to group of n particles is $F = n \times 3 \times 10^{-10} \text{ N}$.

- (b) Here, $F = 6 \times 10^{-6} \text{ N}$

$$n = \frac{F}{3 \times 10^{-10}} = \frac{6 \times 10^{-6}}{3 \times 10^{-10}} = 2 \times 10^4$$

4. As $F = k_e \frac{q_1 q_2}{r^2}$
 $3.7 \times 10^{-9} = (9 \times 10^9) \frac{q^2}{(5 \times 10^{-10})^2}$ hence $q = 3.2 \times 10^{-19} \text{ C}$

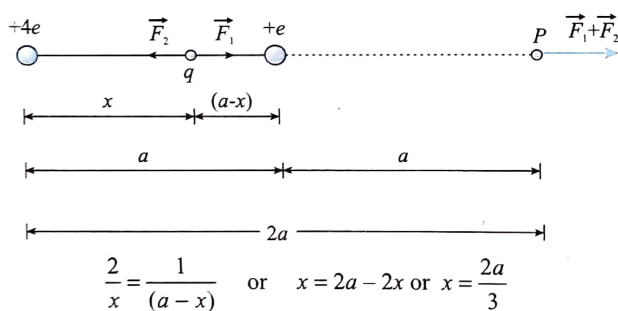
$$\text{As } q = ne, n = \frac{q}{e} = \frac{3.2 \times 10^{-19} \text{ C}}{1.6 \times 10^{-19} \text{ C}} = 2$$

5. Let the third point charge (say q) be at a distance x from $+4e$ as shown in figure. Since the distance between $+4e$ and $+e$ is a , distance between q and $+e$ is $(a-x)$.

For q to be in equilibrium, force between q and $+4e$, i.e., F_1 = force between q and $+e$, i.e., F_2

$$k_e \frac{q \times 4e}{x^2} = k_e \frac{q \times e}{(a-x)^2}$$

$$\text{or } \frac{4}{x^2} = \frac{1}{(a-x)^2} \text{ or } \frac{2}{x} = \pm \frac{1}{(a-x)}$$



Thus, q should be at a distance of $2a/3$ from $+4e$ charge.

6. We are given that, charge on sphere A = charge on sphere B = $q = 6.5 \times 10^{-7} \text{ C}$

Force of repulsion between A and B is

$$F = k_e \frac{q \times q}{r^2} = k_e \frac{q^2}{r^2} = (9 \times 10^9) \frac{(6.5 \times 10^{-7})^2}{(0.5)^2} = 1.5 \times 10^{-2} \text{ N}$$

Let the third sphere (say C), of the same size but uncharged, be brought in contact with A . Due to the flow of electrons, the two spheres share the charge equally. Therefore, charge on A = charge on C = $\frac{q+0}{2} = \frac{q}{2}$

When the sphere C (having charge $q/2$) touches the sphere B (having charge q), total charge on B and C ($q + q/2 = 3q/2$) is equally distributed between them. Thus, charge on B = charge on C = $3q/4$. If F' is the new force of repulsion between A and B ,

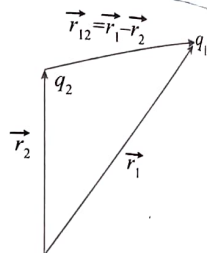
$$F' = k_e \frac{(q/2)(3q/4)}{r^2} = k_e \frac{3q^2}{8r^2} = \frac{3}{8} F = \frac{3}{8} (1.5 \times 10^{-2} \text{ N}) = 5.7 \times 10^{-3} \text{ N}$$

$$7. \vec{F}_{12} = \frac{q_1 q_2 \vec{r}_{12}}{4\pi\epsilon_0 r_{12}^3}$$

$$\text{where } \vec{r}_{12} = \vec{r}_1 - \vec{r}_2 = (\hat{i} - \hat{j} + 2\hat{k}) - (-\hat{i} + \hat{j} + \hat{k}) = 2\hat{i} - 2\hat{j} + \hat{k}$$

$$\text{and } r_{12} = |\vec{r}_{12}| = \sqrt{4 + 4 + 1} = 3$$

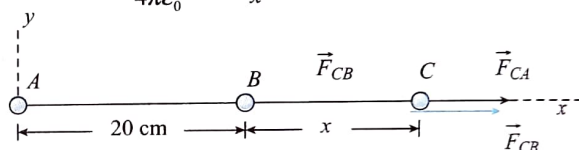
$$\text{or } \vec{F}_{12} = \frac{(9 \times 10^9)(20 \times 10^{-6})(-10 \times 10^{-6})}{(3)^3} (2\hat{i} - 2\hat{j} + \hat{k}) = -0.06(2\hat{i} - 2\hat{j} + \hat{k}) \text{ N}$$



8. As the net electric force on C should be equal to zero, the forces due to A and B must be opposite in direction. Hence, the particle should be placed on the line AB . As A and B have charges of opposite signs, C cannot be between A and B . Also, A has larger magnitude than B . Hence, C should be placed closer to B than A . The situation is shown in figure. Suppose $BC = x$ and the charge on C is Q .

$$\vec{F}_{CA} = \frac{1}{4\pi\epsilon_0} \frac{(8.0 \times 10^{-6})Q}{(0.2+x)^2} \hat{i}$$

$$\text{and } \vec{F}_{CB} = \frac{-1}{4\pi\epsilon_0} \frac{(2.0 \times 10^{-6})Q}{x^2} \hat{i}$$



$$\vec{F}_C = \vec{F}_{CA} + \vec{F}_{CB} = \frac{1}{4\pi\epsilon_0} \left[\frac{(8.0 \times 10^{-6})Q}{(0.2+x)^2} - \frac{(2.0 \times 10^{-6})Q}{x^2} \right] \hat{i}$$

$$\text{But } |\vec{F}_C| = 0$$

$$\text{Hence, } \frac{1}{4\pi\epsilon_0} \left[\frac{(8.0 \times 10^{-6})Q}{(0.2+x)^2} - \frac{(2.0 \times 10^{-6})Q}{x^2} \right] = 0$$

which gives $x = 0.2 \text{ m}$

9. Electrostatic force on $-q_1$ due to q_2 will provide the necessary centripetal force. Hence,

$$\frac{kq_1 q_2}{r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{kq_1 q_2}{mr}}$$

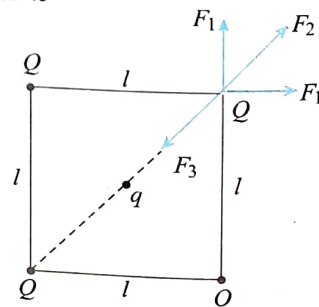
$$\text{Now } T = \frac{2\pi r}{v} = \sqrt{\frac{16\pi^3 \epsilon_0 m r^3}{q_1 q_2}}$$

10. (a) $F = \frac{9 \times 10^9 \times 12 \times 10^{-9} \times 18 \times 10^{-9}}{(0.30)^2} = 2.16 \times 10^{-5} \text{ N (attractive)}$

- (b) When they are connected by a conducting wire, finally charge on each will be half of total charge on both. Let q be the final charge on each, then $q = (12 - 18)/2 = -3 \text{ nC}$.

$$F' = \frac{9 \times 10^9 \times (3 \times 10^{-9})^2}{(0.30)^2} = 9 \times 10^{-7} \text{ N (repulsive)}$$

11. $F_3 = F_2 + 2F_1 \cos 45^\circ$



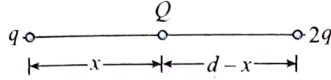
$$\text{or } \frac{kqQ}{(l/\sqrt{2})^2} = \frac{kQQ}{(\sqrt{2}l)^2} + 2 \frac{kQQ}{l^2} \frac{1}{\sqrt{2}}$$

$$\text{or } q = \frac{Q(2\sqrt{2}+1)}{4}$$

Q should be negative as there is attraction between Q and q . So,

$$q = \frac{-Q(2\sqrt{2}+1)}{4}$$

12. Q should be negative and it should be placed somewhere between q and $2q$.



For net force on q to be zero:

$$\frac{kqQ}{x^2} = \frac{kq2q}{d^2} \quad \dots(i)$$

For net force on $2q$ to be zero:

$$\frac{kQ2q}{(d-x)^2} = \frac{kq2q}{d^2} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{1}{x^2} = \frac{2}{(d-x)^2} \text{ or } x = \frac{d}{1+\sqrt{2}}$$

13. As there is attraction (here we have assumed Q_1 and Q_2 in μC), so we get

$$\frac{9 \times 10^9 \times Q_1 Q_2 \times 10^{-12}}{1^2} = -0.027 \text{ or } Q_1 Q_2 = -3$$

Negative sign is due to attraction.

For repulsion:

$$\frac{9 \times 10^9 \times \left(\frac{Q_1 + Q_2}{2}\right)^2 \times 10^{-12}}{1^2} = 0.009$$

$$\text{or } (Q_1 + Q_2)^2 = 4 \text{ or } Q_1 + Q_2 = \pm 2$$

Here, we have two sets of equations.

(i) From $Q_1 Q_2 = -3$ and $Q_1 + Q_2 = 2$ we get $3 \mu\text{C}$ and $-1 \mu\text{C}$.

(ii) From $Q_1 Q_2 = -3$ and $Q_1 + Q_2 = -2$ we get $-3 \mu\text{C}$ and $1 \mu\text{C}$.

Both are valid.

14. Number of atoms in 10 g of copper is $n = \frac{10}{63.5} \times 6 \times 10^{23} = 9.448 \times 10^{22}$

Number of electrons transferred is $N = \frac{n \times 1}{1000} = 9.448 \times 10^{19}$

(a) Magnitude of charge appearing on either piece is

$$q = Ne = 9.448 \times 10^{19} \times 1.6 \times 10^{-19} = 15.12 \text{ C}$$

$$(b) F = \frac{kq^2}{r^2} = \frac{9 \times 10^9 \times (15.12)^2}{(0.10)^2} = 2.05 \times 10^{14} \text{ N}$$

15. As we see

$$T \sin \theta = F = \frac{1}{4\pi\epsilon_0} \frac{q^2}{a^2} \quad \dots(i)$$

$$T \cos \theta = mg \quad \dots(ii)$$

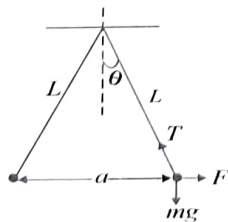
From Eqs. (i) and (ii), we get

$$\tan \theta = \frac{q^2}{4\pi\epsilon_0 a^2 mg}$$

$$\frac{a/2}{L} = \frac{q^2}{4\pi\epsilon_0 a^2 mg}$$

$$\text{or } \frac{a^3}{L} = \frac{q^2}{2\pi\epsilon_0 mg} \quad \dots(iii)$$

When one of the balls is discharged, the balls come closer and touch each other and again separate due to repulsion. The charge



on each ball after touching each other is $q/2$. Replacing q with $q/2$ in Eq. (iii), we get

$$\frac{b^3}{L} = \frac{(q/2)^2}{2\pi\epsilon_0 mg} \quad \dots(iv)$$

From Eqs. (iii) and (iv), we get

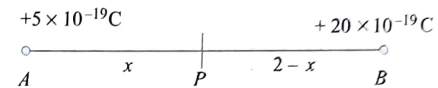
$$\frac{b^3}{a^3} = \frac{1}{4} \text{ or } b = \frac{a}{2^{2/3}}$$

Exercise 1.3

1. (i) Both the charges cannot be of same sign and magnitudes of charge should be different.

(ii) The observation point (where electric field intensity is zero) has to be closer to the charge smaller in magnitude.

2. Let electric field at point P is zero, then



$$\frac{1}{4\pi\epsilon_0} \frac{5 \times 10^{-19}}{x^2} = \frac{1}{4\pi\epsilon_0} \frac{20 \times 10^{-19}}{(2-x)^2}$$

$$\text{or } \frac{2-x}{x} = 2 \text{ or } x = \frac{2}{3} \text{ m}$$

3. (a) As electric field tends away at a and toward at b , there should be positive charge at a and negative charge at b , i.e., q_a is '+' and q_b is '-'.

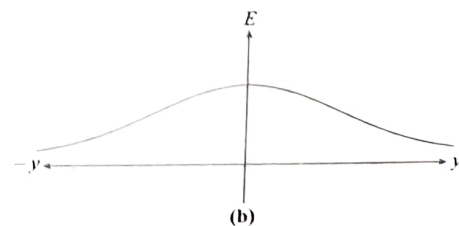
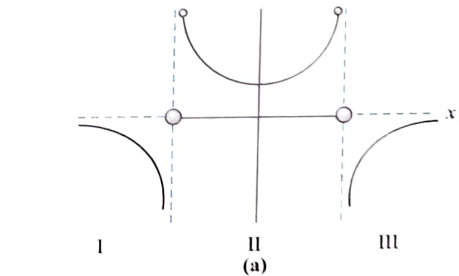
(b) The neutral point exists between a and b only when both q_a and q_b are of same sign. As direction of electric field is away from both, both charges are positive, i.e., q_a is '+' and q_b is '+'.

(c) q_a is '-' and q_b is '+'

(d) q_a is '-' and q_b is '-'

4. (a) Variation of E along the x -axis is shown in Fig. (a).

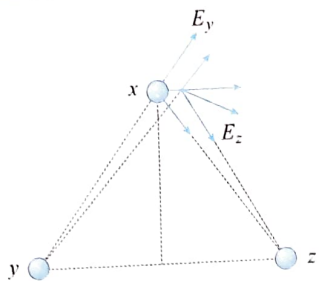
(b) Variation of E along the y -axis is shown in Fig. (b). In this case, field will be maximum at origin because at origin, field due to both charges is directly added.



5. (a) Along C .

(b) Path D . As x starts moving, the magnitude of electric field due to charge on y decreases while the magnitude of electric field due to charge on z increases. The resultant electric field changes the direction as shown in the figure. Hence, we can say the charge particle follows path D .

5.4 Electrostatics and Current Electricity



6. We have to find intensity at P .

$$E = \frac{kQ}{a^2}, \frac{a/2}{x} = \cos 30^\circ \text{ or } x = \frac{a}{\sqrt{3}}$$

$$y = \sqrt{a^2 - x^2} = \sqrt{a^2 - \frac{a^2}{3}} = \left(\frac{\sqrt{2}}{\sqrt{3}}\right)a$$

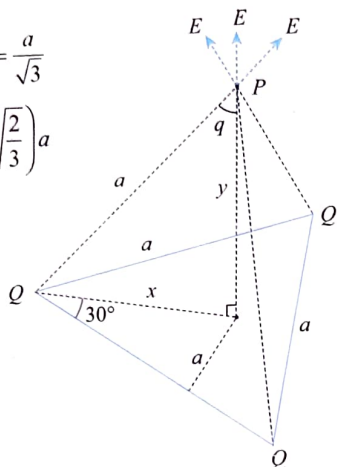
$$\cos \theta = \frac{y}{a} = \frac{\sqrt{2}}{\sqrt{3}}$$

Net intensity at P is

$$E_0 = 3E \cos \theta = \frac{3kQ}{a^2} \sqrt{\frac{2}{3}}$$

$$= \frac{3}{4\pi\epsilon_0} \frac{Q}{a^2} \sqrt{\frac{2}{3}}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} \frac{Q}{\pi\epsilon_0 a^2}$$



7. Electric field at $P = x$ is

$$E = 2 \left[\frac{1}{4\pi\epsilon_0} \frac{q}{(a^2 + x^2)} \right] \cos \theta$$

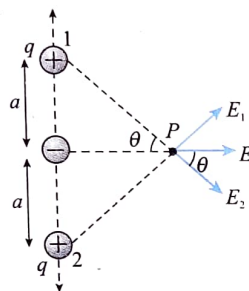
$$= \frac{q}{2\pi\epsilon_0} \frac{x}{(a^2 + x^2)^{3/2}}$$

Force on particle is

$$F = -qE = \frac{-q^2}{2\pi\epsilon_0} \frac{x}{(a^2 + x^2)^{3/2}}$$

For $x \ll a$, particle will execute SHM with time period

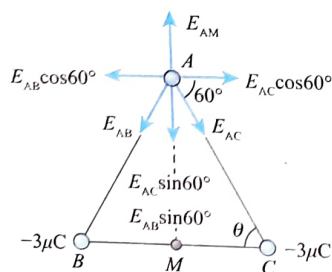
$$T = 2\pi \sqrt{\frac{2\pi\epsilon_0 m a^3}{q^2}}$$



8. Net electric field at A due to charges at B and C is

$$E_A = 2E_{AC} \sin 60^\circ = 2 \times 9 \times 10^9 \times \frac{3 \times 10^{-6}}{(0.20)^2} \times \frac{\sqrt{3}}{2}$$

$$= \sqrt{3} \times 6.75 \times 10^5 \text{ N/C}$$



$$AM = \sqrt{(20)^2 - (10)^2} = \sqrt{400 - 100} = 10\sqrt{3} \text{ cm}$$

Let the charge at M be q . Charge q should be positive so that there can be repulsion between the charges at A and M .

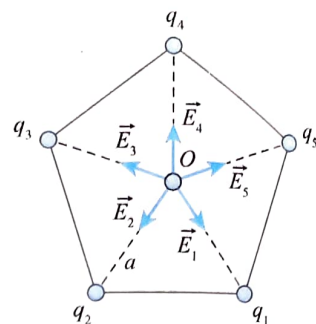
$$E_{AM} = 9 \times 10^9 \times \frac{q}{(10\sqrt{3}/100)^2} = 3q \times 10^{11} \text{ N/C}$$

For A to be in equilibrium: $E_A = E_{AM}$

$$\Rightarrow \sqrt{3} \times 6.75 \times 10^5 = 3q \times 10^{11} \text{ N/C}$$

$$\text{or } q = \frac{9\sqrt{3}}{4} \times 10^{-6} = \frac{9\sqrt{3}}{4} \mu\text{C}$$

9. We can calculate the electric field at centre by the superposition method. Consider pentagon with charges on all vertex. The electric field at centre must be zero due to symmetry.



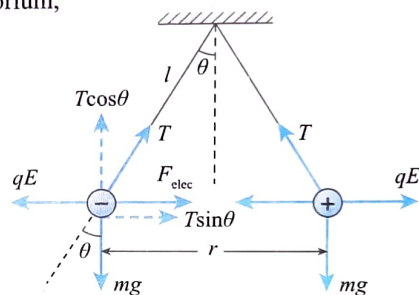
$$\vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 + \vec{E}_5 = 0$$

Thus electric field due to 4 charge + electric field due to 1 charge = 0 or electric field due to 4 charge = - electric field due to 1 charge or $\vec{E}_5 = -(\vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4)$

Hence the magnitude of electric field due to four charges should be equal to the magnitude of electric field due to point charged particle at the vertex 'S'.

Thus |electric field due to 4 charge| = |electric field due to 1 charge| $= \frac{kq}{a^2}$

10. For equilibrium,



$$T \sin \theta = qE - F_{\text{elec}} \quad \dots(i)$$

$$T \cos \theta = mg \quad \dots(ii)$$

$$\text{From (i) and (ii), } \tan \theta = \frac{qE - F_{\text{elec}}}{mg}$$

$$\text{and } mg \tan \theta = qE - \frac{kq^2}{r^2} \quad mg \tan \theta = qE - \frac{kq^2}{(2l \sin \theta)^2}$$

$$\text{or } E = \left(\frac{mg \tan \theta}{q} + \frac{a}{16\pi\epsilon_0 l^2 \sin^2 \theta} \right)$$

Exercise 1.4

1. Electric field at the center of the full ring will be zero, because field at center due to some element will be canceled by the opposite element of same length. But in half ring, there will be a net field at the center of the ring. So field due to half ring has greater magnitude.

$$2. \text{ Electric field due to } MA: \vec{E}_1 = \frac{\lambda}{4\pi\epsilon_0 R} (\hat{i} - \hat{k})$$

$$\text{Electric field due to } ADC: \vec{E}_2 = \frac{\lambda}{2\pi\epsilon_0 R} (-\hat{k})$$

$$\text{Electric field due to } ABC: \vec{E}_3 = \frac{\lambda}{2\pi\epsilon_0 R} (-\hat{j})$$

$$\text{Electric field due to } NC: \vec{E}_4 = \frac{\lambda}{4\pi\epsilon_0 R} (-\hat{i} + \hat{k})$$

$$\text{Net electric field is } \vec{E}_0 = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 = -\frac{\lambda}{2\pi\epsilon_0 R} (\hat{j} + \hat{k})$$

3. Electric field due to straight wires will cancel out. The net electric field will be due to semicircular wire only. Hence,

$$\vec{E}_{\text{net}} = \vec{E}_{\text{circular}} = \frac{\lambda}{2\pi\epsilon_0 R}(-\hat{j})$$

So, acceleration is $\vec{a} = \frac{q\vec{E}_{\text{net}}}{m} = \frac{q\lambda}{2\pi\epsilon_0 m R}(-\hat{j})$

4. Electric field at point P due to charge of ring is

$$E = \frac{kQx}{(R^2 + x^2)^{3/2}}$$

At $x = R$, $E = \frac{kQ}{2\sqrt{2}R^2}$ directed toward the center.

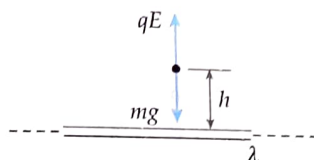
Electric field at P due to charge at center $= kq/R^2$

For net field to be zero at $P = \frac{kq}{R^2} = \frac{kQ}{2\sqrt{2}R^2}$ or $q = \frac{Q}{2\sqrt{2}}$

5. At equilibrium $mg = qE$

$$mg = \frac{q2k\lambda}{h}$$

...(i)



When particle is displaced to a distance x in upward direction, then net force in upward direction is

$$F = q\left(\frac{2k\lambda}{h+x}\right) - mg$$

$$= \frac{2kq\lambda}{h}\left(1 + \frac{x}{h}\right)^{-1} - mg = \frac{2kq\lambda}{h}\left(1 - \frac{x}{h}\right) - mg$$

$$= -\left(\frac{2kq\lambda}{h^2}\right)x = -k_0x$$

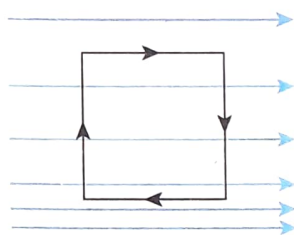
$\therefore F \propto -x$ (Motion is SHM)

Time period of motion is

$$T = 2\pi\sqrt{\frac{m}{K_0}} = 2\pi\sqrt{\frac{mh^2}{2kq\lambda}}$$

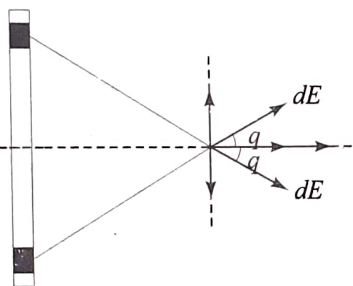
Exercise 1.5

1. No, in given hypothetical field lines electric field is strong at bottom and weak at top. If we move a charge in a close path some work will be done by electric field, but in an electric field, no work is done in close path.



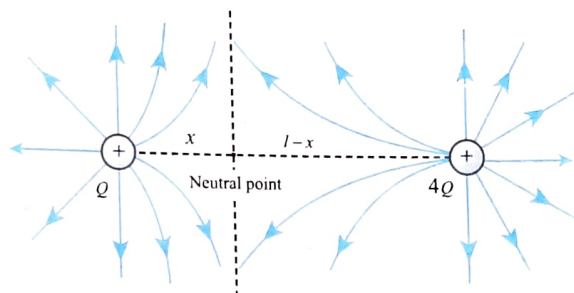
2. (a) Along the tangents at points A and B .
(b) At A , as intensity of electric field is more at A .

3. It is clear from the diagram that the components perpendicular to normal to plate will cancel each other and along the normal $dE \cos\theta$, they will get added. As the distance decreases, dE increases but $\cos\theta$ also decreases in such a way that $dE \cos\theta$ remains constant for all positions.



It can be observed that near an infinite sheet, electric field is constant everywhere.

4.



For its location,

$$\frac{1}{4\pi\epsilon_0} \frac{Q}{x^2} = \frac{1}{4\pi\epsilon_0} \frac{4Q}{(l-x)^2}$$

$$\left(\frac{l-x}{x}\right)^2 = 4 \quad \text{or} \quad x = \frac{l}{3} = \frac{12}{3} = 4 \text{ cm}$$

Hence, neutral point will be at 4 cm from charge Q .

5. The acceleration experienced by a charged particle under the action

of an electric field E is $a = \frac{eE}{m}$

If it falls through a distance h , starting from rest,

$$h = \frac{1}{2}at^2 \quad (\text{as } u = 0)$$

$$\text{or } t^2 = \frac{2h}{a} = \frac{2h}{eE/m} = \frac{2mh}{eE}$$

$$\text{or } t \propto \sqrt{m}$$

...(i)

From Eq. (i) the time of fall will be

$$\frac{t_{\text{electron}}}{t_{\text{proton}}} = \sqrt{\frac{m_e}{m_p}}$$

Now since $m_p > m_e$, so $t_p > t_e$, i.e., time of fall through the same distance is greater for a heavier particle (i.e., proton), which is in contrast with the situation of free fall under gravity where the time of fall is independent of the mass of the body.

6. (1) and (2) are negative while (3) is positive.

Acceleration of each particle is $a = \frac{qE}{m}$

Deflection suffered by a particle is $y = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}\frac{qE}{m}t^2$
or $y \propto \frac{q}{m}$

Hence, (3) will have the highest q/m ratio.

7. (a) The force F experienced by the electron toward the positive plate is given by

$$F = qE = 1.6 \times 10^{-19} \times (45/16) \times 10^3 = 4.5 \times 10^{-16} \text{ N}$$

Hence, the acceleration experienced by the electron is given by

$$a = \frac{F}{m} = \frac{4.5 \times 10^{-16} \text{ N}}{9.0 \times 10^{-31} \text{ kg}} = 5.0 \times 10^{14} \text{ ms}^{-2}$$

Now, the electron is released from the negative plate and is traveling toward the positive plate. Hence we have $u = 0$, $x = 0.10 \text{ m}$, and $a = 5.0 \times 10^{14} \text{ ms}^{-2}$.

According to equation $x = ut + \frac{1}{2}at^2$,

$$t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2 \times 0.10}{5.0 \times 10^{14}}} = 2 \times 10^{-8} \text{ s}$$

(b) $v = u + at = 0 + 5.0 \times 10^{14} \times 2.0 \times 10^{-8} = 10^7 \text{ ms}^{-2}$

8. (a) The electron will move with constant velocity in the x -direction. Time taken by the electron to come out of the plate is $t = l/v_0$.

$$\text{Hence } h = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{l}{v_0}\right)^2 = \left(\frac{eEl}{2mv_0^2}\right)l^2$$

- (b) Velocity of electron when it comes out of the plate is

$$\begin{aligned}\vec{v} &= \vec{u} + \vec{at} = v_0\hat{i} + \left(\frac{eE}{m}\right)\hat{j} \cdot t \\ &= v_0\hat{i} + \left(\frac{eE}{m}\right)\hat{j}\left(\frac{l}{v_0}\right) = v_0\hat{i} + \left(\frac{eEl}{mv_0}\right)\hat{j}\end{aligned}$$

- (c) After coming out of field, the electron moves in straight line with constant velocity $\vec{v}_0$. Further time taken by electron to hit the screen is $t' = l/v_0$. Hence, further displacement of electron outside the electric field is

$$y' = (v_y) \cdot t' = \left(\frac{eEl}{mv_0}\right)\frac{l}{v_0} = \frac{eEl^2}{mv_0^2}$$

$$\text{Hence } H = h + y' = \frac{3eEl^2}{2mv_0^2}$$

$$9. v_i = 6.0 \times 10^3 \text{ ms}^{-1}$$

$$a_y = \frac{eE}{m} = \frac{(1.6 \times 10^{-19})(900)}{(1.6 \times 10^{-27})} = 9.0 \times 10^{10} \text{ ms}^{-2}$$

$$R = \frac{v_i^2 \sin 2\theta}{a_y} = 2 \times 10^{-3} \text{ m}$$

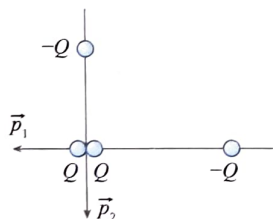
$$\text{So that } \frac{(6 \times 10^3)^2 \sin 2\theta}{9 \times 10^{10}} = 2 \times 10^{-3}$$

$$\sin 2\theta = \frac{1}{2} \quad \text{or} \quad 2\theta = 30^\circ$$

$$\text{or } \theta = 15^\circ, 90^\circ - \theta = 75^\circ$$

Exercise 1.6

1. We can make the arrangement of the charges as shown.

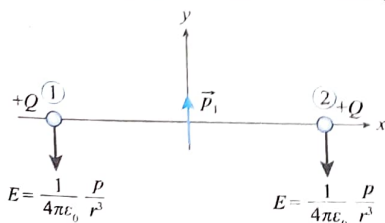


The dipole moments of the dipoles $\vec{p}_1 = Qa(-\hat{i})$ and $\vec{p}_2 = Qa(-\hat{j})$

$$\vec{p}_{\text{net}} = \vec{p}_1 + \vec{p}_2 = Qa(-\hat{i}) + Qa(-\hat{j}) = -Qa(\hat{i} + \hat{j})$$

$$|\vec{p}_{\text{net}}| = \sqrt{2} Q \cdot a$$

2. The charged particles are placed at the equatorial position of the dipole. The force on the charged particles because of the dipole should be equal to the force on the dipole because of the charged particle as both the forces form the action reaction pair.



From the above figure it is clear. Force on the charged particles are

$$\vec{F}_{\text{charged particles}} = 2Q\vec{E} = 2Q \cdot \frac{1}{4\pi\epsilon_0} \frac{p}{r^3} (-\hat{j})$$

$$\text{or } \vec{F}_{\text{charged particles}} = \frac{1}{2\pi\epsilon_0} \frac{pQ}{r^3} (-\hat{j})$$

$$\text{Hence the force on the dipole } \vec{F}_{\text{dipole}} = \frac{1}{2\pi\epsilon_0} \frac{pQ}{r^3} (\hat{j})$$

3. The electric field due to first dipole on the second is $E_1 = \frac{1}{4\pi\epsilon_0} \frac{p_1}{r^3}$

$$\text{The gradient of electric field on the axis is } \frac{dE_1}{dr} = -\frac{1}{4\pi\epsilon_0} \frac{3p_1}{r^4}$$

The magnitude of force of interaction between the two dipoles is

$$|\vec{F}| = p_2 \left| \frac{dE_1}{dr} \right| = \frac{1}{4\pi\epsilon_0} \frac{3p_1 p_2}{r^4}$$

4. Suppose, the dipole axis makes an angle θ with the electric field at an instant. The magnitude of the torque on it is $\tau = -qlE \sin \theta$.

This torque will tend to rotate the dipole back towards the electric field. Also, for small angular displacement $\sin \theta \approx \theta$ so that $\tau = -qlE\theta$

The moment of inertia of the system about the axis of rotation is

$$I = 2 \times m \left(\frac{l}{2} \right)^2 = \frac{ml^2}{2}$$

$$\text{Thus, the angular acceleration is } \alpha = \frac{\tau}{I} = -\frac{2qEl}{ml} \theta = -\omega^2 \theta$$

$$\text{where, } \omega^2 = \frac{2qEl}{ml}$$

Thus, the motion is angular simple harmonic and the time period is

$$T = 2\pi \sqrt{\frac{ml}{2qEl}}$$

5. The force on a dipole is

$$\vec{F} = p \left(\frac{\partial \vec{E}}{\partial x} \right) = p \frac{\partial}{\partial x} \left\{ \frac{Qx}{4\pi\epsilon_0 (R^2 + x^2)^{3/2}} \right\}$$

$$= \frac{pQ}{4\pi\epsilon_0} \cdot \frac{d}{dx} \left\{ \frac{x}{(R^2 + x^2)^{3/2}} \right\}$$

$$= \frac{Qp}{4\pi\epsilon_0} \left\{ \frac{(R^2 + x^2)^{3/2} - \frac{3}{2}x(R^2 + x^2)^{1/2} \cdot 2x}{(R^2 + x^2)^3} \right\}$$

$$F = \frac{Qp}{4\pi\epsilon_0} \left(\frac{R^2 - 2x^2}{(R^2 + x^2)^{5/2}} \right)$$

Exercises

Single Correct Answer Type

1. (3) 1 2 3 4 5

+ - +

If 1 is positively charged, then 2 should be negatively charged, then 4 should be positively charged. Now, 1 and 4 cannot attract. It means ball 1 should be neutral (2 and 4 cannot be neutral because they are showing repulsion with 3 and 5, respectively).

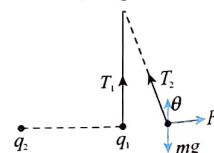
2. (2) Initial tension: $T_1 = mg$

$$\text{Final tension: } T_2 \cos \theta = mg$$

$$\text{or } T_2 = \frac{mg}{\cos \theta}$$

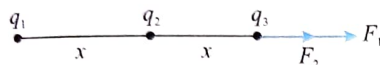
Obviously, $T_2 > mg$.

Here we have assumed θ to be small so that F is almost horizontal.

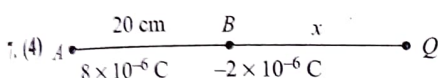


- (1) The object should be positively charged. Being positively charged, it repels positive charge from top plate toward leaves causing to separate more.
- (3) Electric field is perpendicular to the equipotential surfaces and is zero everywhere inside the metal.
- (1) is incorrect because '1' and '2' are repelled by '3'. So '1' and '2' have charge of same nature. Hence they should repel each other. For this reason, (II) is correct.
- (III) is correct if charge on '2' and '3' is of same nature and on '1' opposite of them.
- (IV) All attraction is not possible.

- (3) Net force on q_3 is $F_1 + F_2 = 0$



$$\text{or } \Rightarrow \frac{kq_1q_3}{(2x)^2} + \frac{kq_2q_3}{x^2} = 0 \Rightarrow q_1 = -4q_2$$



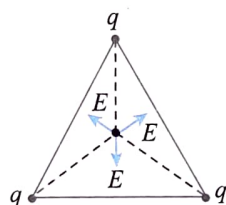
Let the third charge Q be placed at a distance x to the right of B . Then

$$\frac{kQ \times 8 \times 10^{-6}}{(20+x)^2} = \frac{kQ(2 \times 10^{-6})}{x^2}$$

$$\text{or } x = 20 \text{ cm}$$

- (3) $100 = kq_1q_2/r^2$ and $F = k(1.1q_1)(0.9q_2)/r^2$
 $\Rightarrow \frac{F}{100} = 0.99$ or $F = 99 \text{ N}$

- (4) Each charge will produce the same magnitude of intensity, say E , at the centroid. These are directed at angles of 120° with each other. So, their vector sum will be zero.

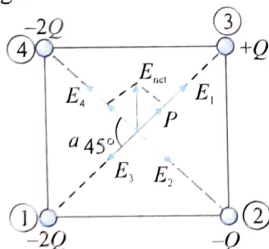


- (4) $\frac{kQ_1}{x^2} = \frac{kQ_2}{(x+R)^2}$ or $x = \frac{R}{2}$

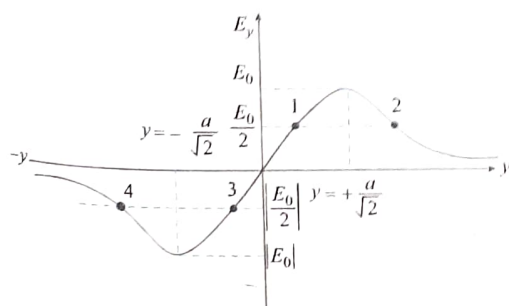


- (4) Force between two charges does not depend upon the presence or absence of third charge.

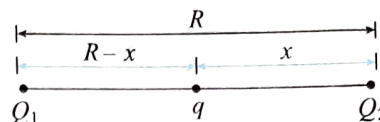
- (2) $E_1 = E_4 = 2E_2 = 2E_3$



- Horizontal components will be canceled, net field will be upward.
- (4) Four points as shown.



- (3) Force on Q_2 is zero (q should be negative)



$$\frac{kQ_1Q_2}{R^2} = \frac{kqQ_2}{x^2} \text{ or } \frac{x}{R} = \sqrt{\frac{q}{Q_1}}$$

Force on q is zero:

$$\frac{kQ_1q}{(R-x)^2} = \frac{kqQ_2}{x^2}$$

$$\text{or } \frac{R-x}{x} = \frac{\sqrt{Q_1}}{\sqrt{Q_2}}$$

$$\text{or } \frac{R}{x} = \frac{\sqrt{Q_1} + \sqrt{Q_2}}{\sqrt{Q_2}} \text{ or } \frac{\sqrt{Q_1}}{\sqrt{q}} = \frac{\sqrt{Q_1} + \sqrt{Q_2}}{\sqrt{Q_2}}$$

$$\text{or } q = \frac{Q_1Q_2}{(\sqrt{Q_1} + \sqrt{Q_2})^2}$$

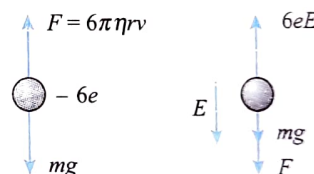
- (2) $\vec{r} = (9-3)\hat{i} + (12-4)\hat{j} = 6\hat{i} + 8\hat{j}$

$$\text{or } r = \sqrt{6^2 + 8^2} = 10 \text{ m}$$

$$E = \frac{9 \times 10^9 \times 100 \times 10^{-6}}{10^2} = 9000 \text{ Vm}^{-1}$$

- (3) For the first case, $F = mg$.

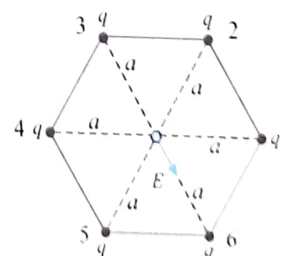
$$\text{For the second case, } F + mg = 6eE \text{ or } 2mg = 6eE$$



$$\text{or } E = \frac{mg}{3e} = \frac{1.6 \times 10^{-15} \times 10}{3 \times 1.6 \times 10^{-19}} = 3.3 \times 10^4 \text{ NC}^{-1}$$

- (1) $E = \frac{1}{4\pi\epsilon_0} \frac{q}{a^2}$

Suppose the charge is present at the sixth vertex also, then electric field at center would be zero. Now, if charge is not present at this vertex, the electric field at center would be because of other five charges, which should be equal and opposite to the field produced due to single charge at the sixth vertex.

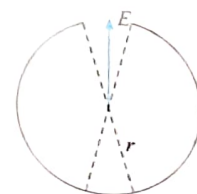


- (2) Charge on the element opposite to the gap is

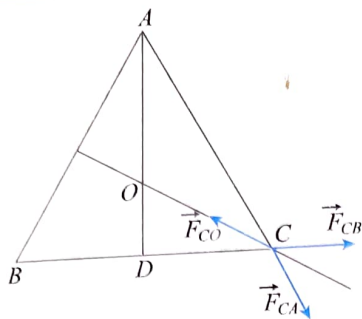
$$dq = \frac{Q}{2\pi r} (0.002\pi)$$

$$= \frac{1}{2\pi(0.5)} \times \frac{2\pi}{1000} = 2 \times 10^{-3} \text{ C}$$

$$E = \frac{9 \times 10^9 \times 2 \times 10^{-3}}{(0.5)^2} = 7.2 \times 10^7 \text{ NC}^{-1}$$



19. (1) To keep the system in equilibrium, net force experienced by charges at 'A', 'B', and 'C' should be zero. For this, another charge of opposite sign should be placed at the centroid of the triangle. Let this charge be $-Q$.

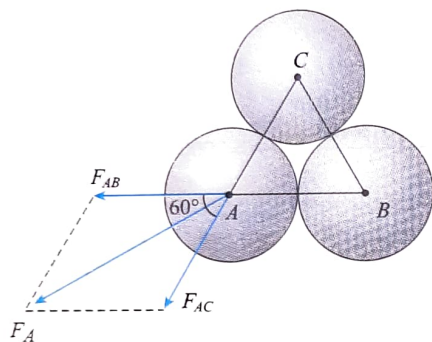


$$AD = l \cos 30^\circ = \frac{l\sqrt{3}}{2}, AO = \frac{2}{3} AD = \frac{l}{\sqrt{3}}$$

$$2|\vec{F}_{CA}| \cos 30^\circ = |\vec{F}_{CO}|$$

$$2 \times \frac{1}{4\pi\epsilon_0} \times \frac{q^2}{l^2} \times \frac{\sqrt{3}}{2} = \frac{1}{4\pi\epsilon_0} \frac{Qq}{(l/\sqrt{3})^2} \quad \text{or} \quad Q = -\frac{q}{\sqrt{3}}$$

20. (3) For external points, a charged sphere behaves as if the whole of its charge is concentrated at its center.



Force on A due to B,

$$F_{AB} = \frac{1}{4\pi\epsilon_0} \frac{q^2}{(2R)^2} = \frac{1}{4\pi\epsilon_0} \frac{q^2}{4R^2} \text{ along } \overline{BA}$$

And force on A due to C,

$$F_{AC} = \frac{1}{4\pi\epsilon_0} \frac{q^2}{(2R)^2} = \frac{1}{4\pi\epsilon_0} \frac{q^2}{4R^2} \text{ along } \overline{CA}$$

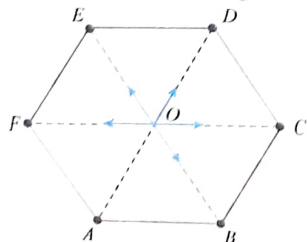
Now as angle between BA and CA is 60° and

$$|F_{AB}| = |F_{AC}| = F$$

$$\therefore F_A = \sqrt{F^2 + F^2 + 2FF \cos 60^\circ} = \sqrt{3}F$$

$$= \frac{1}{4\pi\epsilon_0} \frac{\sqrt{3}}{4} \left(\frac{q}{R} \right)^2$$

21. (4) If there had been a sixth charge $+q$ at the remaining vertex of hexagon, force due to all the six charges on $-q$ at O will be zero.



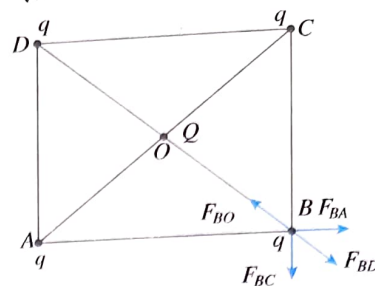
Now if f is the force due to the sixth charge and F due to the remaining five charges, then

$$\vec{F} + \vec{f} = 0, \quad \text{i.e., } \vec{F} = -\vec{f}$$

$$|\vec{F}| = |\vec{f}| = \frac{1}{4\pi\epsilon_0} \frac{q \times q}{L^2} = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{L} \right)^2$$

$$22. (3) AC = \sqrt{2}l = BD$$

$$\text{or } BO = \frac{1}{\sqrt{2}}$$



$$F_{BO} = F_{BD} + (F_{BA} + F_{BC}) \cos 45^\circ$$

Solving, we get,

$$Q = \frac{q}{4}(1 + 2\sqrt{2}) \quad Q \text{ should be negative of } q.$$

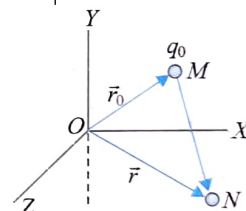
$$23. (3) \vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

$$\hat{r} = \frac{\vec{r}}{r} = \frac{(1.2-0)\hat{i} + (-1.6-0)\hat{j}}{\sqrt{(1.2)^2 + (1.6)^2}} = \frac{1}{2}(1.2\hat{i} - 1.6\hat{j})$$

$$\vec{E} = 9 \times 10^9 \times \frac{(-8 \times 10^{-9})}{2^2} \times \left[\frac{1}{2}(1.2\hat{i} - 1.6\hat{j}) \right]$$

$$= -10.8\hat{i} + 14.4\hat{j}$$

$$24. (3) \text{ As } \vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{|\vec{r} - \vec{r}_0|^3} (\vec{r} - \vec{r}_0)$$

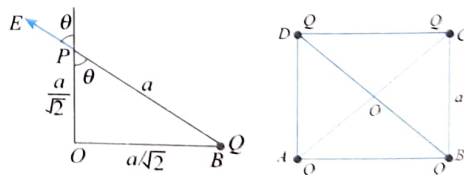


$$\overline{MN} = \vec{r} - \vec{r}_0 = (8\hat{i} - 5\hat{j}) - (2\hat{i} + 3\hat{j}) = (6\hat{i} - 8\hat{j})$$

$$|\vec{r} - \vec{r}_0| = \sqrt{6^2 + 8^2} = 10 \text{ m}$$

$$\vec{E} = 9 \times 10^9 \times \frac{50 \times 10^{-6}}{(10)^3} (6\hat{i} - 8\hat{j}) E = (2.7\hat{i} - 3.6\hat{j}) \text{ kNC}^{-1}$$

25. (2)



$$E_{\text{net}} = 4E \cos \theta = \frac{4}{4\pi\epsilon_0} \frac{Q}{a^2} \frac{1}{\sqrt{2}} = \frac{Q}{\sqrt{2} \pi \epsilon_0 a^2}$$

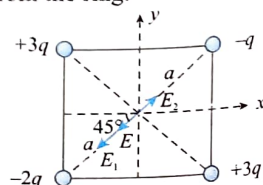
26. (3) The following two arguments shall lead us to the right choice.
(i) Electric field at the center of the ring is zero.
(ii) Electric field is directed away from the ring.

27. (2) At center electric field due to both $3q$ will be canceled. Now net field at center is

$$E = E_1 - E_2 = \frac{k2q}{(a)^2} - \frac{kq}{(a)^2} = \frac{kq}{(a)^2}$$

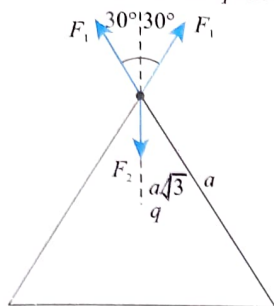
This is 45° below negative x-axis.

28. (4) Both angles will remain same at any time. It is because, force on the balls will be equal and opposite, although they have



different charges. $\phi > \theta$, because force has increased. Angles would have been different if masses were different.

29. (2) Net force on q will be zero for any values of q and Q . Net force on each Q should also be zero. For this q should be negative of Q .



$$F_2 = 2F_1 \cos 30^\circ$$

$$\text{or } \frac{kqQ}{(a/\sqrt{3})^2} = 2 \frac{kQ^2}{a^2} \frac{\sqrt{3}}{2} \quad \text{or } \frac{q}{Q} = \frac{1}{\sqrt{3}}$$

30. (1) Time period is independent of a constant force acting on the block of spring-block system. Its time period will remain same

$$\text{as } T = 2\pi \sqrt{\frac{m}{k}}$$

31. (3) From the directions of fields from graph, it is clear that q_1 is negative and q_2 is positive. Now since electric field is zero to the right of q_2 , q_2 should be smaller in magnitude.

32. (1) $F_e = F_2 + F_3$

$$= \frac{kq^2}{(L \sin \theta)^2} + \frac{kq^2}{(2L \sin \theta)^2}$$

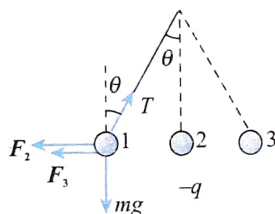
$$F_e = \frac{5}{4} \frac{kq^2}{L^2 \sin^2 \theta} \quad \dots(i)$$

$$T \sin \theta = F_e \quad \dots(ii)$$

$$T \cos \theta = mg \quad \dots(iii)$$

From (i), (ii), and (iii)

$$q = \sqrt{\frac{16}{5} \pi \epsilon_0 mg L^2 \sin^2 \theta \tan \theta}$$



33. (3) Electric field due to dipole at axial position is

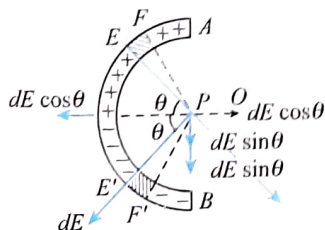
$$E_{\text{axial}} = \frac{1}{4\pi\epsilon_0} \frac{2p}{r^3} \quad (\text{along the dipole moment})$$

Electric field due to dipole at equatorial position

$$E_{\text{eq}} = \frac{1}{4\pi\epsilon_0} \frac{p}{r^3} \quad (\text{opposite to dipole moment})$$

$$\text{Hence } \vec{E}_A = -2\vec{E}_B$$

34. (1) Take PO as the x -axis and PA as the y -axis. Consider two elements EF and $E'F'$ of width $d\theta$ at angular distance θ above and below PO , respectively.



The magnitude of the field at P due to either element is

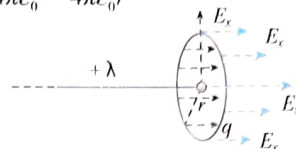
$$dE = \frac{1}{4\pi\epsilon_0} \frac{rd\theta \times Q/(\pi r/2)}{r^2} = \frac{Q}{2\pi^2\epsilon_0 r^2} d\theta$$

Resolving the fields, we find that the components along PO sum up to zero, and hence the resultant field is along PB . Therefore, field at P due to pair of elements is $2dE \sin \theta$

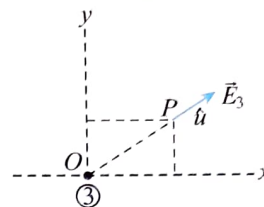
$$E = \int_0^{\pi/2} 2dE \sin \theta = 2 \int_0^{\pi/2} \frac{Q}{2\pi\epsilon_0 r^2} \sin \theta d\theta = \frac{Q}{\pi^2\epsilon_0 r^2}$$

35. (1) Net force $F_{\text{net}} = qE_x$

$$F = q \frac{\lambda}{4\pi\epsilon_0 r} = \frac{\lambda q}{4\pi\epsilon_0 r}$$



36. (2) Let us consider the electric field due to wire (3) only.



$$\vec{E}_3 = E\hat{u} = \frac{\lambda}{2\pi\epsilon_0(a^2 + a^2)^{1/2}} (\hat{i} \cos 45^\circ + \hat{j} \cos 45^\circ)$$

$$= \frac{\lambda}{2\sqrt{2}\pi\epsilon_0 a} (\hat{i} + \hat{j}) = \frac{\lambda}{4\pi\epsilon_0 a} (\hat{i} + \hat{j})$$

Similarly, electric field due to wires (1) and (2)

$$\vec{E}_1 = \frac{\lambda}{4\pi\epsilon_0 a} (\hat{j} + \hat{k}) \quad \text{and} \quad \vec{E}_2 = \frac{\lambda}{4\pi\epsilon_0 a} (\hat{i} + \hat{k})$$

$$\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 \quad \vec{E}_{\text{net}} = \frac{\lambda}{2\pi\epsilon_0 a} (\hat{i} + \hat{j} + \hat{k})$$

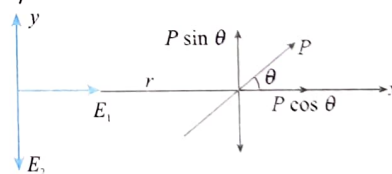
37. (1) Mean position: $x = l$

$$F = -k(x - l) \quad \text{where } k = m\omega^2$$

$$\text{or } qE = -m\omega^2(x - l) \quad \text{At } x = l, E = 0$$

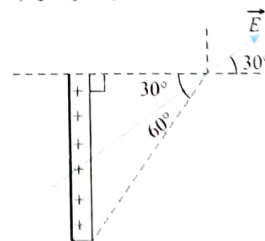
To the right of $x = l$, E is negative, so toward left and to the left of $x = l$, E is positive.

$$38. (3) \quad \vec{E}_1 = \frac{2kP \cos \theta}{r^3} \hat{i}$$



$$\vec{F}_A = -Q\vec{E}_1 = \frac{-2kPQ}{r^3} \cos \theta \hat{i}$$

39. (1) Using symmetry property.



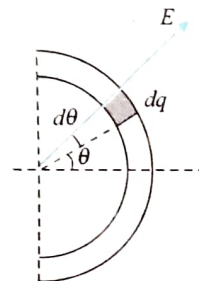
$$40. (2) \quad F_{\text{net}} = \int dq E \cos \theta$$

$$= \int_{-\pi/2}^{\pi/2} \left(\frac{q}{\pi R} \right) R d\theta \frac{\lambda}{2\pi\epsilon_0 R} \cos \theta$$

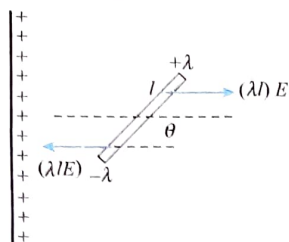
$$= \frac{\lambda q}{2\pi^2\epsilon_0 R} \int_{-\pi/2}^{\pi/2} \cos \theta d\theta$$

$$= \frac{\lambda q}{2\pi^2\epsilon_0 R} [\sin \theta]_{-\pi/2}^{\pi/2}$$

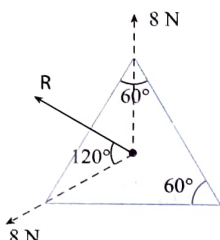
$$= \frac{\lambda q}{2\pi^2\epsilon_0 R} [1 - (-1)] = \frac{\lambda q}{\pi^2\epsilon_0 R}$$



$$41. (2) \tau = (\lambda l E) \sin \theta = \lambda l^2 \frac{\sigma}{2\epsilon_0} \sin \theta = \frac{\sigma \lambda l^2}{2\epsilon_0} \sin \theta$$



$$42. (4) R = \sqrt{8^2 + 8^2 + 2 \times 8 \times 8 \cos 120^\circ} = 8 \text{ N}$$



43. (3) From (i), A and C both are charged, either positively or negatively. From (ii), B is charged and D and E have no charge. From (iii), A is positively charged. Therefore, from Eq. (i), B is negatively charged.

$$44. (1) a_x = \frac{qE}{m} = \frac{10^{-6} \times 2 \times 10^7}{2} = 10 \text{ ms}^{-2}$$

$$\text{Time of flight } T = \frac{2u \sin \theta}{g} = \frac{2 \times 10 \times \frac{1}{\sqrt{2}}}{10} = \sqrt{2} \text{ s}$$

$$\text{Hence, } R = u_x T + \frac{1}{2} a_x T^2$$

$$= 10 \cos 45^\circ \times T + \frac{1}{2} a_x T^2$$

$$= \frac{10}{\sqrt{2}} \times \sqrt{2} + \frac{1}{2} \times 10 \times 2 = 20 \text{ m}$$

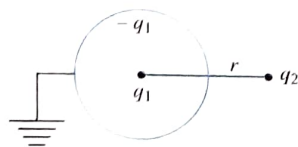
45. (4) l is the length of the each tube. $u_x = u = \text{constant}$. Time of travel between the plates is $t = l/u$. Let a be constant acceleration in y -direction. So $v_y = at$ when the particle emerges from the plates so,

$$v^2 = u_x^2 + v_y^2 = u^2 + a^2 t^2$$

$$= u^2 + a^2 \frac{l^2}{u^2} = u^2 + \frac{C}{u^2} \quad (\text{where } a^2 l^2 = C)$$

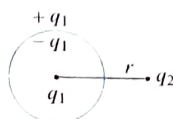
$$\text{So, } v = \sqrt{u^2 + \frac{C}{u^2}}$$

46. (1) There is no charge on the outer surface. Hence, no force on q_2 (see figure).



47. (2) The charge on outer surface of shell will apply a force on q_2

$$F = \frac{k q_1 q_2}{r^2}$$



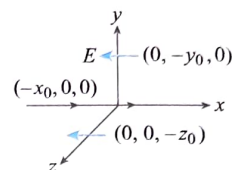
$$48. (4) \frac{\lambda q}{2\pi\epsilon_0 r} = \frac{mv^2}{r} \text{ or } v = \sqrt{\frac{\lambda}{2\pi\epsilon_0 m q}} \text{ independent of } r$$

$$49. (3) P = 2 \times 10^{-8} \times 2 \times 10^{-3} = 4 \times 10^{-11} \text{ Cm}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 x}, F = p \cos \theta \frac{dE}{dx}$$

$$F = -\frac{p \cos \theta \lambda}{2\pi\epsilon_0 x^2} = \frac{-4 \times 10^{-11} \times \cos 60^\circ \times 4 \times 10^{-4}}{2 \times 9 \times 10^9 \times \left(\frac{6}{100}\right)^2} = 0.04 \text{ N}$$

50. (4) Field on the axis of the dipole lies in the direction of dipole.



Field on the bisector of the dipole lies opposite to the direction of dipole.

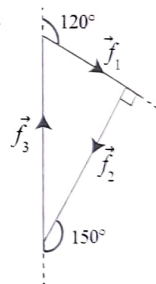
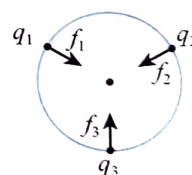
51. (3) Net dipole moment = $\sqrt{3} qa$

$$\tau = -\sqrt{3} qa \frac{|\sigma_1 - \sigma_2| \theta}{2\epsilon_0} = l \alpha$$

$$\frac{\sqrt{3} qa |\sigma_1 - \sigma_2|}{2\epsilon_0} = l \left(\frac{2\pi}{T} \right)^2$$

$$T = 2\pi \sqrt{\frac{2\epsilon_0 l}{\sqrt{3} qa |\sigma_1 - \sigma_2|}}$$

52. (3) Since the charges exert force on one another that are in action-reaction pair, sum of forces $\vec{f}_1$, $\vec{f}_2$, and $\vec{f}_3$ is zero. So force vectors form a triangle, as shown in the figure.



Applying sine rule, we have

$$\frac{f_1}{\sin 30^\circ} = \frac{f_2}{\sin 60^\circ} = \frac{f_3}{\sin 90^\circ}$$

$$\text{or } 2f_1 = \frac{2f_2}{\sqrt{3}} = f_3 = k \text{ (suppose)}$$

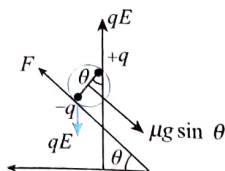
$$f_1 = \frac{k}{2}, f_2 = \frac{\sqrt{3}}{2} k, f_3 = k$$

$$\text{So } f_1 : f_2 : f_3 = \frac{1}{2} : \frac{\sqrt{3}}{2} : 1 = 1 : \sqrt{3} : 2$$

53. (3) Since magnitude of forces is proportional to m as $f = mv^2/r$, $m_1 : m_2 : m_3 = 1 : \sqrt{3} : 2$.

54. (4) External applied field's effect is neutralized by charge induced on outer surface, i.e., their combined effect is zero. Hence, force applied by induced charges on outer surface should be equal and opposite to that applied by electric field. This is equal to $qE = 1 \times 20 = 20 \text{ N}$ toward left. 15 N force is only due to induced charge on the inside surface of the hollow sphere. Hence, net force by induced charges is $15 + 20 = 35 \text{ N}$.

55. (2)

Balancing torque about $-q$ is

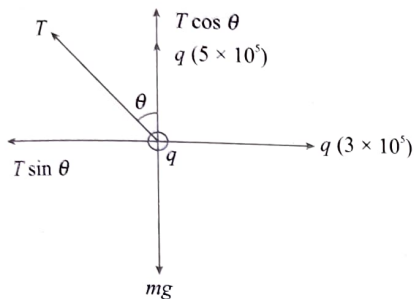
$$qE \cdot 2R \sin \theta = \mu g \sin \theta \cdot R \quad \text{or} \quad E = \frac{\mu g}{2q}$$

$$56. (3) \quad \vec{\tau} = \vec{p} \times \vec{E} \quad \text{or} \quad \tau_{\max} = pE$$

$$\text{or} \quad 50 \times 10^{-28} = p \cdot 40 \quad \text{or} \quad p = 1.25 \times 10^{-28} \text{ Cm}$$

57. (3) If another shell is kept upside down over it complete a sphere, net field should become zero.

58. (4)

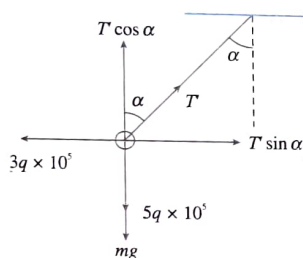


$$T \sin \theta = 3q \times 10^5$$

$$T \cos \theta = mg - 5q \times 10^5$$

$$\text{Solve to get } q = 100 \mu\text{C}, T = 50 \text{ N.}$$

After the reversal of direction of electric field



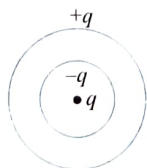
$$T' \sin \alpha = 3q \times 10^5 \quad \text{or} \quad T' \cos \alpha = mg + 5q \times 10^5$$

$$\tan \alpha = \frac{3q \times 10^5}{mg + 5q \times 10^5} = \frac{3 \times 10^{-4} \times 10^5}{9 \times 10 + 5 \times 10^{-4} \times 10^5} = \frac{3}{14}$$

$$\text{or} \quad \alpha = \tan^{-1} \left(\frac{3}{14} \right)$$

59. (3) In (I) and (II), forces on P due to all three charges are in same directions, but in case (II) distances are minimum. Hence forces (II > I). In case (III) force on P will be toward left and in (IV), force will be toward right. But on calculation forces (IV > III).

60. (2) Let the charge on the ball at center is q , then $-q$ charge will be induced on the inner surface and $+q$ on the outer surface. Electric field at any point outside the sphere due to inner charges q and $-q$ will combine to be zero. Outer charge q will produce the electric field at outside points as it would be produced by a charge at the center. Hence presence of sphere does not affect the electric field outside the sphere.



$$61. (4) \quad F_x = p_x \frac{\partial(E_x)}{\partial x} = p \cdot 4y^2;$$

$$F_y = p_y \frac{\partial(E_y)}{\partial x} = p \cdot 8xy$$

$$F = \sqrt{F_x^2 + F_y^2} = 4py \sqrt{y^2 + 4x^2}$$

62. (2) θ is equilibrium position. Net force in equilibrium position

$$F = \sqrt{(mg)^2 + (qE)^2}$$

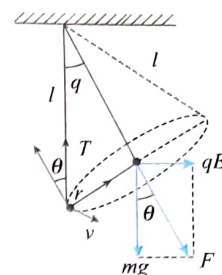
$$\tan \theta = \frac{qE}{mg}$$

$$T \cos \theta = F$$

$$T \sin \theta = \frac{mv^2}{r}$$

where $r = l \sin \theta$

$$\text{Solve to get } v = \frac{qE}{m} \sqrt{\frac{l}{g}}$$



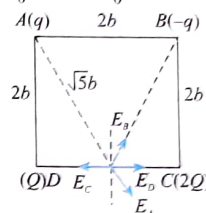
63. (1) Since the field at midpoint of CD is zero. Hence,
 $EB \cos \theta + ED + EA \cos \theta - EC = 0$

$$\left(\frac{q}{4\pi\epsilon_0(\sqrt{5}b)^2} + \frac{q}{4\pi\epsilon_0(\sqrt{5}b)^2} \right) \cos \theta + \frac{Q}{4\pi\epsilon_0 b^2} = \frac{2Q}{4\pi\epsilon_0 b^2}$$

On solving, we get

$$\frac{2q}{5} \cos \theta = Q \quad \text{or} \quad \frac{2q}{5} \left(\frac{b}{\sqrt{5}b} \right) = Q$$

$$\text{or} \quad \frac{2q}{5\sqrt{5}} = Q \quad \text{or} \quad \frac{q}{Q} = \frac{5\sqrt{5}}{2}$$



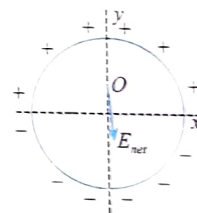
64. (2) Electric lines of force crossing per unit area are assumed to be directly proportion to the intensity of the electric field.

$$65. (3) \quad F_x = P_x \frac{\partial E_x}{\partial x} = 0$$

$$F_y = P_y \frac{\partial E_y}{\partial y} = P_0(4by + 2c)$$

$$\therefore F = 2cP_0 \hat{j}$$

66. (2) For $\theta \leq \pi$, λ is positive and symmetrically distributed with respect to y -axis. Similarly for $\pi \leq \theta < 2\pi$, λ is negative and symmetrically distributed about y -axis. Hence direction of electric field E_{net} at O due to charge distribution is in downward direction. If q_0 is $-$, F_{q_0} upward. If q_0 is $+$, F_{q_0} downward.



Multiple Correct Answers Type

1. (1), (2), (3)

Due to induction there will be shifting of charge on ball and force can be of any type depending upon the relative charge on them.

2. (1), (2), (3)

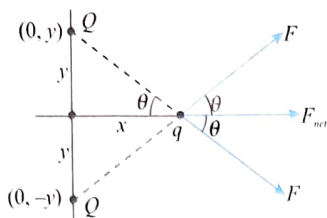
Acceleration, velocity, and electric field all change in direction although they remain same in magnitude.

3. (2), (3)

Just draw the free-body diagram of both charge particles and see whether force can be zero or not on the particle. At least, one force should be opposite to others.

4. (1), (2), (4)

$$\begin{aligned} \text{Net force on } q, F_{\text{net}} &= 2F \cos \theta = \frac{2}{4\pi\epsilon_0} \frac{Qq}{(x^2 + y^2)} \frac{x}{\sqrt{x^2 + y^2}} \\ &= \frac{Qq}{2\pi\epsilon_0} \frac{x}{(x^2 + y^2)^{3/2}} \end{aligned}$$



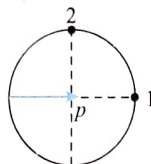
To maximize F_{net} , $\frac{dF_{\text{net}}}{dx} = 0$ or $x = \pm \frac{y}{\sqrt{2}}$. Hence (1) is correct.

At origin $x = 0$, $F_{\text{net}} = 0$. Hence (2) is correct. Since net force is away from origin, particle will not perform oscillatory motion. Hence (3) is incorrect and (4) is correct.

5. (1), (4)

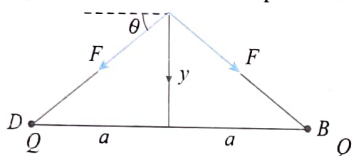
(i) Let electric field at '1' be 10 NC^{-1} , then at '2' it is 5 NC^{-1} and at any other point on surface of sphere electric field will be from 5 NC^{-1} to 10 NC^{-1} .

(ii) Let electric field at '2' is 10 NC^{-1} , then at '1' it is 20 NC^{-1} and at any other point on surface of sphere, electric field will be from 10 NC^{-1} to 20 NC^{-1} .



6. (3), (4)

The particle will oscillate or perform SHM if equilibrium is a stable one. For negative charge, equilibrium is stable if the particles are displaced along CD and unstable for displacement along AB.



For $y \ll a$, in displaced position, net resultant force toward equilibrium position is

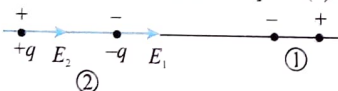
$$F_{\text{res}} = 2F \sin \theta = \frac{2 \times Q}{4\pi\epsilon_0(a^2 + y^2)} \times \frac{y}{(a^2 + y^2)^{1/2}} = \frac{2Qy}{4\pi\epsilon_0 a^3} [a \gg y]$$

$F_{\text{res}} \propto y$, so SHM for $y \ll a$.

For $y = a$, there would be restoring force, but F is not proportional to y . So there is oscillatory motion but not SHM.

7. (1), (4)

E_1 and E_2 are the electric fields due to dipole (a) so $E_1 > E_2$



Force on dipole (b): $qE_1(\text{left}) + qE_2(\text{right}) = q(E_1 - E_2)$ left. Dipole (b) will not experience any torque, because lines of action of both forces are same.

8. (1), (2), (4)

At a center, the force experienced by the charge particle is zero, so it is a position of equilibrium. As we displace the charge from the equilibrium position, electric force starts acting on it toward equilibrium position and hence equilibrium is a stable one. At a distance x from the center of sphere (equilibrium position), force experienced by the charge particle is $F = \frac{qQx}{4\pi\epsilon_0 R^3}$ [for $x < R$]

As $F \propto x$, it performs SHM about the center. Time period can be calculated by using $F = m\omega^2 x$.

9. (1), (2), (3)

$$\text{Time of flight } (t) = \frac{2u}{g} = \frac{2 \times 10}{10} = 2 \text{ s}$$

$$H = \frac{u^2}{2g} = \frac{10^2}{2 \times 10} = 5 \text{ m}$$

$$R = 0 + \frac{1}{2} \left(\frac{qE}{m} \right) t^2 = \frac{1}{2} \times \frac{10^{-3} \times 10^4 \times 2 \times 2}{2} = 10 \text{ m}$$

10. (1), (4)

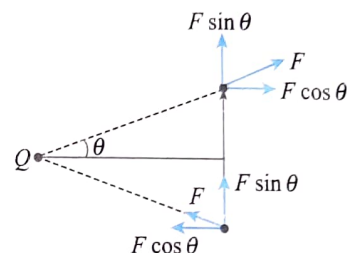
Suppose a positively charged sphere is brought near an uncharged metallic sphere, then on nearer surface of the uncharged sphere, negative charge is induced and on farther surface, positive charge is induced. Hence, a force of attraction will be observed between these two spheres. Therefore, if a force of attraction is observed between a positively charged sphere and a metallic sphere, it cannot be concluded that the metallic sphere is necessarily negatively charged. Therefore, option (2) is wrong and options (1) and (4) are correct.

11. (1), (3)

If positive test charge is displaced along x -axis, then net force will always act in a direction opposite to that of displacement and the test charge will always come back to its original position. But if test charge is displaced along y -axis, it will never come back to its original position and will fly away along y -axis.

12. (2), (3)

The situation is shown in figure.



$$\vec{F}_{\text{net}} = 2F \sin \theta \hat{j} = \frac{pQ}{4\pi\epsilon_0 r^3}$$

$$\vec{\tau} = F \cos \theta \times 2a \text{ in clockwise direction}$$

$$= \frac{pQ}{4\pi\epsilon_0 r^2}$$

13. (3), (4)

Torque will be perpendicular to the line $y = 2x$ and it should be in xy plane, because electric field is in z -direction. The lines in options (3) and (4) are perpendicular to $y = 2x$.

14. (1), (3)

$$\vec{E}_A \text{ is along } \vec{OA} \text{ and } \vec{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{E}_B \text{ is along } \vec{OB} = \hat{i} + \hat{j} - \hat{k}$$

$$\text{Since } \vec{OA} \times \vec{OB} = (\hat{i} + 2\hat{j} + 3\hat{k}) \times (\hat{i} + \hat{j} - \hat{k}) = 0$$

$$\text{So } \vec{OA} \perp \vec{OB} \Rightarrow \vec{E}_A \perp \vec{E}_B$$

So, option (1) is correct.

$$\text{Since } E_B = \frac{kq}{|OB|^2} = \frac{kq}{3}$$

$$E_C = \frac{kq}{|OC|^2} = \frac{kq}{12}$$

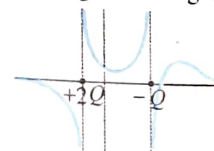
$$\text{So } \frac{E_B}{E_C} = 4 \text{ or } |\vec{E}_B| = 4|\vec{E}_C|$$

So, option (3) is correct.

Options (2) and (4) are wrong from the explanation.

15. (1), (4)

Electric field is zero only at a point in region III. Equilibrium at this point will be stable for a negative charge.



16. (1), (4)
Resultant force on arrangement is $2QE$, thus acceleration of center of mass is given by $a_c = 2QE/3m$.

Net torque on system is

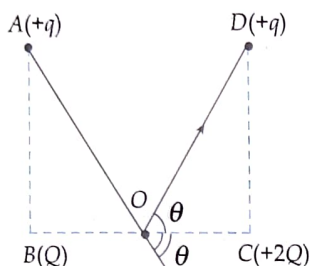
$$QE\left(\frac{2L}{3}\right) - QE\left(\frac{L}{3}\right) = \left[m \times \frac{4L^2}{9} + 2m \times \frac{L^2}{9}\right] \alpha$$

$$\alpha = \frac{QE}{2mL} \text{ (clockwise)}$$

From constraint equation, we get

$$a_A = a_c + \alpha\left(\frac{2L}{3}\right) = \frac{2QE}{3m} + \frac{QE}{2mL}\left(\frac{2L}{3}\right) = \frac{3QE}{3m} = \frac{QE}{m}$$

17. (3), (4)
For ball O (positively charged) to be in equilibrium at the middle of BC



$$\left[\frac{q}{4\pi\epsilon_0(a\sqrt{5})^2} + \frac{q}{4\pi\epsilon_0(a\sqrt{5})^2} \right] \cos\theta + \frac{Q}{4\pi\epsilon_0 a^2} = \frac{2Q}{4\pi\epsilon_0 a^2}$$

$$q = \frac{5\sqrt{5}Q}{2} \quad \left(\text{as } \cos\theta = \frac{a}{a\sqrt{5}} \right)$$

If $q = 5\sqrt{5}Q/2$, ball will be in equilibrium, even if charge are interchanged. (1) is wrong and (3) and (4) are correct. If charge at C is $+Q$, then

$$\frac{2q \cos\theta}{5} + Q = Q \quad \text{or} \quad \frac{2q \cos\theta}{5} = 0$$

$Q \neq 0$, $\theta = 90^\circ$, which is not possible. (2) is wrong.

18. (1), (2)

$$E_x = E_y = 10^5 \text{ NC}^{-1}$$

$$qE_x = T \sin \alpha,$$

$$qE_y = mg - T \cos \alpha$$

$$\text{Dividing } \frac{E_x}{E_y} = \frac{T \sin \alpha}{mg - T \cos \alpha}$$

$$\text{or } mg - T \cos \alpha = T \sin \alpha$$

$$\text{or } mg = \frac{2mg}{1 + \sqrt{3}} (\cos \alpha + \sin \alpha)$$

$$\text{or } \cos \alpha + \sin \alpha = \frac{\sqrt{3} + 1}{2} \Rightarrow \alpha = 30^\circ \text{ or } 60^\circ$$

19. (1), (2), (3), (4)

(1) Only equilibrium position changes. Time period remains same

$$T = 2\pi\sqrt{m/k}$$

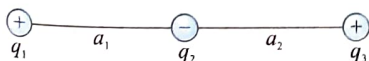
$$(2) \frac{1}{2} kx^2 = \frac{1}{2} mv^2$$

So v at mean position is same amplitude will be same.

(3) In case 4 equilibrium position is $x_0 = 3mg/k$.

20. (1), (2), (3), (4)

Sign of q_2 should be different from q_1 and q_3 .



Linked Comprehension Type

For Problems 1-2

1. (1), 2. (1)

$$qE = mg \text{ or } q \times 1.68 \times 10^5 = 1.08 \times 10^{-14} g$$

$$\text{or } q = 6.40 \times 10^{-19} \text{ C}$$

$$\therefore q = ne \text{ or } 6.4 \times 10^{-19} = n \times 1.6 \times 10^{-19} \text{ or } n = 4$$

For Problems 3-5

3. (1), 4. (1), 5. (2)

Electric field at (b) tends to $-\infty$, hence the charge at (b) should be negative. There is a neutral point to the right of charges. This is possible only when the charge at (1) should be positive. Hence, Q_1 is positive and Q_2 is negative. At neutral point, $\vec{E}_1 = \vec{E}_2$.

$$\frac{1}{4\pi\epsilon_0} \frac{Q_1}{(a+l)^2} = \frac{1}{4\pi\epsilon_0} \frac{Q_2}{a^2} \text{ or } \frac{Q_1}{Q_2} = \left(\frac{a+l}{a} \right)^2$$

Electric field at any position to the right of charges is

$$E = E_1 - E_2 = \frac{1}{4\pi\epsilon_0} \frac{Q_1}{(l+x)^2} - \frac{1}{4\pi\epsilon_0} \frac{Q_2}{x^2}$$

For the maximum value of E ,

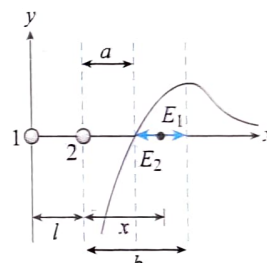
$$\frac{dE}{dx} = 0 \text{ or } \frac{d}{dx} \left[\frac{1}{4\pi\epsilon_0} \frac{Q_1}{(l+x)^2} - \frac{1}{4\pi\epsilon_0} \frac{Q_2}{x^2} \right] = 0$$

$$\text{or } Q_1 \frac{(-2)}{(l+x)^3} - Q_2 \frac{(-2)}{x^3} = 0$$

$$\text{or } \frac{Q_1}{(l+x)^3} = \frac{Q_2}{x^3} \text{ or } \left(\frac{l+x}{x} \right)^3 = \frac{Q_1}{Q_2}$$

$$\text{or } \frac{l+x}{x} = \left(\frac{Q_1}{Q_2} \right)^{1/3} \text{ or } \frac{l}{x} = \left(\frac{Q_1}{Q_2} \right)^{1/3} - 1$$

$$\text{or } x = \frac{l}{\left(\frac{Q_1}{Q_2} \right)^{1/3} - 1}$$



For Problems 6-7

6. (3), 7. (2)

$$T = 2\pi \sqrt{\frac{l}{(mg - qE)/m}}$$

$$T' = 2\pi \sqrt{\frac{l}{g}}$$

$$\text{or } T' < T$$

Obviously change in tension is qE . Tension will increase by qE .

For Problems 8-11

8. (3), 9. (2), 10. (3), 11. (4)

Passing between the charged plates, the electron feels a force upward and just misses the top plate. The distance it travels in the y -direction is 0.005 m.

$$\text{Time of flight is } t = \frac{0.0200 \text{ m}}{1.60 \times 10^6 \text{ ms}^{-1}} = 1.25 \times 10^{-8} \text{ s}$$

The initial y -velocity is zero.

$$\text{Now, } y = v_{0y}t + \frac{1}{2}at^2$$

$$\text{So, } 0.005 \text{ m} = \frac{1}{2}a(1.25 \times 10^{-8} \text{ s})^2$$

$$\text{or } a = 6.40 \times 10^{13} \text{ ms}^{-2}$$

But also $a = \frac{F}{m} = \frac{eE}{m_e}$

$$E = \frac{(9.1 \times 10^{-31} \text{ kg})(6.40 \times 10^{13} \text{ ms}^{-2})}{1.60 \times 10^{-19} \text{ C}} = 364 \text{ NC}^{-1}$$

Since the proton is more massive, it will accelerate less and not hit the plates. To find the vertical displacement when it exits the plates, we use the kinematic equations again.

$$y = \frac{1}{2}at^2 = \frac{1}{2} \frac{eE}{m_p} (1.25 \times 10^{-8} \text{ s})^2 = 2.73 \times 10^{-6} \text{ m}$$

As mentioned in (2), the proton will not hit one of the plates because although the electric force felt by the proton is the same as the electron felt, a smaller acceleration results for the more massive proton. The acceleration produced by the electric force is much greater than g ; it is reasonable to ignore gravity.

For Problems 12–13

12. (2), 13. (1)

$$OA = OB = OC = OD = a$$

The magnitude of the electric force due to each charge is

$$\frac{1}{4\pi\epsilon_0} \frac{Q}{a^2 + h^2}$$

The components of the force perpendicular to OP sum up to zero because of the symmetrical distribution of charges about OP . Hence, the resultant force at P is upward along OP . The magnitude of the force is given by

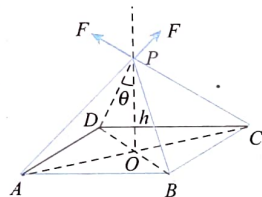
$$\begin{aligned} F_{\text{up}} &= 4 \times \frac{1}{4\pi\epsilon_0} \frac{Q}{h^2 + a^2} \cos \theta \\ &= \frac{Q}{\pi\epsilon_0(h^2 + a^2)} \frac{h}{\sqrt{h^2 + a^2}} \\ &= \frac{Qh}{\pi\epsilon_0(h^2 + a^2)^{3/2}} \end{aligned}$$

$$F_{\text{down}} = \text{weight} = mg$$

$$\text{For equilibrium, } mg = \frac{Qh}{\pi\epsilon_0(h^2 + a^2)^{3/2}}$$

$$\text{or } Q = \pi\epsilon_0 \frac{mg}{h} (h^2 + a^2)^{3/2}$$

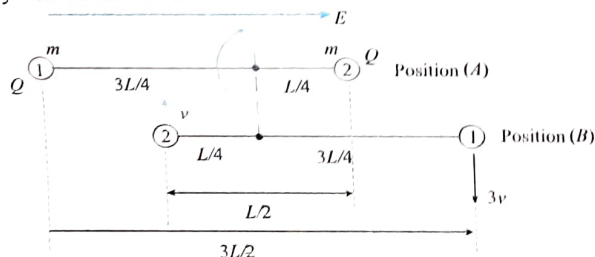
Yes, this is a stable equilibrium as it will regain its position if displaced a little.



For Problems 14–16

14. (3), 15. (1), 16. (4)

Position (A) is of unstable equilibrium, because if displaced from this position, resultant torque will be in the direction of displacement and finally it will attain equilibrium position (B). At position (B), velocity will be maximum.

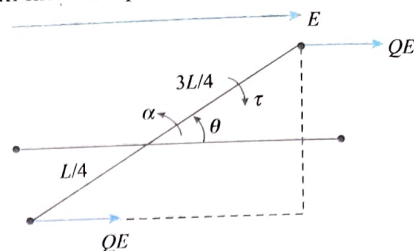


$$W = \Delta KE$$

$$\text{or } QE \frac{3L}{2} - QE \frac{L}{2} = \frac{1}{2} m[v^2 + (3v)^2]$$

$$\text{or } v = \sqrt{\frac{QEL}{5m}}$$

Now we will find time period.



$$\tau = QE \frac{3L}{4} \sin \theta - QE \frac{L}{4} \sin \theta = -I\alpha$$

$$\text{or } QE \frac{L}{2} \sin \theta = -m \left[\left(\frac{L}{4} \right)^2 + \left(\frac{3L}{4} \right)^2 \right] \alpha$$

$$\text{or } QE \frac{L}{2} \theta = \frac{-10mL^2}{16} \alpha \quad \text{or } \alpha = -\left(\frac{4QE}{5mL} \right) \theta$$

$$\omega^2 = \frac{4QE}{5mL} \quad \text{or } \omega = \sqrt{\frac{4QE}{5mL}}$$

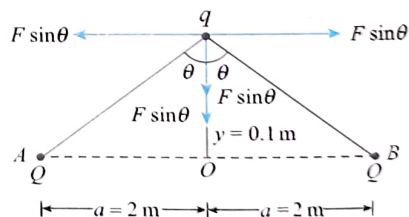
$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{5mL}{4QE}}$$

For Problems 17–20

17. (3), 18. (1), 19. (3), 20. (2)

Since $Q = +8 \mu\text{C}$, if q is a positive charge, resultant force on it due to Q at A and Q at B will be along positive y -axis and it would move away along y -axis. But the charge q here is observed to oscillate. This is possible only if q is a negative charge so that resultant force on it due to Q at A and Q at B is toward O . Under the action of this force, q moves toward O , crosses O and as it is moving along negative Y direction, resultant force on it will again be toward O . This force retards the motion of q along negative y -axis. It comes to rest at some point and then moves back toward O and so on (see figure). Force applied by Q on q has a magnitude

$$F = \frac{1}{4\pi\epsilon_0} \frac{Qq}{y^2 + a^2}$$



Force applied by Q at A on q can be resolved into rectangular components: $F \cos \theta$ and $F \sin \theta$. Similarly, force applied by Q at B on q can be resolved into components: $F \cos \theta$ and $F \sin \theta$. $F \sin \theta$ components of the two forces balance each other so that the net force on q is $2F \cos \theta$ toward O . Therefore, net force on q ,

$$F_n = 2 \frac{1}{4\pi\epsilon_0} \frac{Qq}{y^2 + a^2} \cos \theta$$

$$F_n = \frac{1}{4\pi\epsilon_0} \frac{2Qq}{y^2 + a^2} \frac{y}{(y^2 + a^2)^{1/2}} = \frac{1}{4\pi\epsilon_0} \frac{2Qq y}{(y^2 + a^2)^{3/2}} \quad \dots(i)$$

For $y \ll a$, net force on q ,

$$F_n = \frac{1}{4\pi\epsilon_0} \frac{2Qq y}{a^3} \quad \dots(ii)$$

Here, $Q = 8 \mu\text{C} = 8 \times 10^{-6} \text{ C}$

At $t = 0$, q is at $y = 0.1$ m. Obviously, $y < a$ ($= 2$ m). Since the motion is simple harmonic, we can use the approximation $y \ll a$ so that net force, from Eq. (ii), will be proportional to displacement (y). Initially, i.e., at $y = 0$ m, force on q is 9×10^3 N.

Using Eq. (i), we get

$$\text{or } 9 \times 10^{-3} = (9 \times 10^9) \frac{2(8 \times 10^{-6})q(0.1)}{(2)^3}$$

$$\text{or } q = 5 \times 10^{-6} \text{ C} = 5 \mu\text{C}$$

This, in fact, is the magnitude of q . We know that q , as explained earlier, is a negative charge. Hence, $q = -5 \mu\text{C}$. So, correct option is (3).

At $t = 0$, q is released at a point 0.1 m from O on y -axis. As it oscillates, its other extreme position will be 0.1 m from O on the negative y -axis, assuming undamped simple harmonic motion. Hence, amplitude of oscillations is 0.1 m or 10 cm. So, correct option is (1). From figure, we get

$$F_n = \frac{1}{4\pi\epsilon_0} \frac{2Qqy}{a^3} = ky$$

$$\text{where } k = \frac{1}{4\pi\epsilon_0} \frac{2Qq}{a^3}$$

Thus, $F_n \propto y$. We also know that F_n always acts toward O (mean position). Time period of resulting SHM will be $T = 2\pi\sqrt{m/k}$ of

$$\text{frequency is } \frac{1}{2\pi} \sqrt{k/m}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{1}{4\pi\epsilon_0} \frac{2Qq}{ma^3}}$$

$$= \frac{1}{(2 \times 3.14)} \sqrt{\frac{(9 \times 10^9)(2)(8 \times 10^{-6})(5 \times 10^{-6})}{(91 \times 10^{-6})(2)^3}} = 5$$

$$[m = 91 \text{ mg} = 91 \times 10^{-6} \text{ kg}]$$

Thus, the correct option is (3).

In SHM, equation of displacement from mean position can be expressed as $y = a \sin(\omega t + \phi)$. Here, $a = 0.1$ m; $\omega = 2\pi f = 2\pi \times 5 = 10\pi$; $y = 0.1 \sin(10\pi t + \phi)$

But at $t = 0$, $y = 0.1$ (given). Hence, $0.1 = 0.1 \sin \phi$ or $\sin \phi = 1$

$$\text{or } \phi = \frac{\pi}{2}; y = 0.1 \sin(10\pi t + \pi/2)$$

Thus, the correct option is (2).

For Problems 21–24

21. (4), 22. (1), 23. (3), 24. (3)

The two particles move in different circles. The mutual interaction force provides the required centripetal force to the particle. As magnitude of the interaction force is same, we get

$$F_{12} = \frac{m_1 v_1^2}{r_1} \text{ and } F_{21} = \frac{m_2 v_2^2}{r_2}$$

$$\left| \vec{F}_1 \right| = \left| \vec{F}_2 \right| \text{ or } \frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2}$$

Putting values, we get $r_2 = 2r_1$

Also, $r_1 + r_2 = 12 \times 10^{-12}$ m (given)

$$r_1 = 4 \times 10^{-12} \text{ m}; r_2 = 8 \times 10^{-12} \text{ m}$$

$$\text{Acceleration of first particle} = \frac{v_1^2}{r_1} = \frac{(10^3)^2}{(4 \times 10^{-12})} = 2.5 \times 10^{15} \text{ ms}^{-2}$$

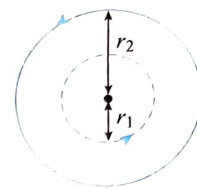
22. (1) Acceleration of second particle is

$$\frac{v_2^2}{r_2} = \frac{(2 \times 10^3)^2}{(8 \times 10^{-12})} = 5 \times 10^{15} \text{ ms}^{-2}$$

23. (3) Just after release,

$$v_{CM} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{(2 \times 10^{-30})(0) + (10^{-30})(2 \times 10^3)}{3 \times 10^{-30}} = \frac{2}{3} \times 10^3 \text{ ms}^{-1}$$

24. (3) Since the distance between them always remains constant, but they move with different velocities, they must move in different circles with common center as shown in figure.



For Problems 25–26

25. (3), 26. (2)

Concentric cylindrical electrodes will produce radial electric field. As ion is entering at O and leaving at B , hence the path followed by ion should be circular and centered at A . Required centripetal force should be provided by force on the ion due to electric field. Hence

$$qE = \frac{mv^2}{R} \text{ or } E = \frac{mv^2}{qR}$$

As final velocity along x -axis becomes zero and finally the ion starts moving toward y direction, the electric field should have component toward x - and y -directions, respectively. For x -component of electric field (using $v_x^2 = u_x^2 + 2a_x \Delta x$),

$$0 = v^2 - 2\left(\frac{qE_x}{m}\right)R \text{ or } E_x = \frac{mv^2}{2qR}$$

For y -component of electric field (again using $v_y^2 = u_y^2 + 2a_y \Delta y$)

$$v^2 = 0 + 2\left(\frac{qE_y}{m}\right)R \text{ or } E_y = \frac{mv^2}{2qR}$$

Hence, net electric field is

$$\vec{E} = -\frac{mv^2}{2qR} \hat{i} + \frac{mv^2}{2qR} \hat{j} = \frac{mv^2}{2qR} (-\hat{i} + \hat{j})$$

For Problems 27–28

27. (3), 28. (3)

When it hits ground, displacement in y -direction will be zero.

$$5t - \frac{1}{2}15t^2 = 0 \text{ or } t = \frac{2}{3} \text{ s}$$

At time $t = 2/3$ s the displacement in x -direction

$$x = 5\sqrt{3}\left(\frac{2}{3}\right) - \frac{1}{5}5\sqrt{3}\left(\frac{2}{3}\right)^2 = \frac{20}{3\sqrt{3}} \text{ m}$$

After impact with the surface, velocity along vertical direction

$$u'_y = eu_y = 0.2 \times 5 = 1 \text{ ms}^{-1}$$

Velocity along the horizontal direction after time T_1

$$u'_x = eu_x + a_x T_1 = 5\sqrt{3} - 5\sqrt{3} \times \frac{2}{3} = \frac{5}{\sqrt{3}} \text{ ms}^{-1}$$

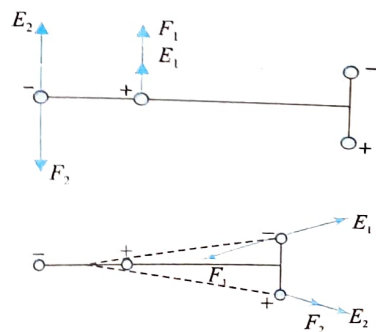
The ball rebounds in the vertical direction with velocity 1 ms^{-1} .

For Problems 29–30

29. (2), 30. (2)

Electric field due to right dipole at left dipole is shown. There are E_1 and E_2 . But $E_1 > E_2$. So $F_1 > F_2$. So net force on the dipole is upward. Also F_1 and F_2 will produce anticlockwise torque.

Electric field on right dipole due to left dipole are shown. F_1 and F_2 are the forces on this dipole. Net forces will be downwards. Also F_1 and F_2 will produce anticlockwise torque.



Matrix Match Type

1. (4)

When two objects are charged by rubbing them against each other (or charging by friction) they acquired equal and opposite charge.

$$|q_A| = |q_B|$$

$$\text{Or } q_A = q_B$$

Induced charge may be less than inducing charge in case of dielectric or non-conducting bodies

$$q_A \geq q_B$$

If two bodies (say two spheres) of same material and same size are taken, 1st is charge ($q = q_A$) is touched with 2nd (initially charged),

or charging by conduction, then $q_B = \frac{q_A}{2}$.

2. (3)

B is negatively charged, so mass of B increases due to gain of electrons. As A loses electrons its mass decreases. Work function (minimum energy required by an e^- to come onto a surface) of A is less, so it readily loses e^- .

3. i. $\rightarrow$ d., ii. $\rightarrow$ a., c., d.; iii. $\rightarrow$ b.; iv. $\rightarrow$ a., c.

In this case, electric field is totally zero because in (a) and (b) particle P can be at origin. For points P, P' in Fig. (c), electric field will be along x-axis only. Also, for point P in Figs. (a) and (b), E_x will remain nonzero while other two components of electric field will be zero.

iii. $\rightarrow$ b. Only in the case of disk xy components of electric field will be present as per given statements.

iv. $\rightarrow$ a., c. For the two points P and P' in Fig. (c), the field will be same as these points are at equal distances from infinite line of charge.

4. i. $\rightarrow$ a., c.; ii. $\rightarrow$ a., b., c., d.; iii. $\rightarrow$ a., b., c.; iv. $\rightarrow$ a., d.

The electric field due to one dipole at center of other dipole is parallel to the dipole in all cases. Hence, torque on dipole is zero in all cases.

In cases (ii) and (iii), the electric field at second dipole due to first is along the second dipole, hence electrostatic potential energy of second dipole is negative.

In cases (i) and (ii), x-axis is the line of zero potential. In case (iii), y-axis is the line of zero potential. In cases (ii) and (iii), electric field at origin is zero.

5. i. $\rightarrow$ b.; ii. $\rightarrow$ b.; iii. $\rightarrow$ b., c.; iv. $\rightarrow$ b., d.

i. Electric field inside due to Q_2 and induced charges on shell will be zero. But Q_1 will produce electric field inside.

ii. Same as (i).

iii. again electric field inside sphere will be non-zero. Due to the movement of Q_1 , electric field inside will change but it will have no effect on the electric field outside.

iv. due to the movement of Q_2 , electric field outside changes but has no effect on the field inside.

Numerical Value Type

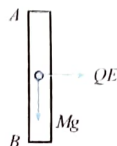
1. (2) For the equilibrium of charge at $z = a$, the net electric field at this point due to both the rings should be zero.

$$E_A + E_B = 0 \Rightarrow \frac{kq_A a}{(a^2 + a^2)^{3/2}} + \frac{kq_B a}{(b^2 + a^2)^{3/2}} = 0$$

Put the values and solve to get $b/a = 2$.

2. (1) After turning through 90° , the rod comes at rest momentarily. So there is no change in KE. Net work done by all the forces should be zero.

$$QE \frac{L}{2} - Mg \frac{L}{2} = 0 \quad \text{or} \quad E = \frac{Mg}{Q}$$



3. (6) Charges at A, B, C, and D are placed at equilateral position of dipole. Hence, force on each of them due to dipole is

$$F_1 = \frac{Qp}{4\pi\epsilon_0(a/\sqrt{2})^3}$$

This force is downward on charges. Hence, force due to these charges on dipole is $4F_1$ upward. Force on dipole due to charge at P is

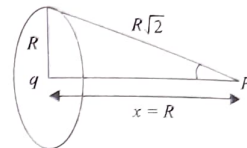
$$F_2 = \frac{2pQ}{4\pi\epsilon_0(a/\sqrt{2})^3} \quad (\text{upward})$$

Net force on dipole is $F = 4F_1 + F_2 = \frac{3\sqrt{2}Qp}{\pi\epsilon_0 a^3}$ upward

After substituting given values, we get $F = \frac{6}{4\pi\epsilon_0} \text{ N}$

4. (2) Electric field at point P due to charge of ring is

$$E = \frac{kQx}{(R^2 + x^2)^{3/2}}$$



At $x = R$, $E = \frac{kQ}{2\sqrt{2}R^2}$ directed toward the center.

Electric field at P due to center charge is kq/R^2 .

For net field to be zero at P

$$\frac{kq}{R^2} = \frac{kQ}{2\sqrt{2}R^2} \quad \text{or} \quad q = \frac{Q}{2\sqrt{2}} = \frac{Q}{4}\sqrt{2}$$

$$5. (4) T \sin \theta = F = \frac{1}{4\pi\epsilon_0} \frac{q^2}{a^2} \quad \dots(i)$$

$$T \cos \theta = mg \quad \dots(ii)$$

$$\text{From (i) and (ii), } \tan \theta = \frac{q^2}{4\pi\epsilon_0 a^2 mg}$$

$$\text{or } \frac{a/2}{L} = \frac{q^2}{4\pi\epsilon_0 a^2 mg} \quad (\text{Since for small } \theta, \tan \theta \approx a/2L)$$

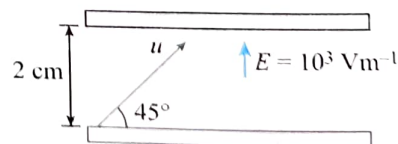
$$\text{or } \frac{a^3}{L} = \frac{q^2}{2\pi\epsilon_0 mg} \quad \dots(iii)$$

When one of the balls is discharged, the balls come closer and touch each other and again separate due to repulsion. The charge on each ball after touching each other is $q/2$. Replacing q with $q/2$ in Eq. (iii), we get

$$\frac{b^3}{L} = \frac{(q/2)^2}{2\pi\epsilon_0 mg} \quad \dots(iv)$$

$$\text{From Eqs. (iii) and (iv), } \frac{b^3}{a^3} = \frac{1}{4} \quad \text{or} \quad \frac{a^3}{b^3} = 4$$

6. (8) Resolving the velocity of particle parallel and perpendicular to the plate.



$$u_{\parallel} = u \cos 45^\circ = \frac{u}{\sqrt{2}} \quad \text{and} \quad u_{\perp} = u \sin 45^\circ = \frac{u}{\sqrt{2}}$$

Force on the charged particle in the downward direction normal is to the plate eE . Acceleration is $a = eE/m$, where m is the mass of charged particle. The particle will not hit the upper plate, if the velocity component normal to plate becomes zero before reaching it, i.e., $0 = u_{\perp}^2 - 2ay$, with $y \leq d$, where d , distance between the plates. Therefore, the maximum velocity for the particle not to hit the upper plate (for this $y = d = 2 \text{ cm}$) is

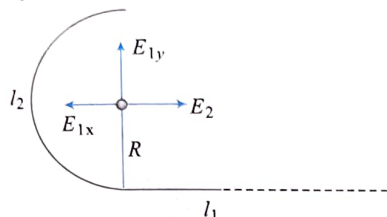
$$u_{\perp} = \sqrt{2ay} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 10^3 \times 2 \times 10^{-2}}{1.6 \times 10^{-30}}} \\ = 2 \times 10^6 \text{ ms}^{-1}$$

$$u_{\text{max}} = u_{\perp} / \cos 45^\circ = 2\sqrt{2} \times 10^6 \text{ ms}^{-1}$$

$$7. (2) q = n_e e \text{ or } n_e = \frac{-3.20 \times 10^{-9}}{-1.6 \times 10^{-19}} = 2.0 \times 10^{10} \text{ s}$$

8. (2) Electric field due to semicircular part:

$$E_2 = \frac{\lambda_2}{2\pi\epsilon_0 R}$$



Electric field due to straight part:

$$E_{1x} = E_{1y} = \frac{\lambda_1}{4\pi\epsilon_0 R}$$

For net field to be along y-axis:

$$E_2 = E_{1x} \Rightarrow \frac{\lambda_2}{2\pi\epsilon_0 R} = \frac{\lambda_1}{4\pi\epsilon_0 R} \Rightarrow \frac{\lambda_1}{\lambda_2} = 2$$

$$9. (3) \text{ At equilibrium, } \frac{q^2}{4\pi\epsilon_0 x^2} \cos \theta = mg \sin \theta$$

$$\text{For small } \theta, \cos \theta \rightarrow 1 \text{ and } \sin \theta \approx \theta = \frac{x}{l}$$

$$\Rightarrow \frac{q^2}{4\pi\epsilon_0 x^2} = \frac{mgx}{2l} \Rightarrow q = x^{3/2} \cdot \sqrt{\frac{2\pi\epsilon_0 mg}{l}}$$

$$\Rightarrow \frac{dq}{q} = \frac{3}{2} \left(x^{3/2} \cdot \sqrt{\frac{2\pi\epsilon_0 mg}{l}} \right) \cdot \left(\frac{dx}{dt} \right)$$

$$\text{Put } v = \frac{dx}{dt} \propto x^{1/2} \Rightarrow \frac{dq}{dt} \propto \frac{3}{2} \sqrt{\frac{2\pi\epsilon_0 mg}{l}}$$

Hence $N = 3$.

10. (5) As particle is moving horizontally, only forces acting on it are Stoke's force and electric force.

If $v = \text{constant}$, then $6\pi\eta v = qE$

$$\Rightarrow 6\pi\eta rv = neE$$

$$\Rightarrow n = \frac{6\pi\eta rv}{Ee}$$

$$= \frac{6 \times 3.14 \times 1.6 \times 10^{-5} \times 5 \times 10^{-7} \times 0.02}{6.28 \times 10^5 \times 1.6 \times 10^{-19}} = 30 = 6k$$

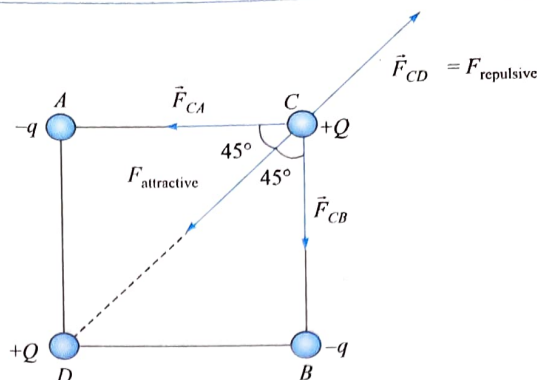
$$\Rightarrow k = 5$$

Archives

JEE Main

1. (1) For electric force to be zero at charge Q , the sign of charge q should be negative. The force of repulsion by Q is cancelled by the resultant attracting force due to $-q$ and $-q$ at A and B .
Force on charge at C due to charge at D ,

$$F_{\text{repulsive}} = \frac{1}{4\pi\epsilon_0} \frac{Q^2}{(\sqrt{a^2 + a^2})^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q^2}{2a^2}$$



Net force on charge placed at C due to charges placed at A and B ,

$$F_{\text{attractive}} = 2 \cdot \frac{1}{4\pi\epsilon_0} \frac{Qq}{a^2} \cos 45^\circ = \frac{1}{4\pi\epsilon_0} \left\{ \frac{Qq\sqrt{2}}{a^2} \right\}$$

For resultant force zero at C ,

$$\frac{1}{4\pi\epsilon_0} \cdot \frac{Q^2}{2a^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Qq\sqrt{2}}{a^2}$$

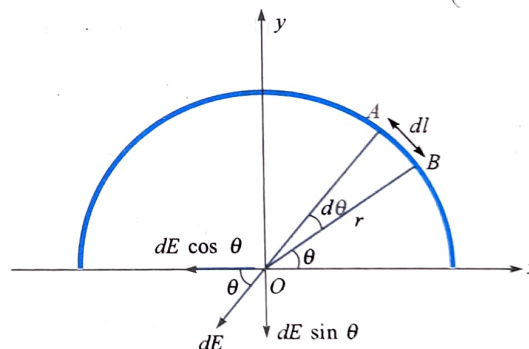
$$\frac{Q^2}{2a^2} = \frac{Qq\sqrt{2}}{a^2} \Rightarrow \frac{Q}{q} = 2\sqrt{2}$$

As q is negative charge, hence $\frac{Q}{q} = -2\sqrt{2}$.

2. (3) Linear charge density, $\lambda = \frac{q}{\pi r}$

Consider a small element AB of length dl subtending an angle $d\theta$ at the centre O as shown in the figure.

$$\therefore \text{Charge on the element, } dq = \lambda dl = \lambda r d\theta \left(\because d\theta = \frac{dl}{r} \right)$$



The electric field at the centre O due to the charge element,

$$dE = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} = \frac{\lambda r d\theta}{4\pi\epsilon_0 r^2}$$

Resolve dE into two rectangular components.

By symmetry, $\int dE \cos \theta = 0$

The net electric field at O ,

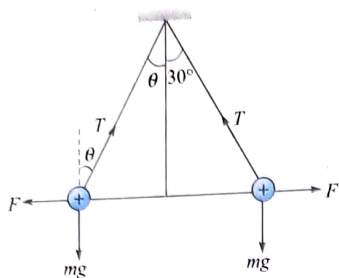
$$\vec{E} = \int_0^\pi dE \sin \theta (-\hat{j}) = \int_0^\pi \frac{\lambda r d\theta}{4\pi\epsilon_0 r^2} \sin \theta (-\hat{j}) \\ = - \int_0^\pi \frac{qr \sin \theta d\theta}{4\pi^2 \epsilon_0 r^3} \hat{j} \quad \left(\because \lambda = \frac{q}{\pi r} \right) \\ = - \int_0^\pi \frac{q \sin \theta d\theta}{4\pi^2 \epsilon_0 r^2} \hat{j} = - \frac{q}{4\pi^2 \epsilon_0 r^2} [-\cos \theta]_0^\pi \hat{j} \\ = - \frac{q}{2\pi^2 \epsilon_0 r^2} \hat{j}$$

3. (3) Initially, the forces acting on each ball are shown in figure.

For its equilibrium along vertical, $T \cos \theta = mg$... (i)

and along horizontal, $T \sin \theta = F$... (ii)

Dividing Eq. (ii) by (i), we get $\tan \theta = \frac{F}{mg}$... (iii)



When the balls are suspended in a liquid of density σ and dielectric constant K , the electrostatic force will become $(1/K)$ times, i.e. $F' = (F/K)$ while weight

$$mg' = mg - \text{Upthrust}$$

$$= mg - V\sigma g \quad [\text{As Upthrust} = V\sigma g]$$

$$= mg \left[1 - \frac{\sigma}{\rho} \right] \quad \left[\text{As } V = \frac{m}{\rho} \right]$$

For equilibrium of balls,

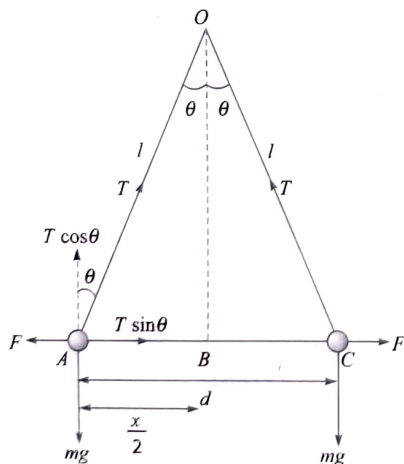
$$\tan \theta' = \frac{F'}{mg'} = \frac{F}{Kmg \left[1 - (\sigma/\rho) \right]} \quad \dots (iv)$$

According to given problem, $\theta' = \theta$.

From Eqs. (iv) and (iii), we get

$$K = \frac{1}{\left(1 - \frac{\sigma}{\rho} \right)} = \frac{\rho}{(\rho - \sigma)} = \frac{1.6}{(1.6 - 0.8)} = 2$$

4. (1) The forces acting on each spheres are shown in FBD



$$\therefore T \cos \theta = mg$$

$$\text{and } T \sin \theta = F = \frac{1}{4\pi\epsilon_0} \frac{q^2}{d^2}$$

$$\therefore \tan \theta = \frac{1}{4\pi\epsilon_0} \frac{q^2}{d^2 mg}$$

When charge begins to leak from both the spheres at a constant rate, then

$$\tan \theta = \frac{1}{4\pi\epsilon_0} \frac{q^2}{x^2 mg}$$

$$\text{or } \frac{x}{2l} = \frac{1}{4\pi\epsilon_0} \frac{q^2}{x^2 mg} \quad \left(\because \tan \theta = \frac{x}{2l} \right)$$

$$\text{or } \frac{x}{2l} \propto \frac{q^2}{x^2}$$

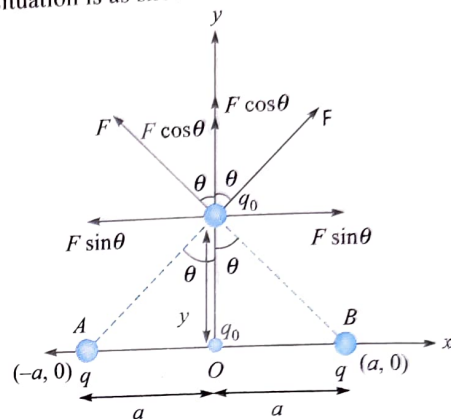
$$\text{or } q^2 \propto x^3$$

$$\Rightarrow q \propto x^{3/2}$$

$$\text{or } \frac{dq}{dt} \propto \frac{3}{2} x^{1/2} \frac{dx}{dt}$$

$$\text{or } v \propto x^{-1/2} \quad \left(\because \frac{dq}{dt} = \text{constant} \right)$$

5. (4) When a particle of mass m and charge $q_0 = \left(\frac{q}{2} \right)$ placed is at the origin and it is given a small displacement along the y -axis, then the situation is as shown in the figure.



By symmetry, the components of forces on the particle of charge q_0 due to charges at A and B along x -axis will cancel each other where along y -axis will add up.

$\therefore$ The net force acting on the particle,

$$F_{\text{net}} = 2F \cos \theta = 2 \frac{1}{4\pi\epsilon_0} \frac{qq_0}{(\sqrt{y^2 + a^2})^2} \frac{y}{\sqrt{y^2 + a^2}}$$

$$= \frac{2}{4\pi\epsilon_0} \frac{q \left(\frac{q}{2} \right)}{(y^2 + a^2)^{3/2}} \frac{y}{\sqrt{y^2 + a^2}} \quad \left(\because q_0 = \frac{q}{2} \text{ (Given)} \right)$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q^2 y}{(y^2 + a^2)^{3/2}}$$

As $y \ll a$

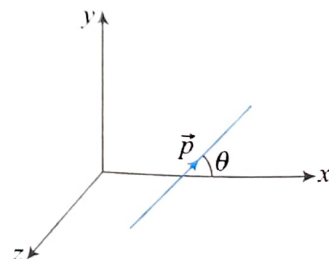
$$\therefore F_{\text{net}} = \frac{1}{4\pi\epsilon_0} \frac{q^2 y}{a^3} \text{ or } F_{\text{net}} \propto y$$

$$6. (1) F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{R^2}$$

$$\Rightarrow \epsilon_0 = \frac{q_1 q_2}{4\pi F R^2} = \frac{C^2}{N \cdot m^2} = \frac{[AT]^2}{MLT^{-2} \cdot L^2} = [M^{-1} L^{-3} T^4 A^2]$$

7. (1) The options (2) and (3) are not possible since field lines should originate from positive and terminate to negative charge. Option (4) is not possible since field lines must be smooth. Also the field lines should resemble that of dipole hence option (1) satisfies all required condition

8. (1) From $\vec{r} = \vec{p} \times \vec{E}$



$$\begin{aligned}\tau \hat{k} - \tau \hat{k} &= (p_x \hat{i} + p_y \hat{j}) \times (E \hat{i} + \sqrt{3} E \hat{j}) \\ &= p_x \times \sqrt{3} E \hat{k} + p_y E (-\hat{k})\end{aligned}$$

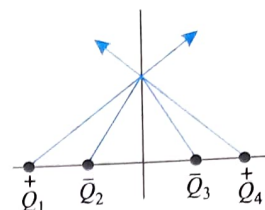
$$0 = E \hat{k} (\sqrt{3} p_x - p_y)$$

$$\frac{p_y}{p_x} = \sqrt{3}$$

$$\therefore \tan \theta = \sqrt{3}$$

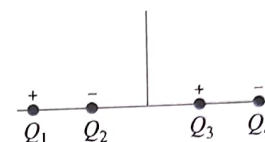
$$\theta = 60^\circ$$

(R) - (4)



Each pair will cancel x of each other and y of +ve pair will be smaller than -ve so net will be along -ve y .

(S) - (2)



Each dipole pair will cancel y of each other and x of $Q_2 - Q_3$ pair along -ve x is stronger than that of $Q_1 - Q_4$ along +ve x .

Numerical Value Type

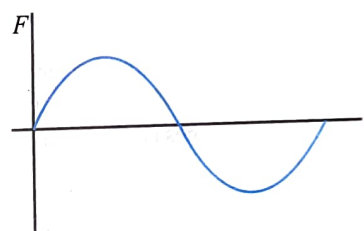
1. (2)

$$m = 10^{-3} \text{ kg and } q = 1 \text{ C and } t = 0, E = E_0 \sin \omega t$$

$$\text{Force on particle will be } E = qE = qE_0 \sin \omega t$$

$$\text{At } v_{\max}, F = 0$$

$$qE_0 \sin \omega t = 0$$



$$\text{At } t = \frac{\pi}{\omega}$$

$$F = qE_0 \sin \omega t$$

$$\text{or } a = \frac{F}{m} = \frac{qE_0}{m} \sin \omega t$$

$$\Rightarrow \frac{dv}{dt} = q \frac{E_0}{m} \sin \omega t$$

$$\int_0^v dv = \int \frac{qE_0}{m} \sin \omega t dt$$

$$v - 0 = \frac{qE_0}{m\omega} [-\cos \omega t]_0^{\pi/\omega}$$

$$v - 0 = \frac{qE_0}{m\omega} [(-\cos \pi) - (-\cos 0)]$$

$$v = \frac{1 \times 1}{10^{-3} 10^3} \times 2 = 2 \text{ m/s}$$

JEE Advanced

Single Correct Answer Type

$$1. (2) \frac{4}{3} \pi R^3 \rho g = qE = 6\pi \eta R v_T$$

$$\therefore q = 8.0 \times 10^{-19} \text{ C}$$

2. (1) The frequency will be the same $\nu = \sqrt{k/m}/2\pi$. Frequency is independent of any external constant force. But due to the constant force qE toward right, the equilibrium position gets shifted by qE/K in forward direction.

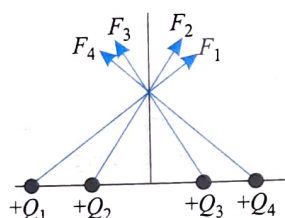
Multiple Correct Answers Type

1. (1), (4)

A number of electric field lines of forces emerging from Q_1 are larger than terminating at Q_2 .

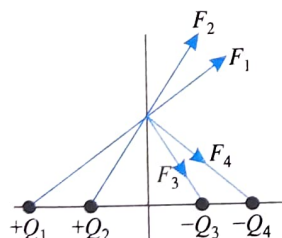
Matrix Match Type

1. (1) (P) - (3)



All x components of force will be cancelled only y will exist, i.e., along +ve direction.

(Q) - (1)



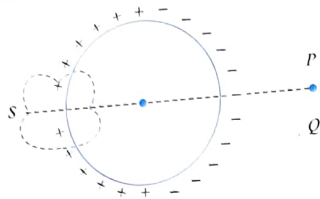
All y components of force will be cancelled and only +ve x will exist.

Chapter 2

Concept Application Exercises

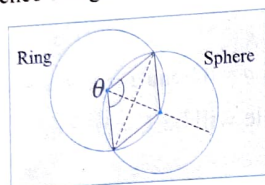
Exercise 2.1

1. The charge induced on left side is positive.



Hence, charge enclosed by Gaussian surface is positive. Thus, the sign of the flux is '+ve'.

2. As $\theta = 120^\circ$, hence charge enclosed within sphere = $Q/3$.



$$\phi = \frac{Q_{\text{enclosed}}}{\epsilon_0} = \frac{Q}{3\epsilon_0}$$

3. As the rod enters the box, the charge increases; hence, flux increases linearly. When rod enters the cube completely, the charge within the cube becomes constant for sometime and hence the flux also becomes constant.

When the rod starts moving out of the cube, the charge inside the cube starts decreasing and hence the flux will also start decreasing linearly. Hence, graph *d* is the current graph.

4. (a) Number of field lines coming to the curved surface is equal to the field lines moving away from the surface. Hence, the flux linked with curved surface is zero.

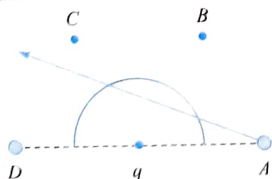
- (b) Flux passing through the plane surface = the flux passing through the curved surface.

$$\phi = E\pi R^2$$

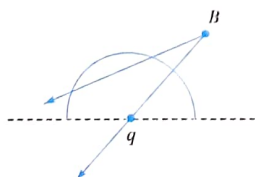
(positive as field lines are away from the surface)

$$(c) \phi = E2\pi R^2$$

5. If the charge is placed at *A* or *D*, any electric field line will cross the curved surface twice.



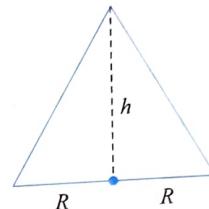
Hence, the flux passing through the curved surface should be unchanged. If the charge is placed at *B*, some of the electric field lines may cut the surface once thereby changing the flux through the surface.



6. (a) and (b): Since the point charge Q is located just above the centre of the flat face, then almost half the lines emitting from Q will pass through the flat surface. Any line passing through the flat surface will also pass through the curved surface below it. Hence the required flux is $\phi = Q/2\epsilon_0$. Here flux through curved surface will be positive and through flat surface will be negative having same magnitude.

- (c) If the charge is placed exactly at the centre, then any line emitting from the charge will be parallel to the flat surface. Hence, there is no flux through the flat surface. But the flux through the curved surface will remain the same as $\phi = Q/2\epsilon_0$.

7. Effective area of the cone through which flux passes is



$$A = 2 \left[\frac{1}{2} Rh \right] = Rh$$

$$\text{Flux } \phi = EA = ERh$$

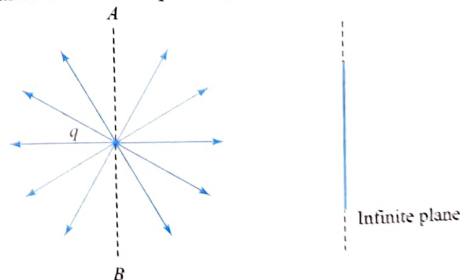
$$8. \vec{E} = a\hat{i} + b\hat{j}$$

$$(a) \vec{A} = A\hat{i} \Rightarrow \phi = \vec{E} \cdot \vec{A} = Aa$$

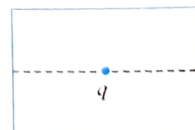
$$(b) \vec{A} = A\hat{j} \Rightarrow \phi = \vec{E} \cdot \vec{A} = Ab$$

$$(c) \vec{A} = A\hat{k} \Rightarrow \phi = \vec{E} \cdot \vec{A} = 0$$

9. (a) Any line emitting from charge q to the right of line *AB* will pass through the infinite plane. It means half flux will pass through the plane. So the flux passing through the plane is $q/2\epsilon_0$.



- (b) q is above the centre at a small distance. So this becomes similar to the case (a).



10. Effective area = $A = \pi r^2$

$$\therefore \phi = E_0 A = E_0 \pi r^2$$

11. (a) Since Q_1 is inside the surface, so flux due to Q_1 is Q_1/ϵ_0 .

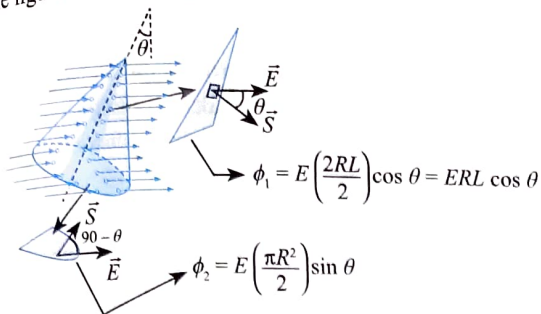
- (b) Since Q_2 is outside, so there is no contribution to the flux due to Q_2 .

12. The flux passing through curved surface

$$= \text{the flux passing through the plane } ABCD$$

$$= E2Rl$$

13. The flux of electric field entering the cone can be calculated as the figure shown in figure.



The flux of electric field entering the cone in this position.

$$\phi_{\text{total}} = \phi_1 + \phi_2 = ERL \cos \theta + \frac{E\pi R^2}{2} \sin \theta$$

14. The flux through the surface, $\phi = \int \vec{E} \cdot d\vec{s}$

Let us take an elemental area ds of the surface, the force on the element $d\vec{F} = dq \vec{E} = (\sigma ds) \vec{E}$

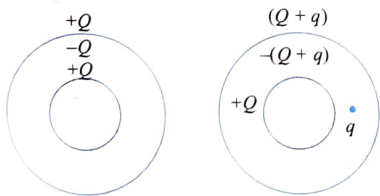
Force on square surface, $F = \int d\vec{F} = \int (\sigma ds) \vec{E} = \sigma \int \vec{E} \cdot d\vec{s} = \sigma \phi$

Exercise 2.2

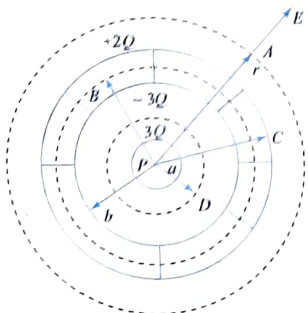
- $\Phi_{S_1} = q_1/\epsilon_0 = (4.00 \times 10^{-9} \text{ C})/\epsilon_0 = 452 \text{ Nm}^2/\text{C}$
- $\Phi_{S_2} = q_2/\epsilon_0 = (-7.80 \times 10^{-9} \text{ C})/\epsilon_0 = -881 \text{ Nm}^2/\text{C}$
- $\Phi_{S_3} = (q_1 + q_2)/\epsilon_0 = ((4.00 - 7.80) \times 10^{-9} \text{ C})/\epsilon_0 = -429 \text{ Nm}^2/\text{C}$
- $\Phi_{S_4} = (q_1 + q_3)/\epsilon_0 = ((4.00 + 2.40) \times 10^{-9} \text{ C})/\epsilon_0 = 723 \text{ Nm}^2/\text{C}$
- $\Phi_{S_5} = \frac{(q_1 + q_2 + q_3)}{\epsilon_0} = \frac{(4.00 - 7.80 + 2.40) \times 10^{-9} \text{ C}}{\epsilon_0} = -158 \text{ Nm}^2/\text{C}$

All that matters for Gauss's law is the total amount of charge enclosed by the surface, not its distribution within the surface.

2. We know that charge on facing surface is equal and opposite.



- Net charge on inner surface is $-Q$.
 - Net charge on the inner surface will remain $-Q$.
 - Net charge on inner surface is $-(Q+q)$.
 - Yes, as location of charge does not matter.
3. The distribution of charge is as shown in figure.



- Net charge enclosed by Gaussian surface ($r > c$) is $3Q - 3Q + 2Q = 2Q$.
- For $r > c$, the direction of electric field is radially outward as shown at point A.
- $E = \frac{k q_{\text{in}}}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{2Q}{r^2}$
- For $c < r < b$, we have point B. If we draw a Gaussian surface through B, net charge enclosed in it will be zero. So electric field at B will be zero.
- As explained above, net charge enclosed is zero.
- For $b > r > a$, we have point D. If we draw a Gaussian surface through D, it will have net charge enclosed $3Q$.

$$(g) E = \frac{k q_{\text{in}}}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{3Q}{r^2}$$

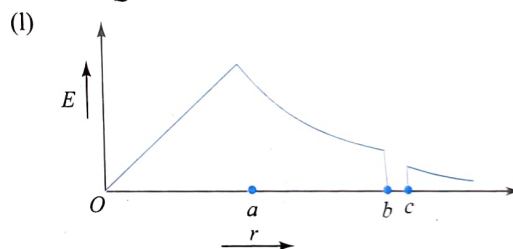
$$(h) \rho = \frac{3Q}{\frac{4}{3}\pi a^3} \text{ (volume charge density)}$$

So, charge within radius r ($< a$) is $q = \rho \frac{4}{3}\pi r^3 = \frac{3Qr^3}{a^3}$

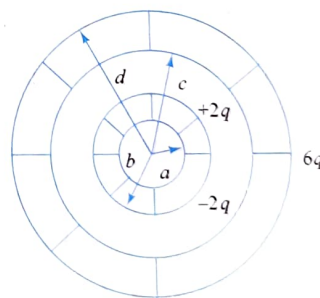
$$(i) E = \frac{k q_{\text{in}}}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{3Qr^3}{a^3 r^2} = \frac{1}{4\pi\epsilon_0} \frac{3Qr}{a^3}$$

- (j) As shown in the figure, the charge on the inner surface of shell is $-3Q$.

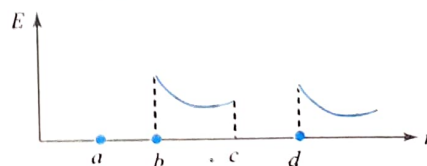
- (k) As shown in the figure, the charge on outer surface of shell is $+2Q$.



4. (a)



- (i) (ii) For $r < b$, electric field is zero, because there is no charge inside.
 - (iii) For $c > r > b$, electric field will be similar to that as if a point charge $2q$ is placed at the centre.
 - (iv) For $d > r > c$, electric field will be zero, as there is no charge inside.
 - (v) For $r > d$, electric field will be similar to that as if a point charge $6q$ is placed at the centre.
- The resulting plot is as shown in figure.



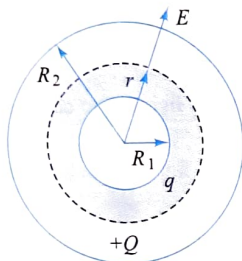
- (c) (i) 0 (ii) $+2q$ (iii) $-2q$ (iv) $+6q$

5. (a) False. Electric field is due to all charges present inside or outside.
 (b) False. Gauss's law is applicable for any distribution of charge. But it is more useful when the charge is distributed symmetrically.
 (c) True. Net flux is due to the charges inside only.
 6. (a) True. If E is zero at each point, then net flux will be zero.
 (b) True. If net flux is zero, then net charge enclosed will be zero.
 7. (a) Yes, as there is no charge for $r < R_1$.

(b) Volume charge density $\rho = \frac{Q}{\frac{4}{3}\pi(R_2^3 - R_1^3)}$

Charge within radius r is $q = \rho \frac{4}{3}\pi(r^3 - R_1^3)$

$$= \frac{Q(r^3 - R_1^3)}{R_2^3 - R_1^3}$$

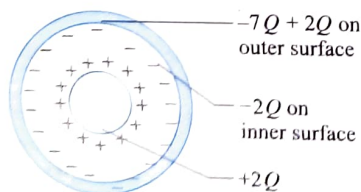


$$E = \frac{kq}{r^2} = \frac{Q}{4\pi\epsilon_0 r^2} \left[\frac{r^3 - R_1^3}{R_2^3 - R_1^3} \right]$$

(c) For $r > R_2$, $E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$

8. $\phi = EA$; $\phi = E \frac{\pi d^2}{4} \Rightarrow E = \frac{4\phi}{\pi d^2}$

9. (a) The charge on the inner sphere induces equal magnitude of charge, but opposite in sign, on the inner surface of the outer sphere. Sum of all the induced charges is always zero. Therefore, an equal amount of charge must appear on the outer surface. Thus outer and inner surface of the outer sphere have charges $-5Q$ and $-2Q$, respectively.

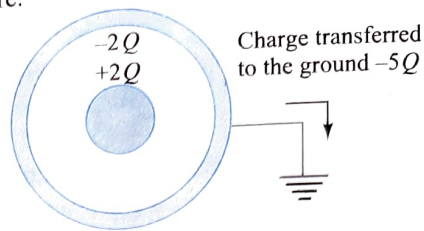


- (b) When outer and inner spheres are connected by a wire, the entire charge is transferred to the outer surface from the inner sphere. In electrostatic equilibrium, charge does not reside inside a conductor. Total charge on the outer surface of the outer sphere is $-5Q$.
 Total charge on the inner surface is 0.
 The electric field at the surface of the inside sphere goes to zero after connection. Consider a Gaussian surface just on the surface of the inner sphere.

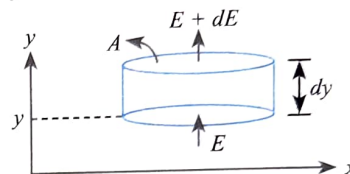
$$\phi_E = E(4\pi r^2) = \frac{Q_{\text{enclosed}}}{\epsilon_0} = 0, \text{ as } Q_{\text{enclosed}} = 0$$

Thus, we have $E = 0$

- (c) When the outer sphere is grounded the charge on the outer surface is transferred to ground, thus charge is reduced to zero on the outer surface. The final charge distribution is shown in figure.



10. Consider a cylindrical Gaussian surface as shown.

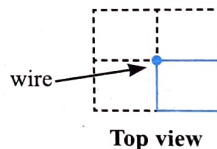


$$(E + dE)A - EA = \frac{\rho A dy}{\epsilon_0}$$

$$dE = \frac{\rho dy}{\epsilon_0}$$

$$\therefore \int_0^E dE = \frac{\rho}{\epsilon_0} \int_0^y y dy$$

11. (a)



If we construct these both cubes as shown (by dotted line)

Net flux through all the four cubes $\frac{q_{\text{in}}}{\epsilon_0} = \frac{\lambda l}{\epsilon_0}$

So flux through any one cube $= \frac{\lambda l}{4\epsilon_0}$

- (b) Field lines will be parallel to the surfaces in contact with wire. Hence flux through these surfaces will be zero. There are four such surfaces.

- (c) There are two surfaces which are not in contact with wire.

So the flux $\frac{\lambda l}{4\epsilon_0}$ will be divided among these two surfaces.

So flux through each surface $= \frac{\lambda l}{8\epsilon_0}$

Exercises

Single Correct Answer Type

1. (2) We should know the total charge inside. There is no contribution in the flux due to outside charges. Also, we should know both magnitude and direction of electric field at any point on the surface.
2. (2) $\phi = \frac{q_{\text{in}}}{\epsilon_0}$ or $0 = \frac{q_{\text{in}}}{\epsilon_0} \Rightarrow q_{\text{in}} = 0$
 Sum of all the charges is zero, so net flux is zero.

1. (1) Charge inside $S_1 = q_1 + q_2 = 3 \times 10^{-6} \text{ C}$
 Charge inside $S_2 = q_2 + q_3 = -1 \times 10^{-6} \text{ C}$
 Charge inside $S_3 = 0$
 Charge inside S_1 is greatest. So, flux through S_1 is maximum.

4. (3) Electric field at any point on Gaussian surface is due to all charges present inside or outside the surface.

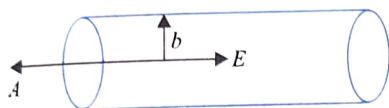
5. (2) Net charge on the conductor will be zero. So, net charge inside S_1 will be the charge on the rod. Hence, flux through S_1 is q/ϵ_0 .

6. (3) Electric field is zero inside the shell due to the outside charges.

7. (3) $\phi = \frac{q}{\epsilon_0}$ (Independent of dimensions)

8. (3) Flux through both will be same as the net charge enclosed by both is same.

9. (4) $\phi = EA \cos \theta = E\pi b^2 \cos 180^\circ = -200\pi b^2$



10. (2) Net flux is due to charges inside S only.

11. (3) $1 - 7 - 4 + 10 + 2 - 5 - 3 + 6 = 0$
 Sum of all the charges is zero, so net flux is zero.

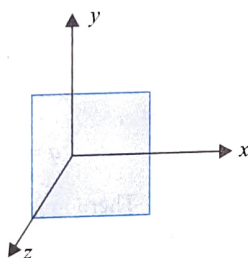
12. (2) $\vec{E} = 8\hat{i} + 4\hat{j} + 3\hat{k}$, $\vec{A} = 100\hat{k}$

So $\phi = \vec{E} \cdot \vec{A} = 300$ units

13. (3) Imagine a charge q at the center of a cube of edge length $2L$ (figure). Then $\phi = q/\epsilon_0$.

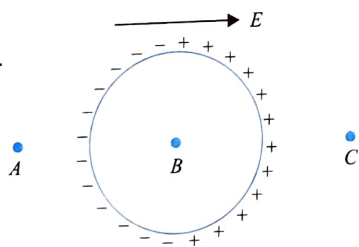
Here, the square is $1/24$ of the surface area of the imaginary cube, so it intercepts $1/24$ of the flux. That is,

$$\Phi = \frac{q}{24\epsilon_0}$$



14. (3) The dielectric gets polarized as shown. Induced charges will also contribute in electric field.

So, intensity at points A and C will increase and at B intensity will decrease.



15. (2) From the figure, it is clear that the plate is placed in an external electric field. Let the electric field due to plate be E and E_0 is the external electric field. Therefore,

$$E_0 + E = 12 \text{ Vm}^{-1} \quad \dots(i)$$

$$E_0 - E = 8 \text{ Vm}^{-1} \quad \dots(ii)$$

Solving Eqs. (i) and (ii), $E = 2 \text{ Vm}^{-1}$ and $E_0 = 10 \text{ Vm}^{-1}$

Now, electric field due to plate is $\sigma/2\epsilon_0 = 2$.

Therefore, $\sigma = 4\epsilon_0$

16. (2) $\rho = Cr^2$

$$q = \int_0^r 4\pi r^2 dr \rho = \int_0^r 4\pi Cr^4 dr = \frac{4}{5} \pi Cr^5$$

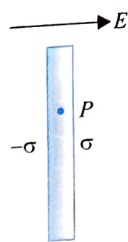
$$E_{r=2R} = \frac{kq_{(r=R)}}{(2R)^2} = \frac{k(4/5)\pi CR^5}{4R^2}$$

$$= \frac{kq_{(r=R/2)}}{(R/2)^2} = \frac{k(4/5)\pi C(R/2)^5}{(R/2)^2}$$

Now solve to get $\frac{E_{r=2R}}{E_{r=R/2}} = 2$

17. (2) Net electric field at point P should be zero. For this electric field due to induced charges is equal to the applied electric field. So

$$\frac{\sigma}{\epsilon_0} = E \text{ or } \sigma = \epsilon_0 E$$



18. (3) Electric field is zero inside a metallic cavity if no charge is present in the cavity, whatever be the field outside.

19. (2) Let us complete the sphere.

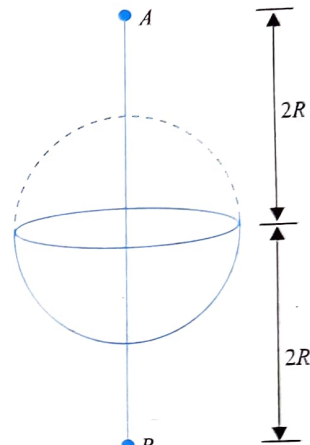
Electric field due to lower part at A is equal to electric field due to upper part at $B = E$ (given)

Electric field due to lower part at $B =$ electric field due to full sphere – electric field due to upper part

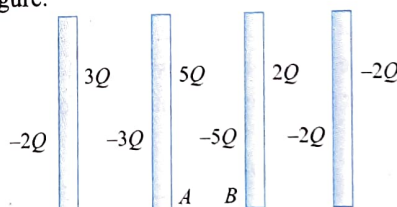
$$= \frac{kQ}{(2R)^2} - E$$

$$= \frac{1}{4\pi\epsilon_0} \frac{\rho(4/3)\pi R^3}{4R^2} - E$$

$$= \frac{\rho R}{12\epsilon_0} - E$$



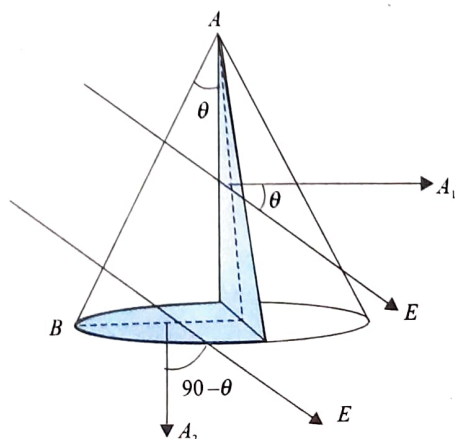
20. (1) Distribution of charge before the wire is connected is shown in figure.



On connecting with the wire, $5Q$ and $-5Q$ will get neutralized. Hence, $5Q$ charge will flow from A to B .

21. (1) Flux entering the cone from side AB will ultimately also pass through areas A_1 and A_2 . So,

$$\phi = EA_1 \cos \theta + EA_2 \cos (90^\circ - \theta)$$



$$= E \left(\frac{1}{2} 2Rh \cos \theta + \frac{\pi}{2} R^2 \sin \theta \right)$$

$$= ER[h \cos \theta + \pi(R/2) \sin \theta]$$

22. (1) Let external field be E_0 and surface charge density of sheet be σ (including both surfaces).

$$E_p = \frac{\sigma}{2\epsilon_0}$$

Electric field due to sheet

$$E_1 = E_0 - E_p \quad \dots(i)$$

$$-E_2 = E_0 + E_p \quad \dots(ii)$$

From Eqs. (i) and (ii), $E_0 = \frac{E_1 - E_2}{2} = 10^5 \text{ Vm}^{-1}$

and $E_p = \frac{-E_1 - E_2}{2} = -4 \times 10^5 \text{ Vm}^{-1}$

$$\frac{\sigma}{2\epsilon_0} = -4 \times 10^5 \text{ or } \sigma = -8\epsilon_0 \times 10^5$$

Force on sheet:

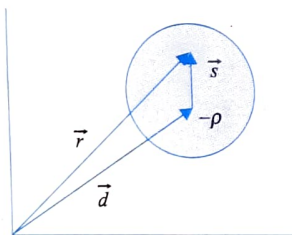
$$F = qE_0 \text{ or } 0.08 = \sigma AE_0$$

$$\text{or } 0.08 = 8\epsilon_0 \times 10^5 A \times 10^5$$

$$\text{or } A = \frac{10^{-12}}{\epsilon_0} = 10^{-12} \times 36 \times 10^9 \pi = 3.6\pi \times 10^{-2} \text{ m}^2$$

23. (3) $\vec{E}_p = \frac{-\rho}{3\epsilon_0} \vec{s} = -\frac{\rho}{3\epsilon_0} (\vec{r} - \vec{d})$

$$= \frac{\rho}{3\epsilon_0} (\vec{d} - \vec{r})$$

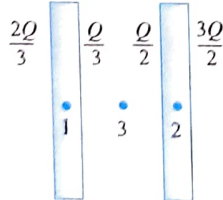


24. (2) Let us take rightward as positive and leftward as negative. These four faces can be considered four thin infinite plane sheets.

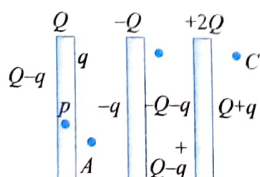
$$\vec{E}_1 = \frac{1}{2\epsilon_0 S} \left[\frac{2Q}{3} - \frac{Q}{3} - \frac{Q}{2} - \frac{3Q}{2} \right] = \frac{-5Q}{6\epsilon_0 S}$$

$$\vec{E}_2 = \frac{1}{2\epsilon_0 S} \left[\frac{2Q}{3} + \frac{Q}{3} - \frac{Q}{2} - \frac{3Q}{2} \right] = \frac{-Q}{2\epsilon_0 S}$$

$$\vec{E}_3 = \frac{1}{2\epsilon_0 S} \left[\frac{2Q}{3} + \frac{Q}{3} + \frac{Q}{2} - \frac{3Q}{2} \right] = 0$$



25. (4)



$E_p = 0$ (since it is a point inside conductor)

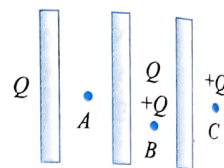
$$\text{or } K \left[\frac{Q-q}{2\epsilon_0} - \frac{q}{2\epsilon_0} + \frac{q}{2\epsilon_0} + \frac{Q-q}{2\epsilon_0} - \frac{(Q+q)}{2\epsilon_0} - \frac{(Q-q)}{2\epsilon_0} \right] = 0$$

$$\text{or } q = 0$$

$$\text{Clearly } E_A = 0$$

$$E_B = -\frac{2KQ}{2\epsilon_0} = \frac{-KQ}{\epsilon_0}$$

$$E_C = \frac{2KQ}{2\epsilon_0} = \frac{KQ}{\epsilon_0}$$



$$\text{In magnitude, } E_B = E_C$$

26. (1) To find the charge enclosed, we need the flux through the parallelepiped:

$$\Phi_1 = AE_1 \cos 60^\circ$$

$$= (0.0500 \text{ m})(0.0600 \text{ m})(2.50 \times 10^4 \text{ NC}^{-1}) \cos 60^\circ$$

$$= 37.5 \text{ Nm}^2\text{C}^{-1}$$

$$\Phi_2 = AE_2 \cos 120^\circ$$

$$= (0.0500 \text{ m})(0.0600 \text{ m})(7.00 \times 10^4 \text{ NC}^{-1}) \cos 120^\circ$$

$$= -105 \text{ Nm}^2\text{C}^{-1}$$

So, the total flux is

$$\Phi = \Phi_1 + \Phi_2 = (37.5 - 105) \text{ Nm}^2\text{C}^{-1} = -67.5 \text{ Nm}^2\text{C}^{-1}$$

$$q = \Phi \epsilon_0 = (-67.5 \text{ Nm}^2\text{C}^{-1}) \epsilon_0 = -5.97 \times 10^{-10} \text{ C}$$

There must be a net charge (negative) in the parallelepiped since there is a net flux flowing into the surface. Also, there must be an external field, otherwise all lines would point toward the slab.

27. (2) The number of field lines crossing an area is directly related to electric flux passing through surface. In first case, the flux crossing the surface is more than the flux crossing through surface in second case, so $n_1 > n_2$.

28. (3) $\alpha = 60^\circ$. Solid angle subtended by BCD is

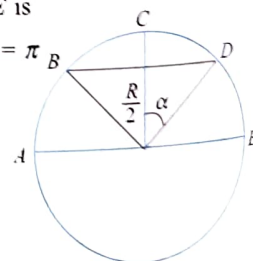
$$\omega = 2\pi (1 - \cos \alpha) = \pi$$

Solid angle subtended by ABDE is

$$\omega_{(ABCDE)} - \omega_{(BCD)} = 2\pi - \pi = \pi$$

Hence, flux through ABDE is

$$\phi = \frac{q}{\epsilon_0} \frac{\pi}{4\pi} = \frac{q}{4\epsilon_0}$$



29. (3) Electric flux $\oint_S \vec{E} \cdot d\vec{s} = \frac{q_{in}}{\epsilon_0}$

q_{in} is the charge enclosed by the Gaussian surface, which, in the present case, is the surface of the given sphere. As shown, length AB of line lies inside the sphere. In $\triangle OOA'$,

$$R^2 = y^2 + (O'A)^2$$

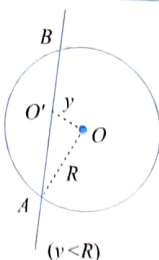
$$\therefore O'A = \sqrt{R^2 - y^2}$$

and $AB = 2\sqrt{R^2 - y^2}$

Charge on length AB is $2\sqrt{R^2 - y^2} \times \lambda$

Therefore, electric flux is

$$\oint_S \vec{E} \cdot d\vec{s} = \frac{2\lambda\sqrt{R^2 - y^2}}{\epsilon_0}$$



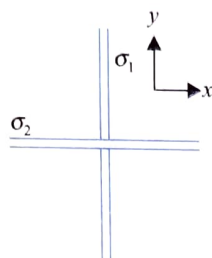
30. (3) Using the principle of superposition $\vec{E}$ due to infinite plane sheet having surface charge density σ on its one face is $\sigma/2\epsilon_0$. So

$$\vec{E}_1 = \frac{\sigma_1}{2\epsilon_0} \hat{i} + \frac{\sigma_2}{2\epsilon_0} \hat{j}$$

$$\vec{E}_2 = -\frac{\sigma_1}{2\epsilon_0} \hat{i} + \frac{\sigma_2}{2\epsilon_0} \hat{j}$$

$$\vec{E}_3 = -\frac{\sigma_1}{2\epsilon_0} \hat{i} - \frac{\sigma_2}{2\epsilon_0} \hat{j}$$

$$\vec{E}_4 = \frac{\sigma_1}{2\epsilon_0} \hat{i} - \frac{\sigma_2}{2\epsilon_0} \hat{j}$$



31. (1) Electric field on surface of a uniformly charged sphere is given by

$$\frac{Q}{4\pi\epsilon_0 R^2} = \frac{\rho R}{3\epsilon_0}$$

Electric field at outside point is given by

$$E = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{\rho R^3}{3\epsilon_0 r^2}$$

$$E_B = E_{\text{whole sphere}} - E_{\text{cavity}}$$

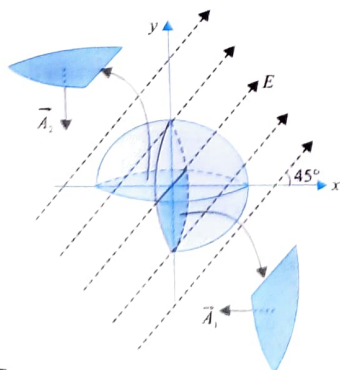
$$= \frac{\rho r_0}{3\epsilon_0} - \frac{\rho \left(\frac{r_0}{2}\right)^3}{3\epsilon_0 \left(\frac{3r_0}{2}\right)^2} = \frac{17\rho r_0}{54\epsilon_0}$$

32. (1) If portion outside the cylinder is removed, electric field at points on curved surface will decrease.

33. (3) $\phi_{\text{plain}} + \phi_{\text{curve}} = 0$ or $\phi_{\text{plain}} = -\phi_{\text{curve}}$

$$\vec{A}_1 = -\frac{\pi R^2}{2} \hat{i}, \vec{A}_2 = -\frac{\pi R^2}{2} \hat{j}$$

$$\vec{E} = E \cos 45^\circ \hat{i} + E \sin 45^\circ \hat{j}$$



$$= \frac{E}{\sqrt{2}} \hat{i} + \frac{E}{\sqrt{2}} \hat{j}$$

$$\phi = \vec{E} \cdot (\vec{A}_1 + \vec{A}_2)$$

$$= \frac{-E \pi R^2}{\sqrt{2}} - \frac{E \pi R^2}{\sqrt{2}} = \frac{-\pi R^2 E}{\sqrt{2}}$$

This is the flux entering. So flux is $\frac{\pi R^2 E}{\sqrt{2}}$

34. (4) $E = \frac{\sigma}{2\epsilon_0}$

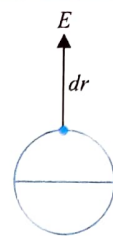
$$F = E dr = \frac{\sigma dA \sigma}{2\epsilon_0}$$

$$P = \frac{F}{dA} = \frac{\sigma^2}{2\epsilon_0}$$

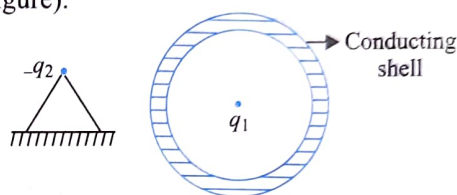
$$\text{Force} = P \pi R^2 = \frac{\sigma^2}{2\epsilon_0} \pi R^2$$

$$= \frac{Q^2}{16\pi^2 R^4} \frac{\pi R^2}{2\epsilon_0} = \frac{Q^2}{32\pi\epsilon_0 R^2}$$

$$= \frac{(20 \times 10^{-6})^2 \times 9 \times 10^9}{8(1.5 \times 10^{-2})^2} = 2000 \text{ N}$$



35. (3) Net force on q_1 is zero, while that on the conducting sphere is toward the left due to attraction of $-q_2$ (refer figure).



36. (2) From Gauss's law, $\phi = \frac{q}{\epsilon_0}$

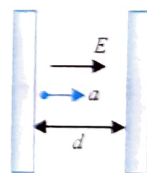
$$\text{So, } \frac{\phi_{\text{max}}}{\phi_{\text{min}}} = \frac{Q_{\text{max}}}{Q_{\text{min}}} = \frac{\lambda(l^2 + b^2 + h^2)^{1/2}}{\lambda h} = \frac{\sqrt{l^2 + b^2 + h^2}}{h}$$

37. (1) According to Gauss's law, electric field at any point on Gaussian surface is due to all the charges present inside or outside.

38. (2) $E = \frac{Q}{2A\epsilon_0}$

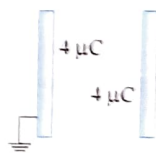
$$a = \frac{qE}{m} = \frac{qQ}{2Am\epsilon_0}$$

$$d = \frac{1}{2} at^2; t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2d \cdot 2Am\epsilon_0}{qQ}}$$



39. (2) Final charges will be as shown in figure.

So charge flowing from earth to plate = final charge - initial charge = $-4 - 10 = -14 \mu\text{C}$



40. (2) Given

$$\vec{E} = \frac{E_0}{l} \hat{i}, l = 2 \text{ cm}, a = 1 \text{ cm}, E_0 = 5 \times 10^3 \text{ NC}^{-1}$$

We see that flux passes mainly through surface area $ABDC$ and $EFGH$. As the $AEFB$ and $CHGD$ are parallel to the flux again in $ABDC = 0$, the flux only passes through the surface area $EFGHE = 0$.

$$\text{Flux} = E_0 \frac{a}{l} \times \text{area} = 5 \times 10^3 \times \frac{a}{l} \times a^2$$

$$= 5 \times 10^3 \times \frac{a^3}{l}$$

$$= 5 \times 10^3 \times \frac{(0.01)^3}{2 \times 10^{-2}} = 2.5 \times 10^{-1}$$

So, $q = \epsilon_0 \times \text{flux}$

$$= 8.85 \times 10^{-12} \times 2.5 \times 10^{-1}$$

$$= 22.125 \times 10^{-13} = 2.2125 \times 10^{-12} \text{ C}$$

41. (4) Consider the Gaussian surface where the induced charge is as shown in the figure. The net field at P due to all the charges is zero, i.e.,

$$-2Q + \frac{q}{2A\epsilon_0} (\text{left}) + \frac{q}{2A\epsilon_0} (\text{right}) + q - \frac{q}{2A\epsilon_0} (\text{right}) = 0$$

$$\text{or } -2q + q - Q + q = 0$$

$$\text{or } q = \frac{3}{2}Q$$

Charge on the right side of the rightmost plate is

$$-2Q + Q = -2Q + \frac{3}{2}Q = -\frac{Q}{2}$$

42. (1) $\phi_{ABCD} = -acd \text{ unit}$

$\phi_{CDEF} = -bcE_0 \text{ unit}$

$$\phi_{ABEF} = bcE_0 + c \int_0^a (d+x) dy$$

$$= +bcE_0 + acd + c \int_0^a x dy$$

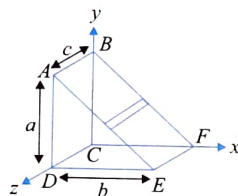
$$= +bcE_0 + acd + \frac{ca}{b} \int_0^b x dx$$

$$\left[\text{Since } \frac{x}{b} + \frac{y}{a} = 1 \text{ or } \frac{dx}{b} = -\frac{dy}{a} \right]$$

$$= \left[+bcE_0 + acd + \frac{acb}{2} \right] \text{ unit}$$

Using Gauss's law, we get

$$\phi_{\text{net}} = \frac{q_{\text{in}}}{\epsilon_0} \text{ or } q_{\text{in}} = \frac{abcE_0}{2}$$

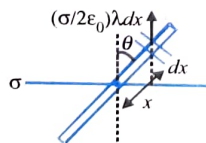


43. (2) $\tau = 2 \left(\frac{\sigma}{2\epsilon_0} \lambda dx \right) x \sin \theta = \frac{\sigma}{\epsilon_0} \lambda \int_0^{l/2} x dx \sin \theta = \frac{\sigma}{2\epsilon_0} \lambda \frac{l^2}{4} \sin \theta$

$$\tau = -I\alpha \text{ or } \alpha = -\tau/I = \frac{\sigma}{2\epsilon_0} \lambda \frac{l^2}{4} \frac{12}{ml^2} \sin \theta$$

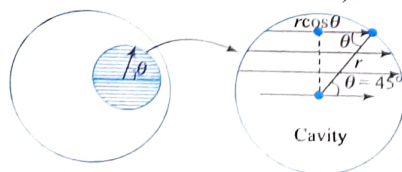
$$\alpha = -\left(\frac{3\sigma\lambda}{2m\epsilon_0} \theta \right) \text{ (for small angle)}$$

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{2m\epsilon_0}{3\sigma\lambda}}$$



44. (1) Acceleration of electron

$$A = eE/m \text{ (in backward direction).}$$



$$\text{and } E = \frac{\rho a}{3\epsilon_0} \text{ or } A = \frac{e\rho a}{3\epsilon_0 m}$$

Distance traveled by electron where it hits the wall

$$\text{of cavity is } S = 2r \cos \theta = \frac{r}{\sqrt{2}}$$

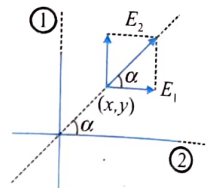
$$\text{Hence using } S = \frac{1}{2} At^2$$

$$t = \sqrt{\frac{6\sqrt{2} r \epsilon_0 m}{e\rho a}}$$

45. (2) $E_1 = \frac{\lambda_1}{2\pi\epsilon_0 x}$ and $E_2 = \frac{\lambda_2}{2\pi\epsilon_0 y}$

$$\frac{E_1}{E_2} = \left(\frac{\lambda_1}{\lambda_2} \right) \left(\frac{y}{x} \right)$$

$$\frac{1}{\tan \alpha} = \left(\frac{\lambda_1}{\lambda_2} \right) \tan \alpha \text{ or } \frac{\lambda_1}{\lambda_2} = \cot^2 \alpha$$



46. (3) Use Gauss theorem.

Electric field inside uniformly distributed dielectric sphere

$$E \propto r \quad (\text{For } 0 < r \leq R)$$

Between the spheres

$$E \propto \frac{Q}{r^2} \quad (\text{For } R < r \leq 2R)$$

Outside the spheres

$$E \propto \frac{3Q}{r^2} \quad (\text{For } 2R < r)$$

47. (1) Force on Q_2 will be due to induced charge on outer surface only which will be same in both cases along with its distribution. Hence the ratio is 1.

Multiple Correct Answers Type

1. (2), (3)

ϕ crossing through Gaussian surface does not depend on the location of charge, while $\vec{E}$ depends on it. If q crosses the boundary, then q_{enclosed} changes and hence the flux and $\vec{E}$ also change.

2. (2), (4)

In LHS of Gauss's law, $\vec{E}$ is due to all point charges present in space and ϕ depends only on the enclosed charges.

3. (1), (2), (4)

(1) Net charge of dipole is zero.

$$(2) \phi_{\text{net}} = \frac{q_{\text{in}}}{\epsilon_0} = 0$$

(3) Electric field is nowhere zero due to a dipole.

$$(4) \int \vec{E} \cdot d\vec{s} = \phi_{\text{net}} = 0$$

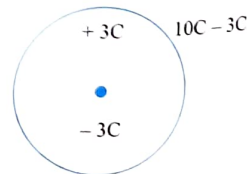
4. (1), (4)

Due to induction, charges on various faces are as shown in figure.

Charge on the inner surface of the shell is +3 C.

Net charge on the outer surface of the shell is -3 C + 10 C = +7 C.

Distribution of charge on the inner surface would be uniform if charge is placed at the center, otherwise nonuniform. On outer surface, charge would be always uniformly distributed as the displacement of inside charges does not affect the distribution of the outer charge.



5. (1), (4)

$$\text{At } R, E_{\text{net}} = E + \frac{Q}{2A\epsilon_0} - \frac{Q}{2A\epsilon_0} = E$$

$$\text{At } S, E_{\text{net}} = E + \frac{Q}{2A\epsilon_0} + \frac{Q}{2A\epsilon_0} = E + \frac{Q}{A\epsilon_0}$$

$$\text{At } T, E_{\text{net}} = E + \frac{Q}{2A\epsilon_0} - \frac{Q}{2A\epsilon_0} = E$$

6. (1), (2), (3)

E depends upon both Q_1 and Q_2 . But ϕ depends only on Q_1 .

7. (2), (3)

$$\text{At } A, E = \frac{9 \times 10^9 \times 0.5 \times 10^{-6}}{(2 \times 10^{-2})^2} = 1.125 \times 10^7 \text{ NC}^{-1}$$

At $B, E = 0$, because in a metal, electric field is zero.

8. (3), (4)

Flux through curved surface will be zero when charge is placed at A, C , or D . When charge is placed at B , flux through curved surface will be $Q/2\epsilon_0$.

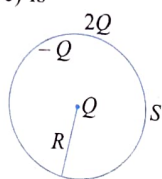
9. (1), (3), (4)

Charge density at the outer surface (refer figure) is

$$\frac{2Q}{4\pi R^2} = \frac{Q}{2\pi R^2}$$

$$E_{\text{inside}} = \frac{Q}{4\pi\epsilon_0 R^2}, E_{\text{outside}} = \frac{2Q}{4\pi\epsilon_0 R^2}$$

$$\therefore E_{\text{outside}} = 2E_{\text{inside}}$$



10. (1), (3)

As A and C are earthed, they are connected to each other. Hence, ' $A+B$ ' and ' $B+C$ ' are two capacitors with the same potential difference. If B is closer to A than to C , then the capacitance $C_{AB} > C_{BC}$. The upper surface of B will have greater charge than the lower surface. As the force of attraction between the plates of a capacitor is proportional to Q^2 , there will be a net upward force on B . This can balance its weight.

11. (2), (4)

The electric field inside any point of the sphere is zero.

12. (1), (2), (3)

As charge enclosed by each Gaussian surfaces is the same, hence net flux enclosed through all the Gaussian surfaces will be the same.

$$\text{Gauss theorem } \oint \vec{E} \cdot d\vec{S} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

The spherical Gaussian surfaces are symmetrical about charge, hence flux will be uniform for these surfaces. But for cubical Gaussian surface, this symmetry does not present as the distance of the charge vary from point to point on the surface.

13. (1), (2)

The electric field is uniform and no charge is enclosed within the Gaussian surfaces, hence the net flux enclosed with each cylindrical midsection will be zero.

As the magnitude of the flux is proportional to number of electric field line. The number of electric field lines passing through the circular base of the cylindrical midsection is equal to the number of lines passing through caps which are the same.

14. (1), (2), (3), (4)

(1) Q lies outside Gaussian surface.

(2) Net charge enclosed by Gaussian surface is zero.

(3) As the charge is displaced towards sphere amount of negative charge induced on right half of sphere will increase, hence net flux through right hemispherical closed Gaussian surface increases.

(4) Same reason as (c) and hence charge distribution on outer surface of sphere will change.

15. (1), (3)

$$\phi_1 = E_1 2\pi r_1^2 \text{ and } \phi_2 = E_2 2\pi r_2^2$$

$$E_1 \neq E_2$$

$$\frac{\phi_1}{\phi_2} = \frac{E_1 r_1^2}{E_2 r_2^2}$$

...(i)

$$\text{But } \phi_1 = \frac{q_0}{2\epsilon_0}; \phi_2 = \frac{q_0}{2\epsilon_0} \Rightarrow \frac{\phi_1}{\phi_2} = 1$$

$$\text{From Eq. (i), } E_1 r_1^2 = E_2 r_2^2$$

$$\frac{r_1}{\sqrt{E_2}} = \frac{r_2}{\sqrt{E_1}}$$

16. (1), (3), (4)

For face CDEF, $A = 4\hat{i} \text{ m}^2$ and $E = 15\hat{i} + 3\hat{j}$,

As $x = 3$

ϕ = flux linked with face

$$= E \cdot A = (15\hat{i} + 3\hat{j}) \cdot 4\hat{i} = 60 \frac{\text{Nm}^2}{\text{C}}$$

For face ABGH, flux linked with face

$$= \phi = E \cdot A = (5\hat{i} + 3\hat{j}) \cdot 4\hat{i} = 20 \frac{\text{Nm}^2}{\text{C}}$$

For face EFGH, x varies from $x = 1 \text{ m}$ to $x = 3 \text{ m}$.

$\therefore \phi$ = Flux linked with face

$$= \int_{x=1}^{x=3} E \cdot A = \int_1^3 (5x\hat{i} + 3\hat{j}) \cdot (2dx \hat{j}) = \int_{x=1}^{x=3} 6dx = 6x \Big|_{x=1}^{x=3}$$

$$= 6 \times 3 - 6 \times 1 = 18 - 6 = 12 \frac{\text{Nm}^2}{\text{C}}$$

Linked Comprehension Type

For Problems 1-4

1. (4) Electric field is always net electric field in the formula

$$\oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$$

Since q is the charge inside the closed surface, E_p is due to q_1, q_2, q_3 , and q_4 .

2. (1) Net flux depends on charge change enclosed, i.e., due to q_1 and q_2 only. Net flux depends on charge.

3. (1)

4. (4) Electric field will change since two of the charges are displaced from their earlier position, while flux will not change since q_1 and q_2 are still inside the surface.

For Problems 5-6

5. (1), 6. (4)

Given that $\vec{E} = -B\hat{i} + C\hat{j} - D\hat{k}$, $\Phi = \vec{E} \cdot \vec{A}$, edge length L , and

$$\hat{n}_{s_1} = -\hat{j} \Rightarrow \Phi_1 = \vec{E} \cdot A\hat{n}_{s_1} = -CL^2$$

$$\hat{n}_{s_2} = +\hat{k} \Rightarrow \Phi_2 = \vec{E} \cdot A\hat{n}_{s_2} = -DL^2$$

$$\hat{n}_{s_3} = +\hat{j} \Rightarrow \Phi_3 = \vec{E} \cdot A\hat{n}_{s_3} = +CL^2$$

$$\hat{n}_{s_4} = -\hat{k} \Rightarrow \Phi_4 = \vec{E} \cdot A\hat{n}_{s_4} = +DL^2$$

$$\hat{n}_{s_5} = +\hat{i} \Rightarrow \Phi_5 = \vec{E} \cdot A\hat{n}_{s_5} = -BL^2$$

$$\hat{n}_{s_6} = -\hat{i} \Rightarrow \Phi_6 = \vec{E} \cdot A\hat{n}_{s_6} = +BL^2$$

$$\text{Total flux is } \sum_{i=1}^6 \Phi_i = 0$$

For Problems 7–9

7. (4) Charges outside cavity of a conductor have no influence on the field inside cavity.
8. (2) There is charge inside cavity, so field lines will be present, but the potential at surface of conductor is same everywhere. So field lines should be perpendicular.
9. (2) We see that the surface of two spherical cavities carries charges $-q_1, -q_2$, respectively, and since the sphere A was not charged originally, its spherical surface must carry induced charges $(q_1 + q_2)$. Therefore, force acting on conductor A is $q_3(q_1 + q_2)/4\pi\epsilon_0 r^2$.

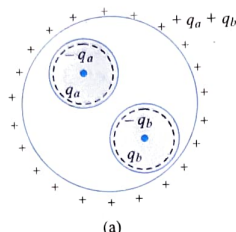
For Problems 10–12

10. (2), 11. (3), 12. (1)

$$(i) \sigma_A = \frac{-q_a}{4\pi a^2}, \sigma_B = \frac{-q_b}{4\pi b^2}, \sigma_R = \frac{q_a + q_b}{4\pi R^2}$$

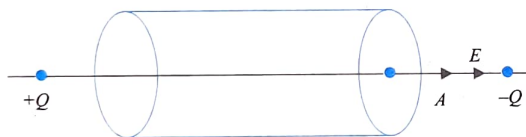
$$(ii) E = \frac{1}{4\pi\epsilon_0} \frac{q_a + q_b}{r^2}$$

$$(iii) E_a = \frac{1}{4\pi\epsilon_0} \frac{q_a}{r^2}$$



For Problems 13–14

13. (1) Net charge enclosed by the cylinder is zero, so flux through the cylinder is zero.
14. (1) Electric field at right end cap due to both the charges is toward right and area vector is also toward right. Hence, flux is positive.

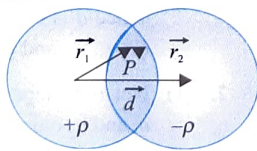


For Problems 15–16

15. (3) Take a point P in overlap region, then

$$\vec{E}_P = \frac{\rho \vec{r}_1}{3\epsilon_0} - \frac{\rho \vec{r}_2}{3\epsilon_0}$$

$$= \frac{\rho}{3\epsilon_0} (\vec{r}_1 - \vec{r}_2) = \frac{\rho \vec{d}}{3\epsilon_0}$$

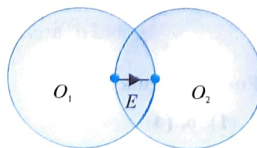


This is the uniform field in overlap region.

16. (1) $d = R$

Electric field from O_1 through O_2 is uniform and is $E = \rho R/3\epsilon_0$. So potential difference is

$$\Delta V = ER = \frac{\rho R^2}{3\epsilon_0} = \frac{\rho d^2}{3\epsilon_0}$$



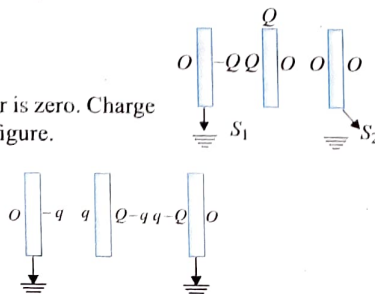
For Problems 17–19

17. (2), 18. (3), 19. (3)

Field inside the conductor is zero. Charge distribution is shown in figure.

$$\frac{(Q-q)}{\epsilon_0 A} 3d = \frac{qd}{\epsilon_0 A}$$

$$\text{or } q = \frac{3Q}{4}$$



For Problems 20–22

20. (3), 21. (2), 22. (4)

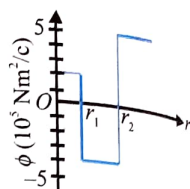
We see that ϕ remains constant up to r_1 . So $r_1 < r_A$. Let q be the charge at center, then

$$\phi = \frac{q}{\epsilon_0} \text{ or } 2 \times 10^5 = \frac{q}{8.85 \times 10^{-12}}$$

$$\text{or } q = 1.77 \mu\text{C}$$

$$\text{Similarly } \phi = -4 \times 10^5 = \frac{q + q_A}{\epsilon_0} \text{ or } q_A = -5.31 \mu\text{C}$$

where q_A is the charge on A . Electric field is zero nowhere as ϕ is not zero anywhere.



For Problems 23–25

23. (4), 24. (1), 25. (4)

For A and B we know that

$$\text{Flux} = \phi = \int \vec{E} \cdot d\vec{s}$$

$$\text{Force} = \int \sigma \vec{E} \cdot d\vec{s} = \sigma \int \vec{E} \cdot d\vec{s} = \sigma \phi$$

As flux due to charge q in each plate is same, so force will be same. For C we have taken the horizontal component of electric field, but for flux, we do not have to take the component of electric field. As force due to one plate on another plate is same, hence ϕ due to 1 in another and due to 2 in 1 will be same. When x and R increased, ϕ in plate due to charge remains same. Hence force due to charge on plates does not change.

For Problems 26–27

26. (3), 27. (2)

In the region between the disks electric field is uniform and in the region outside the disks electric field is zero.

Potential is constant in the region outside the disks, because electric field is zero. In the region between the plates, potential decreases in the direction of electric field. So potential is maximum at positive disk.

For Problems 28–30

28. (2), 29. (1), 30. (1)

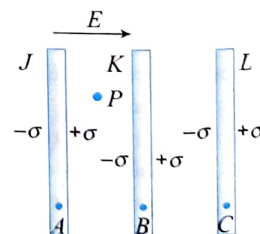
Let charge densities are induced as shown.

Electric field at points A, B, C should be zero.

Let us take point A :

Electric field at A due to plates K and L will be zero, because net charge on these plates is zero. So for field at A to be zero.

Electric field at A due to plate $J = \text{applied field}$



$$\Rightarrow \frac{\sigma}{\epsilon_0} = E \Rightarrow \sigma = \epsilon_0 E$$

Similarly we can find for B and C .

Hence, induced charge density on surface '1' is $-\sigma = -\epsilon_0 E$ on surface '4' is $+\sigma = \epsilon_0 E$

Now electric field at P due to charges induced on plates is zero because net charge on each plate is zero. Hence, electric field at P is equal to applied electric field which is E towards right.

Matrix Match Type

1. i. $\rightarrow$ a., d.; ii. $\rightarrow$ c.; iii. $\rightarrow$ b., d.; iv. $\rightarrow$ c.

Charge enclosed in S is $-2Q + Q = -Q$

So flux is not zero.

Net charge in S_2 is 0, so flux is zero.

Net charge in S_3 is $-2Q + Q - Q = -2Q$.

So flux is not zero.

No charge inside S_4 , hence flux is 0.

2. i. $\rightarrow$ b.; ii. $\rightarrow$ b.; iii. $\rightarrow$ b., c.; iv. $\rightarrow$ b., c.

Electric field inside due to Q_2 and induced charges on shell will be zero.

But Q_1 will produce electric field inside.

Again electric field inside sphere will be nonzero. Due to the movement of Q_1 , electric field inside will change, but it will have no effect on the electric field outside.

Due to the movement of Q_2 , electric field outside changes but has no effect on the field inside.

3. i. $\rightarrow$ a.; ii. $\rightarrow$ a., b.; iii. $\rightarrow$ d.; iv. $\rightarrow$ a., b., c.

After charge distribution according to induction, the charges on various surfaces are as follows:

Surface 1: $7Q$, Surface 2: $-3Q$, Surface 3: $3Q$, Surface 4: $-2Q$, Surface 5: $2Q$, Surface 6: Zero, Surface 7: Zero, Surface 8: $7Q$.

4. i. $\rightarrow$ a., d.; ii. $\rightarrow$ a., d.; iii. $\rightarrow$ b., d.; iv. $\rightarrow$ c.

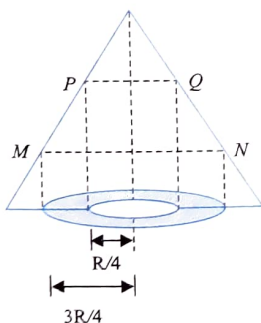
Flux entering through base

Flux outgoing through curved part is same as the flux entering through base.

Flux through $MNOP$ will be same as the flux through base from radius $R/4$ to $3R/4$. So

$$\phi = E\pi \left[\left(\frac{3R}{4} \right)^2 - \left(\frac{R}{4} \right)^2 \right] = \frac{E\pi R^2}{2}$$

Net flux through the entire cone will be zero as in uniform electric field, flux through any closed surface is zero.



5. i. $\rightarrow$ c.; ii. $\rightarrow$ b.; iii. $\rightarrow$ d.; iv. $\rightarrow$ a.

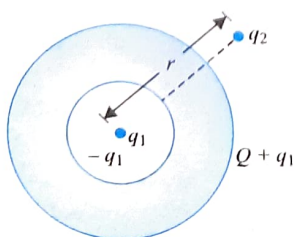
i. Since $q_2 = 0$, so charge $Q + q_1$ on outer surface will be distributed uniformly. Hence, electric field of $Q + q_1$ at center will be zero.

ii. Since q_2 is not zero, so $Q + q_1$ will be induced nonuniformly on outer surface. But due to q_2 and $Q + q_1$, net field at any point inside the outer surface should be zero.

$$E_{q_2} + E_{Q+q_1} = 0$$

$$\frac{1}{4\pi\epsilon_0} \frac{q_2}{(r - R_1)^2} + E_{Q+q_1} = 0$$

$$E_{Q+q_1} = -\frac{1}{4\pi\epsilon_0} \frac{q_2}{(r - R_1)^2}$$



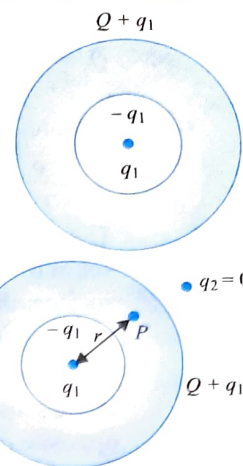
$$= \frac{1}{4\pi\epsilon_0} \frac{q_2}{(r - R_1)^2}, \text{ radially outward}$$

iii. Since $Q + q_1$ will be induced nonuniformly, so it is difficult to determine its electric field at an outside point.

iv. Net field at P due to q_1 and $-q_1$ should be zero.

$$E_{q_1} + E_{(-q_1)} = 0$$

$$E_{(-q_1)} = -E_{q_1} = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r^2}$$



6. i. $\rightarrow$ b., d.; ii. $\rightarrow$ a., d.; iii. $\rightarrow$ b., d.; iv. $\rightarrow$ a., c.

If any charge is present outside, then charge on outer surface will be distributed nonuniformly. But if there is no charge present outside, then charge on outer surface will be distributed uniformly. This is irrespective of location of the charge inside.

If charge is at the center, then charge on inner surface will be distributed uniformly, but if charge is displaced from center, then charge on inner surface will be distributed nonuniformly. This is irrespective of whether the charge is present outside or not.

Numerical Value Type

1. (3) Given $E = \alpha r$. When $r = R$, $ER = \alpha R$.

$$\text{So } \phi = E_R(\text{area}) = \alpha R 4\pi R^2$$

By Gauss's theorem, the net electric flux is

$$\frac{1}{\epsilon_0} \text{ charge enclosed}$$

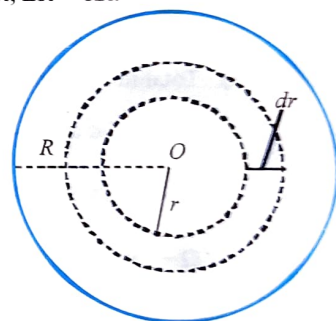
$$\alpha R 4\pi R^2 = \frac{1}{\epsilon_0} Q_{\text{enclosed}}$$

$$\therefore Q_{\text{enclosed}} = (4\pi\epsilon_0) \alpha R^3$$

$$\text{Given } R = 0.30 \text{ m,}$$

$$\alpha = 100 \text{ Vm}^{-2}$$

$$Q_{\text{enclosed}} = \frac{1}{9 \times 10^9} \times 100 \times (0.30)^3 = 3 \times 10^{-10} \text{ C}$$



2. (3) Net electric field is $\vec{P}$.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 \text{ due to vertical wire}$$

$$\vec{E}_1 = \frac{\lambda_1}{2\pi\epsilon_0 y} \text{ in } +y\text{-direction.}$$

$$\vec{E}_2 = \frac{\lambda_2}{2\pi\epsilon_0 x} \text{ due to vertical wire in } x\text{-direction.}$$

α angle of electric field with x direction

$$\tan \alpha = \frac{E_1}{E_2} = \frac{\lambda_1 / 2\pi\epsilon_0 y}{\lambda_2 / 2\pi\epsilon_0 x}$$

$$\text{or } \frac{1}{\sqrt{3}} = \frac{\lambda_1 x}{\lambda_2 y} \text{ or } \frac{\lambda_1}{\lambda_2} = \frac{x}{y} \sqrt{3} \text{ or } \frac{\lambda_1}{\lambda_2} = 3$$

$$3. (9) E_r = \frac{kQ_2}{\left(\frac{3R}{2}\right)^2} - \frac{kQ_1}{R^3} \frac{R}{2} = 0$$

$$\text{or } \frac{4}{9} Q_2 = \frac{Q_1}{2} \text{ or } \frac{Q_2}{Q_1} = \frac{9}{8}$$

$$\therefore Q_2 = \frac{9 \times 64}{8} = 72 \mu\text{C} = 8 \times 9 \mu\text{C}$$

4. (4) According to Gauss' theorem, electric flux:

$$\phi_E = \frac{q}{\epsilon_0} = \int \vec{E} \cdot d\vec{S}$$

The surface integral in the above equation contains six terms—the surface integral over the bottom surface, the surface integral over the top surface and surface integral over the four vertical faces.

For the bottom surface, both the vectors $\vec{E}$ and $d\vec{S}$ are in the same direction. For the top surface, they act in opposite directions while for the vertical faces, they are perpendicular to each other.

$$\begin{aligned} \text{Hence, } \phi_E &= \int_{\text{bottom}} \vec{E}_1 \cdot d\vec{S} + \int_{\text{top}} \vec{E}_2 \cdot d\vec{S} + 4 \int_{\text{faces}} \vec{E} \cdot d\vec{S} \\ &= \int_{\text{bottom}} E_1 \cdot dS \cos 0^\circ + \int_{\text{top}} E_2 \cdot dS \cos 180^\circ + 4 \int_{\text{faces}} E \cdot dS \cos 90^\circ \\ &= \int_{\text{bottom}} E_1 \cdot dS - \int_{\text{top}} E_2 \cdot dS = E_1 S - E_2 S = (E_1 - E_2) S \end{aligned}$$

$$\text{Also, } \phi_E = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

$$\begin{aligned} q &= \epsilon_0 (E_1 - E_2) S = \epsilon_0 (100 \text{ Vm}^{-1} - 60 \text{ Vm}^{-1}) (100 \text{ m})^2 \\ &= 4 \times 10^5 \epsilon_0 \text{ C} \end{aligned}$$

5. (3) Total flux associated with charge

$$Q \text{ for solid angle } 4\pi \text{ is } \frac{Q}{\epsilon_0}$$

Solid angle subtended by disc at Q

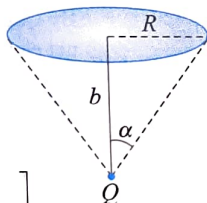
$$\Omega = 2\pi(1 - \cos \alpha) = 2\pi \left[1 - \frac{b}{\sqrt{b^2 + R^2}} \right]$$

$$\text{Flux through disc} = \frac{Q\Omega}{\epsilon_0 4\pi}$$

$$\text{It is given to be } \frac{Q}{4\epsilon_0}$$

$$\text{So } \frac{Q}{4\epsilon_0} = \frac{Q\Omega}{\epsilon_0 4\pi}$$

$$\text{Solve to get } R = \sqrt{3} b$$



6. (9) With air in the space, between S_1 and S_2 ,

$$\phi_{S_1} = \frac{q}{\epsilon_0} \text{ and } \phi_{S_2} = \frac{2q}{\epsilon_0}$$

When a medium of $K = 5$ is filled in space

$$\phi'_{S_1} = \frac{Q}{K\epsilon_0} = \frac{\phi_{S_1}}{5} = \frac{q}{5\epsilon_0}$$

And in this case, flux through S_2

$$\begin{aligned} \phi'_{S_2} &= \text{flux of } S_1 + \text{flux of charge } 2q \\ &= \frac{q}{5\epsilon_0} + \frac{2q}{\epsilon_0} = \frac{q + 10q}{5\epsilon_0} = \frac{11q}{5\epsilon_0} \end{aligned}$$

$$\text{So, ratio } \frac{\phi'_{S_1}}{\phi'_{S_2}} = 11$$

$$\text{So, } \frac{9K}{11} = 9$$

7. (2) Flux linked with a Gaussian sphere of radius r

$$\begin{aligned} &= \text{change enclosed} \times \frac{1}{\epsilon_0} \\ &= \text{volume enclosed} \times \text{charge density} \times \frac{1}{\epsilon_0} \\ &= \frac{4}{3} \pi r^3 \times \rho \times \frac{1}{\epsilon_0} \end{aligned}$$

$$\text{So, rate of change of flux} = \frac{d\phi_E}{dt} = \frac{d}{dt} \frac{4}{3} \pi \frac{\rho}{\epsilon_0} \times r^3$$

$$= \frac{4}{3} \pi \frac{\rho}{\epsilon_0} \times 3r^2 \frac{dr}{dt} = 4\pi r^2 \frac{\rho}{\epsilon_0} \left(\frac{dr}{dt} \right) = 8\pi r^2 \frac{\rho}{\epsilon_0}$$

$$\therefore \frac{d\phi_E}{dt} \propto r^2, \quad k = 2$$

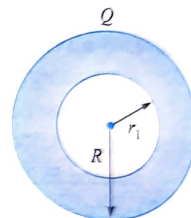
Archives

JEE Main

Single Correct Answer Type

1. (3) If the charge density, $\rho = \frac{Q}{\pi R^4} r$,

The electric field at the point P distant r_1 from the centre, according to Gauss's theorem,



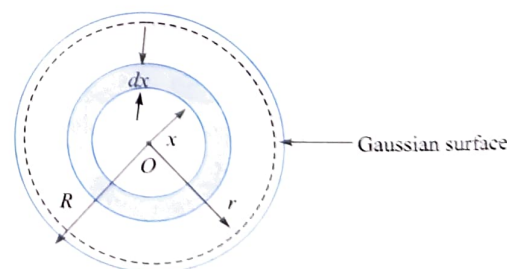
$$\phi = \int \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0} \Rightarrow E \cdot 4\pi r_1^2 = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

$$E \cdot 4\pi r_1^2 = \frac{1}{\epsilon_0} \int \rho dV$$

$$\Rightarrow E \cdot 4\pi r_1^2 = \frac{1}{\epsilon_0} \int_0^{r_1} \frac{Qr}{\pi R^4} \cdot 4\pi r^2 dr$$

$$\Rightarrow E = \frac{Qr_1^2}{4\pi\epsilon_0 R^4}$$

2. (2) Let us take a thin spherical shell of radius x and thickness dx as shown in the figure.



Volume of the shell, $dV = 7\pi x^2 dx$

Let us draw a Gaussian surface of radius r ($r < R$) as shown in the figure above.

Total charge enclosed inside the Gaussian surface,

$$Q_{\text{in}} = \int_0^r \rho dV = \int_0^r \rho_0 \left(\frac{5}{4} - \frac{x}{R} \right) 4\pi x^2 dx$$

$$= 4\pi \rho_0 \int_0^r \left(\frac{5}{4} x^2 - \frac{x^3}{R} \right) dx$$

$$= 4\pi \rho_0 \left[\frac{5}{12} x^3 - \frac{x^4}{4R} \right]_0^r$$

$$= \pi \rho_0 \left[\frac{5}{3} r^3 - \frac{r^4}{R} \right]$$

According to Gauss's law,

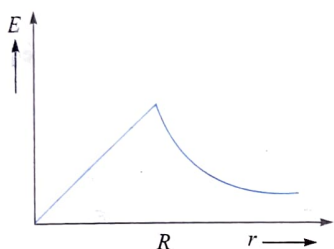
$$\phi = \int \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0} \Rightarrow E \cdot 4\pi r^2 = \frac{Q_{in}}{\epsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{\pi \rho_0}{\epsilon_0} \left[\frac{5}{3} r^3 - \frac{r^4}{R} \right]$$

$$E = \frac{\pi \rho_0 r^3}{4\pi r^2 \epsilon_0} \left[\frac{5}{3} - \frac{r}{R} \right]$$

$$E = \frac{\rho_0 r}{4\epsilon_0} \left[\frac{5}{3} - \frac{r}{R} \right]$$

3. (3) For uniformly charged sphere



$$E = \frac{1}{4\pi\epsilon_0} \frac{Qr}{R^3} \quad (\text{For } r < R)$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{R^2} \quad (\text{For } r = R)$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \quad (\text{For } r > R)$$

The variation of E with distance r from the centre is shown in given figure.

4. (3) The electric field inside the charged sphere,

$$E = \frac{1}{4\pi\epsilon_0} \frac{qr}{R^3} = \frac{\rho r}{3\epsilon_0}$$

Hence, statement 2 is true.

$$\text{Potential inside the sphere, } V = \frac{V_{\text{surface}}}{2} \left[3 - \frac{r^2}{R^2} \right]$$

$$\text{Potential at the centre of the sphere, } V_C = \frac{3V_{\text{surface}}}{2}$$

Potential at the surface of the sphere,

$$V_S = \frac{1}{4\pi\epsilon_0} \frac{q}{R} = \frac{\rho R^2}{3\epsilon_0}$$

When a charge q is taken from the centre to the surface, the charge in potential energy,

$$\Delta U = q(V_C - V_S) = q \left(\frac{3V_S}{2} - V_S \right)$$

$$\Rightarrow \Delta U = q \frac{V_S}{2} = q \left(\frac{\rho R^2}{3\epsilon_0} \right) = \frac{\rho q R^2}{6\epsilon_0}$$

Hence statement 1 is false.

$$5. (1) \oint \vec{E} \cdot d\vec{s} = \frac{q_{in}}{\epsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{Q + \int_{r=a}^r \rho dV}{\epsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{Q + \int_a^r \frac{A}{r} 4\pi r^2 dr}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0 r^2} \left[Q + 4\pi A \left(\frac{r^2 - a^2}{2} \right) \right]$$

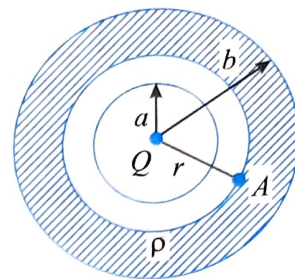
$$E = \frac{1}{4\pi\epsilon_0 r^2} \left[Q + 2\pi A(r^2 - a^2) \right]$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2} + \frac{A(r^2 - a^2)}{2\epsilon_0 r^2} = \frac{Q}{4\pi\epsilon_0 r^2} + \frac{A}{2\epsilon_0} - \frac{Aa^2}{2\epsilon_0 r^2}$$

If E is constant it should be independent of r

$$\therefore \frac{Q}{4\pi\epsilon_0 r^2} = \frac{Aa^2}{2\epsilon_0 r^2}$$

$$\text{which gives, } A = \frac{Q}{2\pi a^2}$$



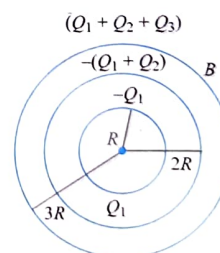
JEE Advanced

Single Correct Answer Type

1. (1) Total charge enclosed by cube is $-2C$. Hence, electric flux through the cube is $-2C/\epsilon_0$.

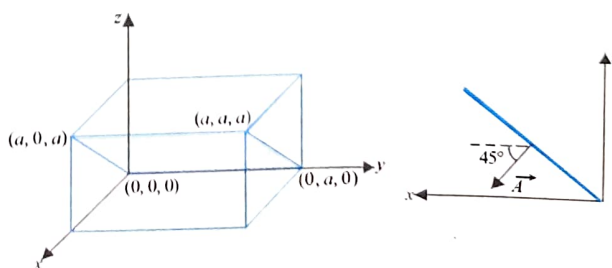
$$2. (2) \frac{Q_1}{4\pi R^2} = \frac{Q_1 + Q_2}{4\pi(2R)^2} = \frac{Q_1 + Q_2 + Q_3}{4\pi(3R)^2}$$

$$\text{or } Q_1 : Q_2 : Q_3 :: 1 : 3 : 5$$



$$3. (1) \text{ Pressure} = \frac{\sigma^2}{2\epsilon_0} \text{ and force} = \frac{\sigma^2}{2\epsilon_0} \times \pi R^2$$

$$4. (3) \vec{A} = A \cos 45^\circ \hat{i} - A \sin 45^\circ \hat{k} \\ = \frac{\sqrt{2}}{2} a^2 \hat{i} - \frac{\sqrt{2}}{2} a^2 \hat{k} = a^2 \hat{i} - a^2 \hat{k}$$



$$\phi = \vec{E} \cdot \vec{A} = E_0 \hat{i} \cdot (a^2 \hat{i} - a^2 \hat{k}) = E_0 a^2$$

5. (3)

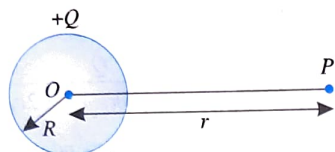
(1) This option is wrong, because flux due a charge through a Gaussian surface (in which the charge is present) will depend upon the shape and size of the surface.

(2) It is almost impossible to calculate the field distribution around an electric dipole using Gauss's law because proper symmetry will not be formed.

(3) This is obvious.

(4) This option is not correct because it is not given that charge is moving slowly.

6. (3) Electric field due to uniformly charged dielectric solid sphere of radius R at a point P is given by $\frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{R^3} r$, $r \leq R$



$$\frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r^2}, r > R$$

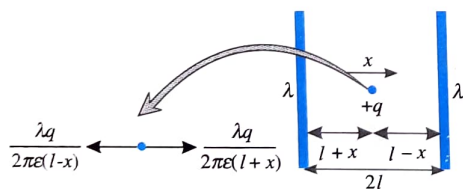
For solid sphere 1, $E_1 = \frac{1}{4\pi\epsilon_0} \frac{Q}{R^2}$

For solid sphere 2, $E_2 = \frac{1}{4\pi\epsilon_0} \frac{2Q}{R^2} = 2E_1$

For solid sphere 3, $E_3 = \frac{1}{4\pi\epsilon_0} \frac{(4Q)R}{(2R)^3} = \frac{E_1}{2}$

$$\therefore E_1 : E_2 : E_3 = 1 : 2 : \frac{1}{2} = 2 : 4 : 1$$

7. (3) For positive charge: If the charge displaced towards right by a distance x



Hence restoring force on the charge

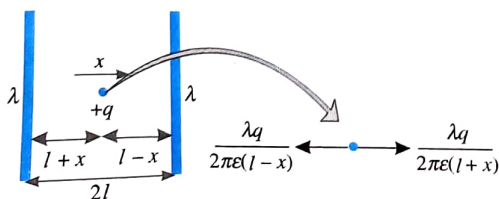
$$F_{\text{rest}} = \frac{\lambda q}{2\pi\epsilon_0(l-x)} - \frac{\lambda q}{2\pi\epsilon_0(l+x)}$$

$$= \frac{\lambda q(2x)}{2\pi\epsilon_0(l^2 - x^2)}$$

$$\Rightarrow F_{\text{rest}} \approx \frac{\lambda q x}{\pi\epsilon_0 l^2} \Rightarrow F_{\text{rest}} \propto x$$

$\Rightarrow$ It executes SHM.

For negative charge:



If we displace the charge towards right. The force in the direction of displacement increases. Hence the net force on the charge particle will be towards right hence the particle keeps going towards right.

8. (4) Since the sphere carries a uniform charge density it will be a non-conducting sphere. Hence, no concept of conducting sphere/cavity should be used.

For a spherical cavity, in a uniformly charged sphere, we use the standard result that:

$$\vec{E} = \frac{\rho \vec{a}}{3\epsilon_0}$$

where $\vec{a}$ = position vector of centre of cavity w.r.t. centre of sphere. Hence option (4) is correct.

Multiple Correct Answers Type

1. (1), (3), (4)

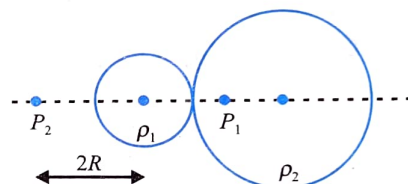
Position of all the charges are symmetric about the planes $x = +a/2$ and $x = -a/2$. So net electric flux through them will be same. Similarly, flux through $y = +a/2$ is equal to flux through $y = -a/2$.

$$\phi = \frac{q_{\text{in}}}{\epsilon_0} = \frac{3q - q - q}{\epsilon_0} = \frac{q}{\epsilon_0}$$

By symmetry flux through $z = +q/2$ is equal to flux through $x = +q/2$.

2. (2), (4)

At point $P_1 = \frac{1}{4\pi\epsilon_0} \frac{\rho_1(4/3)\pi R^3}{4R^2} = \frac{\rho_1 R}{3\epsilon_0}$



$$\frac{\rho_1 R}{12} = \frac{\rho_2 R}{3} \text{ or } \frac{\rho_1}{\rho_2} = 4$$

At point P_2

$$\frac{\rho_1(4/3)\pi R^3}{(2R)^2} + \frac{\rho_2(4/3)\pi 8R^3}{(5R)^2} = 0$$

$$\text{or } \frac{\rho_1}{\rho_2} = -\frac{32}{25}$$

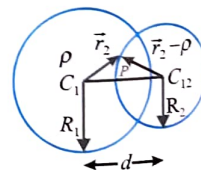
3. (3), (4)

In triangle PC_1C_2 , $\vec{r}_2 = \vec{d} + \vec{r}_1$

The electrostatic field at point P is

$$\vec{E} = \frac{K \left(\rho \frac{4}{3} \pi R_1^3 \right) \vec{r}_2}{R_1^3} - \frac{K \left(\rho \frac{4}{3} \pi R_2^3 \right) (\vec{r}_1)}{R_2^3}$$

$$= K \rho \frac{4}{3} \pi (\vec{r}_2 - \vec{r}_1) = \frac{\rho}{3\epsilon_0} \vec{d}$$



4. (1), (2), (3)

We have $E_1(r_0) = E_2(r_0) = E_3(r_0)$ (given)

$$\therefore \frac{1}{4\pi\epsilon_0} \frac{Q}{r_0^2} = \frac{\lambda}{2\pi\epsilon_0 r_0} = \frac{\sigma}{2\epsilon_0}$$

$$\text{Then } \frac{1}{4\pi\epsilon_0} \frac{Q}{r_0^2} = \frac{\sigma}{2\epsilon_0}$$

$$\text{or } Q = 2\sigma\pi r_0^2$$

Hence, option (1) is correct.

$$\text{Now, } \frac{\lambda}{2\pi\epsilon_0 r_0} = \frac{\sigma}{2\epsilon_0} \text{ or } r_0 = \frac{\lambda}{\pi\sigma}$$

Hence option (2) is correct.

$$\text{At } r = \frac{r_0}{2} \\ E_1\left(\frac{r_0}{2}\right) = \frac{1}{4\pi\epsilon_0} \frac{Q}{(r_0/2)^2} = \frac{4}{4\pi\epsilon_0} \frac{Q}{r_0^2} = 4E_1(r_0)$$

$$\text{or } E_1(r_0) = \frac{1}{4} E_1\left(\frac{r_0}{2}\right) \\ E_2\left(\frac{r_0}{2}\right) = \frac{\lambda}{2\pi\epsilon_0(r_0/2)} = \frac{2\lambda}{2\pi\epsilon_0 r_0} = 2E_2(r_0)$$

$$\text{or } E_2(r_0) = \frac{1}{2} E_2\left(\frac{r_0}{2}\right)$$

$$\therefore E_1(r_0) = E_2(r_0)$$

$$\therefore \frac{1}{4} E_1 = \frac{1}{2} E_2\left(\frac{r_0}{2}\right)$$

$$E_1\left(\frac{r_0}{2}\right) = 2E_2\left(\frac{r_0}{2}\right)$$

Hence, option (3) is correct.

$$E_3\left(\frac{r_0}{2}\right) = \frac{\sigma}{2\epsilon_0} = E_3(r_0)$$

$$E_2(r_0) = E_3(r_0)$$

$$\therefore \frac{1}{2} E_2\left(\frac{r_0}{2}\right) = E_3\left(\frac{r_0}{2}\right)$$

$$\text{or } E_2\left(\frac{r_0}{2}\right) = 2E_3\left(\frac{r_0}{2}\right)$$

Hence, option (4) is incorrect.

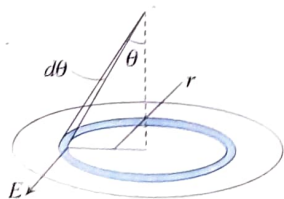
5. (1), (2)

Every point on circumference of flat surface is at equal distance from point charge.

Hence circumference is equipotential.

$\therefore$ Option (1) is correct.

Flux passing through curved surface = - flux passing through flat surface.



$$(d\phi)_{\text{through the ring}} = E \cos\theta \cdot dA$$

$$= \frac{1}{4\pi\epsilon_0} \frac{Q}{(\sqrt{r^2 + R^2})^2} \frac{R}{\sqrt{R^2 + r^2}} \cdot 2\pi r dr$$

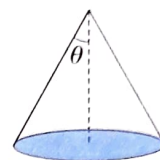
$$\therefore \int d\phi = \frac{QR}{4\pi\epsilon_0} 2\pi \int_0^R \frac{r dr}{(R^2 + r^2)^{3/2}} = \frac{q}{2\epsilon_0} \left(1 - \frac{1}{\sqrt{2}}\right)$$

$$\therefore \text{Flux through curved surface} = -\frac{q}{2\epsilon_0} \left(1 - \frac{1}{\sqrt{2}}\right)$$

Note: Flux through surface can be calculated using concept of solid angle.

$$\Omega = 2\pi(1 - \cos\theta) = 2\pi\left(1 - \frac{1}{\sqrt{2}}\right)$$

$$\therefore \text{Solid angle subtended} = 2\pi\left(1 - \frac{1}{\sqrt{2}}\right)$$



$$\text{The flux } (\phi) \text{ for } 4\pi \text{ solid angle} = \frac{q}{\epsilon_0}$$

$$\text{The flux } (\phi) \text{ for } 2\pi\left(1 - \frac{1}{\sqrt{2}}\right)$$

$$\text{solid angle} = \frac{q}{4\pi\epsilon_0} \cdot 2\pi\left(1 - \frac{1}{\sqrt{2}}\right) = \frac{q}{2\epsilon_0} \left(1 - \frac{1}{\sqrt{2}}\right)$$

$\therefore$ Option (2) is correct.

6. (1), (2)

Field due to infinitely long straight wire is perpendicular to the wire and radially outward. Hence, $E_z = 0$.

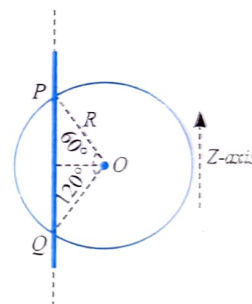
Hence option (B) is correct. Now let us calculate the electric flux through the shell. Hence the required electric flux

$$\phi = \frac{q_{in}}{\epsilon_0} = \frac{\lambda\sqrt{3}R}{\epsilon_0} = \frac{\sqrt{3}R\lambda}{\epsilon_0}$$

Hence option (1) is correct.

According to Gauss's law

$$\text{Total flux} = \oint \vec{E} \cdot d\vec{S} = \frac{q_{in}}{\epsilon_0}$$



Matrix Match Type

1. (2)

(1) Electric field due to point charge

$$E = \frac{KQ}{d^2} \Rightarrow E \propto \frac{1}{d^2}$$

(2) Electric field due to dipole

$$E = \frac{kp}{d^3} \Rightarrow E \propto \frac{1}{d^3} \text{ for dipole}$$

(3) Electric field due to line charge

$$E = \frac{2k\lambda}{d} \Rightarrow E \propto \frac{1}{d}$$

(4) From resonance

(5) Electric field due to sheet

$$E = \frac{\sigma}{2\epsilon_0} = \text{constant}$$

E is independent of d .

Numerical Value Type

1. (2) $\rho = kr^{-a}$

$$E \left(r - \frac{R}{2} \right) = \frac{1}{8} E (r - R)$$

$$\frac{q_{\text{enclosed}}}{4\pi\epsilon_0 (R/2)^2} = \frac{1}{8} \frac{Q}{4\pi\epsilon_0 R^2}$$

or $32q_{\text{enclosed}} = Q$

$$q_{\text{enclosed}} = \frac{Q}{32}$$

or $q_{\text{enclosed}} = \int_0^{R/2} kr^{-a} 4\pi r^2 dr = \frac{4\pi k}{(a+3)} \left(\frac{R}{2} \right)^{(a+3)}$

or $Q = \frac{4\pi k}{(a+3)} R^{(a+3)}$

or $\frac{Q}{q_{\text{enclosed}}} = 2^{(a+3)}$

or $2^{a+3} = 32$

or $a = 2$

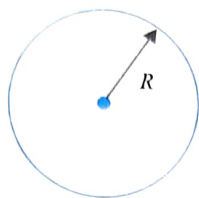
2. (6) $E_1 = \frac{\lambda}{2\pi\epsilon_0 2R} = \frac{\rho\pi R^2}{4\pi\epsilon_0 R} = \frac{\rho R}{4\epsilon_0}$

$$E_2 = \frac{1}{4\pi\epsilon_0} \frac{\rho \cdot \frac{4}{3} \pi \cdot \frac{R^3}{8}}{(2R)^2} = \frac{\rho R}{\epsilon_0 \cdot 24 \times 4}$$

$$E_1 - E_2 = \frac{\rho R}{4\epsilon_0} - \frac{\rho R}{\epsilon_0 \cdot 24 \times 4} = \frac{\rho R}{4\epsilon_0} \left[1 - \frac{1}{24} \right]$$

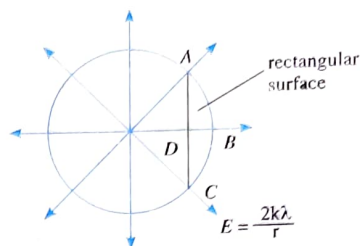
$$= \frac{23\rho R}{96\epsilon_0} = \frac{23\rho R}{16k\epsilon_0} \therefore k = 6$$

3. (6) Suppose that an infinite, charged wire is placed parallel to a rectangular area. Then, number of field lines passing through

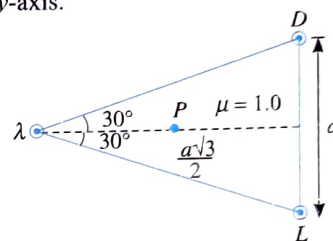


the rectangular area (denoted by ADC in figure) is equal to number of field lines passing through portion ABC of cylindrical surface.

Hence flux through both of them will be the same.



Let us redraw the figure from the point of view of an observer sitting y-axis.



Consider a cylindrical surface of length L , centred on the

charged wire, having a radius $r = \sqrt{\left(\frac{a}{2} \right)^2 + \left(\frac{a\sqrt{3}}{2} \right)^2}$.

From the figure, it is clear that the flux passing through the rectangular area is equal to $\frac{1}{6}$ of the flux passing through this cylindrical surface.

So, $\phi = \frac{1}{6} \cdot \frac{Q_{\text{enc}}}{\epsilon_0}$

$$\Rightarrow \phi = \frac{1}{6} \cdot \frac{\lambda L}{\epsilon_0}$$

$\therefore n = 6$

Concept Application Exercises

Exercise 3.1

1. Yes (a) Potential will increase by 100 V.
(b) No effect on potential difference.

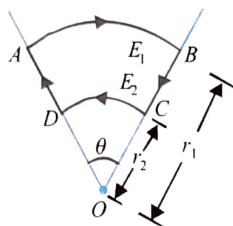
2. For closed path, net work is zero.

$$W_{AB} + W_{BC} + W_{CD} + W_{DA} = 0$$

$$E_1 r_1 \theta + 0 - E_2 r_2 \theta + 0 = 0$$

$$\text{or } E_1 r_1 = E_2 r_2 \text{ or } E \propto \frac{1}{r}$$

$$3. V_f - V_i = \int_i^f \vec{E} \cdot d\vec{l}$$



In a closed path, $V_i = V_f$. Hence, $\int_i^f \vec{E} \cdot d\vec{l}$ in a close path will be zero.

$$4. V = \frac{W_{\text{ext}}}{q} = \frac{-40 \mu\text{J}}{2 \mu\text{C}} = -20 \text{ V}$$

$$5. (W_{\text{ext}})_{\infty P} = -(W_{\text{cl}})_{\infty P} = (W_{\text{cl}})_{P\infty} = 10 \mu\text{J}$$

Because $\Delta KE = 0$,

$$V_P = \frac{(W_{\text{ext}})_{\infty P}}{q} = \frac{10 \mu\text{J}}{10 \mu\text{C}} = 1 \text{ V}$$

So if now the charge is doubled and taken from infinity, then

$$1 = \frac{(W_{\text{ext}})_{\infty P}}{20 \mu\text{C}} \text{ or } (W_{\text{ext}})_{\infty P} = 20 \mu\text{J}$$

$$(W_{\text{cl}})_{\infty P} = -20 \mu\text{J}$$

$$6. W_{\text{cl}} = \Delta K = K_f - 0$$

$$(W_{\text{cl}})_{P \rightarrow \infty} = qV_P = 60 \mu\text{J}$$

$$\text{So } K_f = 60 \mu\text{J}$$

7. (a) The potential difference between two points in electric field is

$$|\Delta V_{AB}| = -\vec{E} \cdot \vec{AB} = Ed \cos \theta = E \cdot d$$

$$= Ed = 20 \times 2 \times 10^{-2} = 0.4$$

$$\text{So } V_A - V_B = 0.4 \text{ V}$$

Negative because in the direction of electric field, potential always decreases.

$$(b) |\Delta V_{BC}| = Ed = 20 \times 2 \times 10^{-2} = 0.4$$

d is the distance between the equipotential passing through B and C , so $V_B - V_C = 0.4 \text{ V}$

$$(c) |\Delta V_{CA}| = Ed = 20 \times 4 \times 10^{-2} = 0.8$$

$$\text{So } V_C - V_A = -0.8 \text{ V}$$

Negative because in the direction of electric field, potential always decreases.

$$(d) |\Delta V_{DC}| = Ed = 20 \times 0 = 0$$

$$\text{So } V_D - V_C = 0$$

Because the effective distance between D and C is zero.

Also potential difference between C and D is zero as both the points lie on same equipotential.

$$(e) |\Delta V_{AD}| = Ed = 20 \times 4 \times 10^{-2} = 0.8$$

Here d is the distance between equipotential passing through A and D .

$$\text{So } V_A - V_D = 0.8 \text{ V}$$

Because in the direction of electric field, potential always decreases.

- (f) The order of potential is

$$V_A > V_B > V_C = V_D$$

8. Here, the given field is uniform (constant). So,

$$dV = -\vec{E} \cdot d\vec{r}$$

$$\text{or } V_{ab} = V_a - V_b = -\int_b^a \vec{E} \cdot d\vec{r}$$

$$= -\int_{(2,1,-2)}^{(1,-2,1)} (2\hat{i} + 3\hat{j} + 4\hat{k}) \cdot (dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$= -\int_{(2,1,-2)}^{(1,-2,1)} (2dx + 3dy + 4dz)$$

$$= -[2x + 3y + 4z]_{(2,1,-2)}^{(1,-2,1)} = -1 \text{ V}$$

9. If r is the distance of the charge Q from the point of observation, then

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} = 32 \quad \dots(i)$$

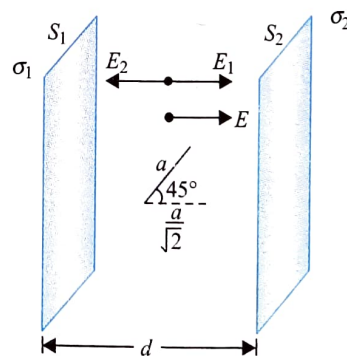
$$\text{and } V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r} = 16 \quad \dots(ii)$$

Dividing Eq. (ii) by Eq. (i), we get $r = 0.5 \text{ m}$ and putting the value of r in Eq. (ii), we get

$$Q = \frac{16 \times 0.5}{9 \times 10^9} = (8/9) \times 10^{-9} \text{ C}$$

$$10. \text{ As we see } E_1 = \frac{\sigma_1}{2\epsilon_0}, E_2 = \frac{\sigma_2}{2\epsilon_0}$$

$$E = E_1 - E_2 = \frac{\sigma_1 - \sigma_2}{2\epsilon_0}$$



$$W = q_0 E \times \frac{a}{\sqrt{2}} = \frac{q_0 a}{2\sqrt{2}\epsilon_0} (\sigma_1 - \sigma_2)$$

11. The potential difference between two points in electric field

$$V_2 - V_1 = \frac{W_{\text{ext}}(1 \rightarrow 2)}{q}$$

$$W_{\text{ext}}(1 \rightarrow 2) = q(V_2 - V_1)$$

$$= -2 \times 10^{-6} ((-25) - (-15))$$

$$= -2 \times 10^{-6} (-10) = 20 \times 10^{-6} \text{ J} = 20 \mu\text{J}$$

$$W_{\text{electrical}} = -W_{\text{external}} = -20 \mu\text{J}$$

12. Work done by external agent while moving from A to B

$$W_{\text{ext}}(A \rightarrow B) = q(V_B - V_A) = -W_{\text{elc}}(A \rightarrow B)$$

$$W_{\text{elc}}(A \rightarrow B) = -q(V_B - V_A)$$

$$4.8 \times 10^{-19} = -(-1.6 \times 10^{-19})(V_B - V_A)$$

$$\Rightarrow (V_B - V_A) = 3.0 \text{ V} = (V_C - V_A)$$

13. As electric field is a conservative field. Work done doesn't depend upon the path.

Here $W_{AB} = W_{AC}$ (As B and C are at same potentials)

$$W_{AC} = -q \int_{3r}^{2r} \vec{E} \cdot d\vec{r} = -q \int_{2r}^{3r} \frac{\lambda}{2\pi\epsilon_0 r} dr \cos 180^\circ$$

$$= \frac{q\lambda}{2\pi\epsilon_0} \int_{2r}^{3r} \frac{1}{r} dr = \frac{q\lambda}{2\pi\epsilon_0} \ln\left(\frac{3}{2}\right)$$

14. Near positive charge the potential will be positive irrespective of other charge in the surrounding.

Similarly, potential will be negative at a point very close to a negative charge. Hence, q_1 is positive and q_2 is negative.

According to the question

$$k \frac{q_2}{d/4} + k \frac{q_1}{3d/4} = 0 \quad \Rightarrow \quad q_1 = -3q_2$$

If $q_2 = -q$; then $q_1 = +3q$

$$\text{At } P \quad \frac{dV}{dx} = 0 \Rightarrow E_p = 0$$

Let distance of P from q_2 be x. For electric field to be zero

$$K \frac{|q_2|}{x^2} = K \frac{|q_1|}{(d+x)^2}$$

$$\Rightarrow (d+x)^2 = 3x^2 \Rightarrow d+x = \pm\sqrt{3}x$$

$$\text{Hence } x = \frac{d}{\sqrt{3}-1} \text{ [As negative sign is not acceptable]}$$

Exercise 3.2

$$1. \quad \vec{E} = -\left[\frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} + \frac{\partial V}{\partial z} \hat{k}\right]$$

$$\text{Here, } \frac{\partial V}{\partial x} = \frac{\partial}{\partial x}(2x+3y-z) = 2$$

$$\frac{\partial V}{\partial y} = \frac{\partial}{\partial y}(2x+3y-z) = 3$$

$$\frac{\partial V}{\partial z} = \frac{\partial}{\partial z}(2x+3y-z) = -1$$

$$\therefore \vec{E} = -2\hat{i} - 3\hat{j} + \hat{k}$$

2. E is larger where equipotential surfaces are closer. Electric field lines of force is perpendicular to equipotential surfaces. In the figure, we can see that for point B they are closer so E at point B is maximum.

3. Here $\vec{E} = E \cos \theta \hat{i} + E \sin \theta \hat{j}$

$$\vec{l} = \vec{OA} = d\hat{i} + d\hat{j}$$

$$-\Delta V = \vec{E} \cdot \vec{l}$$

$$= Ed \cos \theta + Ed \sin \theta = Ed(\cos \theta + \sin \theta)$$

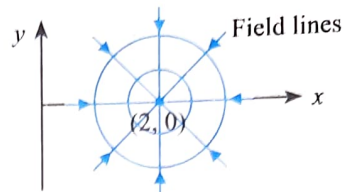
$$\therefore -(V_A - V_0) = Ed(\cos \theta + \sin \theta)$$

$$\text{or } V_0 - V_A = Ed(\cos \theta + \sin \theta)$$

$$4. \quad V = (x-2)^2 + y^2$$

Equipotentials are circles with centre at (2, 0)

Field lines are radial directed towards the centre of the circle.



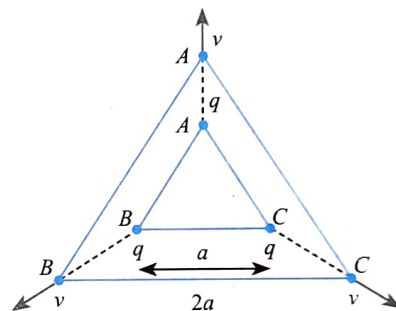
$$5. \quad V = \frac{1}{x} + \frac{1}{y} + \frac{2}{z}$$

$$\vec{E} = -\frac{\partial V}{\partial x} \hat{i} - \frac{\partial V}{\partial y} \hat{j} - \frac{\partial V}{\partial z} \hat{k} = \frac{1}{x^2} \hat{i} + \frac{1}{y^2} \hat{j} + \frac{2}{z^2} \hat{k}$$

$$\text{At } (1, 1, 1) \text{ m, } \vec{E} = \hat{i} + \hat{j} + 2\hat{k}$$

Resultant force on the particle in XY plane is, $\vec{F} = q(\hat{i} + \hat{j})$

$$\vec{a} = \frac{q}{m}(\hat{i} + \hat{j}) = \frac{10^{-12} \text{ C}}{10^{-12} \text{ kg}}(\hat{i} + \hat{j}) = (\hat{i} + \hat{j}) \text{ m/s}^2$$



Exercise 3.3

1. Take the illustration presented with the problem as an initial picture. No external horizontal forces act on the set of four balls, so its center of mass stays fixed at the location of the center of the square. As the charged balls 1 and 2 swing out and away from each other, balls 3 and 4 move up with equal y-components of velocity. The maximum kinetic energy point is illustrated. System energy is conserved, so

$$\frac{k_e q^2}{a} = \frac{k_e q^2}{3a} + \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

$$\text{or } \frac{2k_e q^2}{3a} = 2mv^2 \quad \text{or } v = \sqrt{\frac{k_e q^2}{3am}}$$

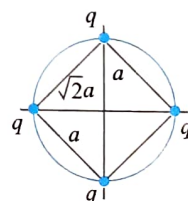
2. U_i is the electrical potential energy in square arrangement. So

$$U_i = \frac{4q^2}{4\pi\epsilon_0 a} + \frac{2q^2}{4\pi\epsilon_0 \sqrt{2}a}$$

U_f is the electric potential energy of circular arrangement of charge.

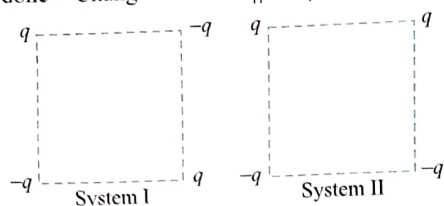
$$U_f = \frac{4q^2}{4\pi\epsilon_0 \sqrt{2}a} + \frac{2q^2}{4\pi\epsilon_0 (2a)}$$

$$W_{\text{ext}} = \Delta U = U_f - U_i = -\frac{q^2}{4\pi\epsilon_0 a}(3 - \sqrt{2})$$



3. Let U_I be the potential energy of system I and U_{II} be the potential energy of system II.

Work done = Change in PE = $U_{II} - U_I$



$$U_I = -\frac{4q^2}{4\pi\epsilon_0 a} + \frac{2q^2}{4\pi\epsilon_0 \sqrt{2}a}$$

$$U_{II} = \frac{-2q^2}{4\pi\epsilon_0 \sqrt{2}a}$$

$$W = \frac{-2q^2}{4\pi\epsilon_0 \sqrt{2}a} + \frac{4q^2}{4\pi\epsilon_0 a} - \frac{2q^2}{4\pi\epsilon_0 \sqrt{2}a}$$

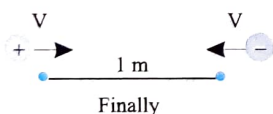
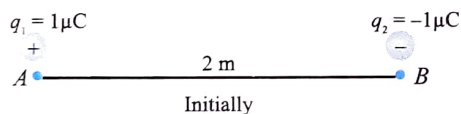
$$= \frac{q^2}{4\pi\epsilon_0 a} (4 - 2\sqrt{2})$$

$$4. W_{\text{external}} = \Delta \text{PE} = \frac{1}{4\pi\epsilon_0} \frac{q^2}{a} \left[-\frac{3}{1} + \frac{3}{\sqrt{2}} - \frac{1}{\sqrt{3}} \right] \times 4$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q^2}{a} \times \frac{4}{\sqrt{6}} [3\sqrt{3} - 3\sqrt{6} - \sqrt{2}]$$

5. Applying conservation of mechanical energy $\Delta K + \Delta U = 0$

$$\text{or } (K_f - K_i) + (U_f - U_i) = 0 \Rightarrow (K_f - K_i) = (U_i - U_f)$$



$$U_i = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_i} = -\frac{1}{4\pi\epsilon_0} \frac{(1 \times 10^{-6})^2}{2} \text{ J}$$

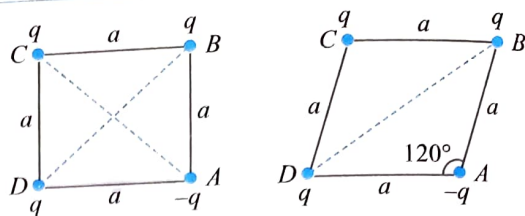
$$\Rightarrow \left(2 \cdot \frac{1}{2} (50 \times 10^{-3})^2 v^2 - 0 \right) = \frac{10^{-12}}{4\pi\epsilon_0} \left(1 - \frac{1}{2} \right)$$

$$= 9 \times 10^9 \times 10^{-12} \cdot \left(\frac{1}{2} \right)$$

$$\Rightarrow v^2 = \frac{9}{100} \quad \text{or} \quad v = \frac{3}{10} \text{ m/s}$$

6. (a) **In case of Fig. (i):** There will be six terms in the expression of potential energy for the system, as there are six pairs of interactions.

U_{AB} and U_{AD} are negative whereas U_{BC} and U_{CD} are positive terms of same magnitude hence they will be cancelled out. Similarly U_{BD} and U_{AC} cancel out. Hence PE of the system is zero.



- (b) **In case of Fig. (ii):** U_{AB} and U_{AD} cancel out with U_{BC} and U_{CD}

Now considering the interactions

$$\text{Length } BD = 2a \cos 30^\circ = \sqrt{3}a$$

$$\text{And } AC = 2a \cos 60^\circ = a$$

$$\therefore U = U_{BD} + U_{AC} = K \frac{q^2}{\sqrt{3}a} - K \frac{q^2}{a} = -\frac{Kq^2}{a} \left(1 - \frac{1}{\sqrt{3}} \right)$$

Hence the work done by the external agent in changing the configuration.

$$W_{\text{ext}} = U_2 - U_1 = -\frac{Kq^2}{a} \left(1 - \frac{1}{\sqrt{3}} \right)$$

7. Applying conservation of mechanical energy

$$\Delta K + \Delta U = 0 \quad \text{or} \quad (K_f - K_i) + (U_f - U_i) = 0$$

$$(0 - 100 \times 1.6 \times 10^{-19}) + 1.6 \times 10^{-19} \Delta V = 0$$

$$\Delta V = Ed = \left(\frac{\sigma}{\epsilon_0} \right) d = \left(\frac{2.2 \times 10^{-6}}{8.8 \times 10^{-12}} \right) d$$

$$(0 - 100 \times 1.6 \times 10^{-19}) + 1.6 \times 10^{-19} \cdot \left(\frac{2.2 \times 10^{-6}}{8.8 \times 10^{-12}} \right) d = 0$$

$$d = 100 \times \frac{8.8 \times 10^{-12}}{2.2 \times 10^{-6}} = 40 \times 10^{-4} \text{ m} = 40 \text{ mm}$$

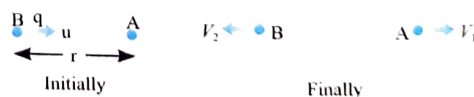
8. (a) KE will be least when PE of the system is maximum, i.e. when charges are closest to each other. In this situation both charges will have same velocity (v).

$$mv + mv = mv \Rightarrow v = \frac{u}{2}$$

$$\therefore KE_{\text{min}} = \frac{1}{2} m \left(\frac{u}{2} \right)^2 + \frac{1}{2} m \left(\frac{u}{2} \right)^2 = \frac{1}{4} mu^2$$

- (b) For making conservation of mechanical energy and linear momentum true, A will move to right with velocity u and B will be at rest.

9. Let the final velocity be V_1 and V_2 as shown.



Conservation of linear momentum

$$mV_1 - mV_2 = mu \Rightarrow V_1 - V_2 = u \quad \dots(i)$$

$$\text{Energy conservation } \frac{1}{2} mV_1^2 + \frac{1}{2} mV_2^2 = \frac{qQ}{4\pi\epsilon_0 r} + \frac{1}{2} mu^2$$

$$\Rightarrow mV_1^2 + m(V_1 - u)^2 = 2mu^2 \quad \left[\because r = \frac{qQ}{2\pi\epsilon_0 mu^2} \right]$$

$$\Rightarrow 2V_1^2 - 2uV_1 - u^2 = 0$$

$$\Rightarrow V_1 = \left(\frac{1 + \sqrt{3}}{2} \right) u$$

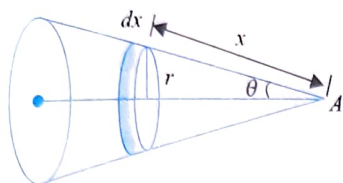
Exercise 3.4

1. $W_{\text{ext}} = \Delta U = U_f - U_i = q(V_A - V_\infty)$

$W_{\text{ext}} = q(V_A - 0) = qV_A$... (i)

Here V_A is electric potential at point A. The electric charge on the considered ring is

$dq = \frac{Q2\pi r dx}{\pi RL}$



Therefore, electric potential due to considered ring is

$dV = \frac{dq}{4\pi\epsilon_0 x}$

$dV = \frac{Q2\pi r dx}{\pi RL4\pi\epsilon_0 x}$

$\sin \theta = \frac{r}{x} = \frac{R}{L} \quad \therefore r = \frac{Rx}{L}$

$\therefore dV = \frac{2\pi Q \left(\frac{R}{L} x\right) dx}{\pi RL4\pi\epsilon_0 x}$

$V_A = \int dV = \frac{Q}{2\pi\epsilon_0 L^2} \int_0^L dx = \frac{Q}{2\pi\epsilon_0} L$

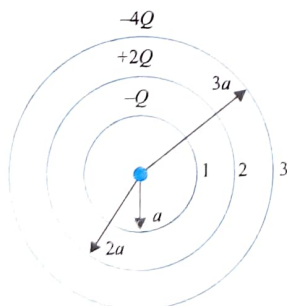
$\therefore W_{\text{ext}} = qV_A = \frac{qQ}{2\pi\epsilon_0 L}$ from Eq. (i)

2. (a) $r < a$

$V = V_{1,\text{in}} + V_{2,\text{in}} + V_{3,\text{in}}$

$= \frac{1}{4\pi\epsilon_0} \frac{(-Q)}{a} + \frac{1}{4\pi\epsilon_0} \frac{(2Q)}{2a} + \frac{1}{4\pi\epsilon_0} \frac{(-4Q)}{3a}$

$= \frac{1}{4\pi\epsilon_0} \frac{Q}{a} \left[-1 + 1 - \frac{4}{3} \right] = \frac{1}{4\pi\epsilon_0} \frac{Q}{a} \left[-\frac{4}{3} \right] = -\frac{Q}{3\pi\epsilon_0 a}$



(b) $a, r < 2a$

$V = V_{1,\text{out}} + V_{2,\text{out}} + V_{3,\text{out}}$

$= \frac{1}{4\pi\epsilon_0} \frac{(-Q)}{r} + \frac{1}{4\pi\epsilon_0} \frac{(2Q)}{2a} + \frac{1}{4\pi\epsilon_0} \frac{(-4Q)}{3a}$

$= \frac{1}{4\pi\epsilon_0} Q \left[-\frac{1}{r} + \frac{1}{a} - \frac{4}{3a} \right] = \frac{1}{4\pi\epsilon_0} Q \left[-\frac{1}{r} - \frac{1}{3a} \right]$

$= -\frac{1}{4\pi\epsilon_0} Q \left[\frac{1}{r} + \frac{1}{3a} \right]$

(c) $2a < r, 3a$

$V = V_{1,\text{out}} + V_{2,\text{out}} + V_{3,\text{in}}$

$= \frac{1}{4\pi\epsilon_0} \frac{(-Q)}{r} + \frac{1}{4\pi\epsilon_0} \frac{(2Q)}{r} + \frac{1}{4\pi\epsilon_0} \frac{(-4Q)}{3a}$

$= \frac{Q}{4\pi\epsilon_0} \left[-\frac{1}{r} + \frac{2}{r} - \frac{4}{3a} \right] = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{r} - \frac{4}{3a} \right]$

(d) $r > 3a$

$V = V_{1,\text{out}} + V_{2,\text{out}} + V_{3,\text{out}}$

$= \frac{1}{4\pi\epsilon_0} \frac{(-Q)}{r} + \frac{1}{4\pi\epsilon_0} \frac{(2Q)}{r} + \frac{1}{4\pi\epsilon_0} \frac{(-4Q)}{r}$

$= \frac{1}{4\pi\epsilon_0} \frac{Q}{r} [-1 + 2 - 4] = -\frac{3}{4\pi\epsilon_0} \frac{Q}{r}$

3. Potential of a drop

$V = \frac{1}{4\pi\epsilon_0 \epsilon_r} \frac{q}{r}$

or $q = 4\pi\epsilon_0 \epsilon_r r V$

Therefore, total charge

$Q = nq = 4\pi\epsilon_0 \epsilon_r r V n$

(n = total number of drops)

Also, $\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3$ or $n = \frac{R^3}{r^3}$

Hence, potential of the hollow conductor

$V' = \frac{1}{4\pi\epsilon_0 \epsilon_r} \frac{Q}{R} = \frac{1}{4\pi\epsilon_0 \epsilon_r} \times \frac{4\pi\epsilon_0 \epsilon_r r V n}{R}$

$= \frac{r}{R} \times V \times \left(\frac{R^3}{r^3} \right)$

$= \left(\frac{R}{r} \right)^2 V$

4. Work done = charge $\times$ difference in potential

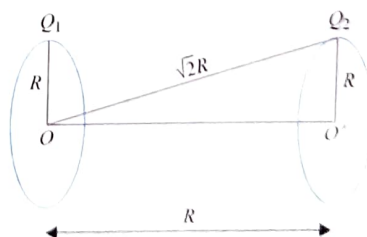
Let the particle move from O to O' (figure)

Potential at point O is

$V = \frac{1}{4\pi\epsilon_0} \frac{Q_1}{R} + \frac{Q_2}{\sqrt{2}R}$

Potential at point O' is

$V' = \frac{1}{4\pi\epsilon_0} \left[\frac{Q_2}{R} + \frac{Q_1}{\sqrt{2}R} \right]$



Difference in potential is

$\Delta V = V' - V$

$= \frac{1}{4\pi\epsilon_0} \left[\frac{Q_1}{R} + \frac{Q_2}{\sqrt{2}R} \right] - \frac{1}{4\pi\epsilon_0} \left[\frac{Q_2}{R} + \frac{Q_1}{\sqrt{2}R} \right]$

$= \frac{1}{4\pi\epsilon_0} \left[\frac{1}{R} (Q_1 - Q_2) + \frac{1}{\sqrt{2}R} (Q_2 - Q_1) \right]$

$$= \frac{1}{4\pi\epsilon_0} \left[\frac{Q_1 - Q_2}{R} \left(1 - \frac{1}{\sqrt{2}} \right) \right]$$

$$\therefore Wd = q\Delta V = q \frac{1}{4\pi\epsilon_0} \left[\frac{(Q_1 - Q_2)}{\sqrt{2}R} (\sqrt{2} - 1) \right]$$

5. (a) Potential of shell A

$$V_A = V_{\text{due to charge on A}} + V_{\text{due to charge on B}} + V_{\text{due to charge on C}}$$

$$= 0 + \frac{Q}{4\pi\epsilon_0 b} - \frac{Q}{4\pi\epsilon_0 c}$$

Potential of shell B

$$V_B = V_{\text{due to charge on A}} + V_{\text{due to charge on B}} + V_{\text{due to charge on C}}$$

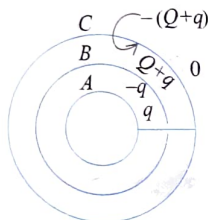
$$= 0 + \frac{Q}{4\pi\epsilon_0 b} - \frac{Q}{4\pi\epsilon_0 c}$$

Potential of shell C

$$V_C = V_{\text{due to charge on A}} + V_{\text{due to charge on B}} + V_{\text{due to charge on C}}$$

$$= 0 + \frac{Q}{4\pi\epsilon_0 c} - \frac{Q}{4\pi\epsilon_0 c} = 0$$

(b) After connecting inner and outer shells, we get



$$V_A = V_C$$

$$\text{or } \frac{q}{a} - \frac{q}{b} + \frac{Q+q}{b} - \frac{Q+q}{c} = \frac{q-q+Q+q}{c} - \frac{(Q+q)}{c}$$

$$\text{or } q = \frac{Qa(b-c)}{b(c-a)}$$

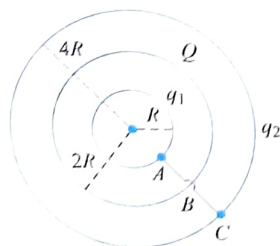
Final potential of A and C is zero.

$$V_B = k \left[\frac{q}{a} - \frac{q}{b} + \frac{Q+q}{b} - \frac{(Q+q)}{c} \right] = k(Q+q) \left[\frac{c-b}{bc} \right]$$

$$= k \left[Q + \frac{Qa(b-c)}{b(c-a)} \right] \left[\frac{c-b}{bc} \right] = kQ \left[\frac{(b-a)(c-b)}{b^2(c-a)} \right]$$

6. (a) Let the charges on A and C be q_1 and q_2 , respectively (figure).

From conservation of charge, we have $q_1 + q_2 = 0$.



Hence, $q_1 = -q_2$. Since A and C are connected by a conducting wire, so they have the same potential.

$$V_A = \text{potential of A}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q_1}{R} + \frac{1}{4\pi\epsilon_0} \frac{Q}{2R} + \frac{1}{4\pi\epsilon_0} \frac{q_2}{4R}$$

$$V_C = \text{potential of C}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q_1}{4R} + \frac{1}{4\pi\epsilon_0} \frac{Q}{4R} + \frac{1}{4\pi\epsilon_0} \frac{q_2}{4R}$$

Equalizing the potentials of A and C, i.e., $V_A = V_C$, we get

$$\frac{q_1}{R} + \frac{Q}{2R} + \frac{q_2}{4R} = \frac{q_1}{4R} + \frac{Q}{4R} + \frac{q_2}{4R}$$

$$\text{or } 4q_1 + 2Q = q_1 + Q$$

$$\text{or } q_1 = -Q/3$$

$$\text{Hence, } q_2 = Q/3.$$

$$(b) V_A = \frac{1}{4\pi\epsilon_0 R} \left[\frac{-Q}{3} + \frac{Q}{2} + \frac{Q}{12} \right] = \frac{Q}{16\pi\epsilon_0 R}$$

$$V_B = \frac{1}{4\pi\epsilon_0} \frac{q_1}{2R} + \frac{1}{4\pi\epsilon_0} \frac{Q}{2R} + \frac{1}{4\pi\epsilon_0} \frac{q_2}{4R}$$

$$= \frac{1}{8\pi\epsilon_0 R} \left[\frac{-Q}{3} + Q + \frac{Q}{6} \right] = \frac{5Q}{48\pi\epsilon_0 R}$$

7. Potential at center O due to induced charges is

$$V_1 = -\frac{q}{4\pi\epsilon_0 a} + \frac{q}{4\pi\epsilon_0 b}$$

$$V_\infty = 0$$

Small work done to transfer a small charge dq from center to infinity is

$$dW = dq(V_\infty - V_1) = \frac{1}{4\pi\epsilon_0} \left[\frac{1}{a} - \frac{1}{b} \right] q dq$$

$$\text{or } W = \int dW = \frac{q^2}{8\pi\epsilon_0} \left[\frac{1}{a} - \frac{1}{b} \right]$$

$$8. W_{\text{net}} = \Delta \text{KE or } -mgl - q_0 El = 0 - \frac{1}{2} mv^2 \text{ or } v = \sqrt{\frac{2Mgl + 2q_0 El}{m}}$$

$$\text{when } E = \frac{\sigma}{\epsilon_0}$$

9. If the ball has to just complete the circle, then the tension must vanish at the topmost point, i.e., A. From Newton's second law

$$T_2 + mg - \frac{q^2}{4\pi\epsilon_0 l^2} = \frac{mv^2}{l} \quad \dots(i)$$

At the topmost point, $T_2 = 0$. So

$$mg - \frac{q^2}{4\pi\epsilon_0 l^2} = \frac{mv^2}{l} \quad \dots(ii)$$

From energy conservation,

Energy at lowest point = Energy at topmost point

$$\text{or } \frac{1}{2} mu^2 = \frac{1}{2} mv^2 + mg2l \quad \dots(iii)$$

$$\text{or } v^2 - u^2 = -4gl \quad \dots(iv)$$

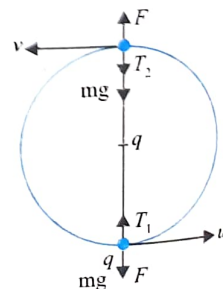
$$\text{From Eq. (ii), } v^2 = gl - \frac{q^2}{4\pi\epsilon_0 ml} \quad \dots(v)$$

From Eqs. (iv) and (v),

$$u = \sqrt{5gl - \frac{q^2}{4\pi\epsilon_0 ml}} = \left(\frac{275}{8} \right)^{1/2} = 5.86 \text{ ms}^{-1}$$

10. Potential at the surface of a sphere $V = \frac{1}{4\pi\epsilon_0} \frac{Q}{R}$

$$Q = V \cdot 4\pi\epsilon_0 R = 9 \times 10^5 \times \frac{1}{9 \times 10^9} \times 2 \times 10^{-2}$$



$$= 2 \times 10^{-6} \text{ C}$$

$$W_{\text{ext}} = \Delta U = q(V_A - V_C)$$

$$= 1 \times \left[\frac{1}{4\pi\epsilon_0} \frac{2 \times 10^{-6}}{25 \times 10^{-2}} - \frac{1}{4\pi\epsilon_0} \frac{2 \times 10^{-6}}{60 \times 10^{-2}} \right]$$

$$= \frac{1}{4\pi\epsilon_0} \times \frac{2 \times 10^{-6}}{10^{-2}} \left[\frac{1}{25} - \frac{1}{60} \right]$$

$$W_{\text{ext}} = \frac{21}{5} \times 10^4 \text{ J}$$

Exercise 3.5

- (a) True
(b) True
- For 2 and 4, $\tau = pE \sin \theta$, $U = -pE \cos \theta$
For 1 and 3, $\tau = pE \sin(180 - \theta) = pE \sin \theta$
 $U = -pE \cos(180 - \theta) = pE \cos \theta$
Hence, torque is same for all, and U is greater for 1 and 3 than 2 and 4.

- (a) Electric field will do positive work, if dipole rotates such that its angle decreases with E .

- (b) 2 and 4 orientations are identical. Hence, work done will be same.

- (a) $\tau_{\text{max}} = pE = 6.2 \times 10^{-30} \times 1.5 \times 10^4 = 9.3 \times 10^{-26} \text{ Nm}$

- (b) $W = 2pE = 2 \times 9.3 \times 10^{-26} = 1.86 \times 10^{-25} \text{ J}$

- Here, $q = \pm 8 \times 10^{-9} \text{ C}$; $2a = 4 \text{ cm} = 0.04 \text{ m}$; $\tau = 4\sqrt{3} \text{ Nm}$, and $\theta = 60^\circ$.

- (a) Now, $p = q(2a) = 8 \times 10^{-9} \times 0.04 = 3.2 \times 10^{-10} \text{ Cm}$

Torque on the electric dipole is $\tau = pE \sin \theta$

$$\text{or } E = \frac{\tau}{p \sin \theta} = \frac{4\sqrt{3}}{3.2 \times 10^{-10} \times \sin 60^\circ}$$

$$= \frac{4\sqrt{3} \times 2}{3.2 \times 10^{-10} \times \sqrt{3}} = 2.5 \times 10^{10} \text{ NC}^{-1}$$

- (b) Potential energy of the electric dipole is

$$U = -pE \cos \theta$$

$$= -3.2 \times 10^{-10} \times 2.5 \times 10^{10} \times \cos 60^\circ$$

$$= -3.2 \times 2.5 \times 0.5 = -4 \text{ J}$$

- Here, $E = 10^5 \text{ NC}^{-1}$; $q = 1 \text{ mC} = 10^{-6} \text{ C}$; $2a = 2 \text{ cm} = 0.02 \text{ m}$

$$p = q(2a) = 10^{-6} \times 0.02 = 2 \times 10^{-8} \text{ Cm}$$

- (a) The torque acting on a dipole is given by

$$\tau = pE \sin \theta$$

Torque is maximum when $\sin \theta = 1$.

$$\tau_{\text{max}} = 2 \times 10^{-8} \times 10^5 \times 1 = 0.002 \text{ Nm}$$

- (b) Now, work done in rotating the dipole from $\theta = \theta_1$ to $\theta = \theta_2$ is given by

$$W = pE (\cos \theta_1 - \cos \theta_2)$$

Here, $\theta_1 = 0^\circ$, $\theta_2 = 180^\circ$. Therefore,

$$W = 2 \times 10^{-8} \times 10^5 (\cos 0^\circ - \cos 180^\circ)$$

$$= 0.002[1 - (-1)] = 0.004 \text{ J}$$

- Potential energy of dipole will be zero because electric field due to point charges at dipole will cancel out. Also electric field due to any point charge is perpendicular to the dipole moment.

- (a) $U = -pE \cos \theta = pE \cos(180 - \theta)$

The more is U , the more is θ . So $\angle 4 > \angle 3 > \angle 1 > \angle 2$.

- (b) If angle between p and E is more closer to 90° , then torque is more. If angle is closer to 0° or 180° , then torque is less. It means the lesser is the magnitude of potential energy, the more is torque and vice versa. So $3 > 1 = 4 > 2$.

$$9. V = \frac{kp \cos \theta}{r^2}$$

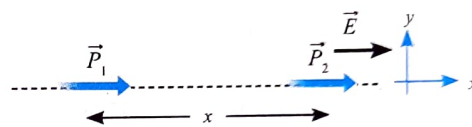
$$\text{For A, } \theta = 135^\circ; V_A = \frac{kp \cos 135^\circ}{r^2} = \frac{-kp}{\sqrt{2} r^2}$$

$$\text{For B, } \theta = 45^\circ; V_B = \frac{kp \cos 45^\circ}{r^2} = \frac{Kp}{\sqrt{2} r^2}$$

$$W = q(V_B - V_A) = \frac{q\sqrt{2} k p}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{\sqrt{2} qp}{r^2}$$

- Potential energy of system is

$$U = -\vec{p}_2 \cdot \vec{E}_{21} = -p_2 \frac{1}{4\pi\epsilon_0} \frac{2p_1}{x^3} \cos 0 = -\frac{1}{4\pi\epsilon_0} \frac{2p_1 p_2}{x^3}$$



$$F = -\frac{\partial U}{\partial x} = -\frac{6}{4\pi\epsilon_0} \frac{p_1 p_2}{x^4} = -\frac{3}{2\pi\epsilon_0} \frac{p_1 p_2}{x^4}$$

F comes out to be negative, so it is an attractive force.

Exercises

Single Correct Answer Type

- (4) V is a scalar quantity, and E is a vector quantity.
- (2) Charge will be induced on A , but total charge on A will remain zero. Negative charge of A will be more closer to B than positive charge on A . So potential of B will decrease.
- (4) Potential decreases in the direction of electric field. So it depends on whether the lines of force are from A to B or from B to A .
- (4) Option (1) is wrong because force will be the same, but acceleration will be different because masses of electrons and protons are different.
Option (2) is wrong because at a point there can be only one potential.
Option (3) is wrong because charge always lies on the outer surface of a conductor.
Option (4) is correct because the whole conductor will be an equipotential body.
- (4) We know, $E = -\frac{dV}{dx}$.
At B , $dV/dx = 0$, hence $E_B = 0$.
At A , dV/dx is positive, hence E_A is negative.
At C , dV/dx is negative, hence E_C is positive.
- (2) If bob is positively charged, it will be repelled by unit positive charge and in second case, distance between charges may increase. Thus final energy in second case may be less than in first case. Hence, less work may be done in second case.
- (2) $W_{\text{el}} = q(V_i - V_f)$
or $6.4 \times 10^{-19} = -1.6 \times 10^{-19} (V_A - V_B)$
or $V_A - V_B = -4 \text{ V}$
or $V_A - V_C = -4 \text{ V} (\because V_B = V_C)$
or $V_C - V_A = 4 \text{ V}$



8. (2) $W_{AB} = W_{AC} + W_{CB}$
 W_{CB} should be zero, because in moving from C to B, we always move perpendicular to field. Hence, force applied by field and displacement will be at 90° .

$$W_{AC} = -e(V_C - V_A)$$

$$V_C - V_A = -E \times AC = -10 \times 4 = -40$$

$$\therefore W_{AC} = -e \times (-40) = 40e$$

$$\text{So } W_{AB} = 40e \text{ J} = 40 \text{ eV}$$

9. (1) $E = -\frac{dV}{dr}$ = negative of the slope of V - R graph.

$$10. (2) E_x = -\frac{-2V}{-2 \times 10^{-2} \text{ m}} = -100 \text{ Vm}^{-1}$$

$$E_y = -\frac{-2V}{1 \times 10^{-2} \text{ m}} = 200 \text{ Vm}^{-1}$$

Time-saving solution

Clearly, x -component is negative and y -component is positive.

11. (4) Let the charge on the smaller sphere be q . As the potential of both will be the same finally,

$$\frac{q}{r'} = \frac{Q-q}{r} \text{ or } q = \frac{Qr'}{r+r'}$$

$$12. (2) W_{eq} = q(V_A - V_B)$$

$$\text{or } 50 \times 10^{-6} = 2 \times 10^{-6} (V_A - V_B)$$

$$\text{or } V_A - V_B = 25 \text{ V}$$

$$13. (1) V_B - V_A = -\int_1^2 2dx + \int_2^1 3dy$$

$$= -[2(2-1) + 3(1-2)]$$

$$= -[2-3] = 1 \text{ V}$$

$$\text{Hence, } V_A - V_B = -1 \text{ V}$$

$$14. (4) E_x = -\frac{dV}{dx} = -4x = -4 \times 2 = -8$$

$$E_y = 0, E_z = 0$$

$$\text{Hence, } \vec{E} = -8\hat{i} \text{ NC}^{-1}$$

15. (1) Their potential will be same, i.e., $V_1 = V_2$

$$\text{or } \frac{kq_1}{R_1} = \frac{kq_2}{R_2} \text{ or } \frac{q_1}{q_2} = \frac{R_1}{R_2}$$

$$\text{or } \frac{E_1}{E_2} = \frac{kq_1/R_1^2}{kq_2/R_2^2} = \frac{q_1}{q_2} \left(\frac{R_2}{R_1} \right)^2$$

$$= \frac{R_1}{R_2} \left(\frac{R_2}{R_1} \right)^2 = \frac{R_2}{R_1}$$

$$16. (4) F = qE$$

$$W = qE \times 2 \cos 60^\circ$$

$$\text{or } 4 = 0.2E \times 2 \times \frac{1}{2} \text{ or } E = 20 \text{ NC}^{-1}$$

17. (3) Apply conservation of mechanical energy between points a and b

$$(KE + PE)_a = (KE + PE)_b$$

$$0 + \frac{k(3 \times 10^{-9})q_0}{0.01} - \frac{k(3 \times 10^{-9})q_0}{0.02}$$

$$= \frac{1}{2}mv^2 + \frac{k(3 \times 10^{-9})q_0}{0.02} - \frac{k(3 \times 10^{-9})q_0}{0.01}$$

$$\text{Put the values and get } v = 12\sqrt{15} = 46 \text{ ms}^{-1}$$

18. (3) Applying conservation of mechanical energy, we get

$$\frac{-9 \times 10^9 \times 10^{-12}}{2} = \frac{-9 \times 10^9 \times 10^{-12}}{1} + \frac{1}{2}(2m)v^2$$

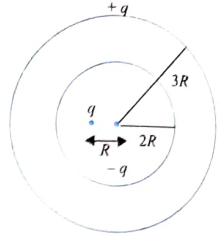
$$\text{or } v = 3/10 \text{ ms}^{-1}$$

19. (1) Electric field due to large metal plate is σ/ϵ_0 . Since the electron has energy of 100 eV, so it can cross a potential difference of 100 V only. So

$$100 = \frac{\sigma}{\epsilon_0} d \text{ (solve to get } d)$$

$$20. (3) V = k \frac{q}{R} - \frac{kq}{2R} + \frac{kq}{3R} = \frac{kq}{R} \left[1 - \frac{1}{2} + \frac{1}{3} \right]$$

$$= \frac{k5q}{6R}$$



$$21. (4) Q = nq, R = n^{1/3}r$$

$$E_{\text{small}} = \frac{kq}{r^2}, E_{\text{big}} = \frac{kQ}{R^2}$$

$$E_{\text{big}} = \frac{k n q}{n^{2/3} r^2} = (n^{1/3}) \frac{kq}{r^2} = n^{1/3} E_{\text{small}}$$

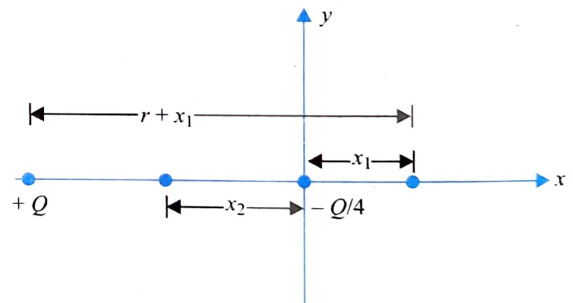
$$V_{\text{small}} = \frac{kq}{r}, V_{\text{big}} = \frac{kQ}{R}$$

$$V_{\text{big}} = \frac{k n q}{n^{1/3} r} = n^{2/3} \frac{kq}{r} = n^{2/3} V_{\text{small}}$$

22. (4) All the charges are placed in the xy plane such that they form a square with origin as its center. So electric field at the origin will be zero, but at other points on z -axis, field will be along z -axis.

$$23. (3) \frac{kQ}{4x_1} = \frac{kQ}{r+x_1} \text{ or } r+x_1 = 4x_1 \text{ or } x_1 = \frac{r}{3}$$

$$\text{or } \frac{kQ}{4x_2} = \frac{kQ}{r-x_2} \text{ or } x_2 = \frac{r}{5}$$



$$24. (3) E_x = \frac{dV}{dx} = \frac{120-80}{2} = 20 \text{ Vcm}^{-1}$$

There may be the y - and z -components of field also.

$$25. (3) F_c = mg \tan \theta$$

$$= (1.20 \times 10^{-3} \text{ kg})(10 \text{ ms}^{-2}) \tan (37^\circ) = 0.0090 \text{ N}$$

$$F_c = Eq = \frac{Vq}{d}$$

$$\therefore V = \frac{F_c d}{q} = \frac{(0.009 \text{ N})(0.0500 \text{ m})}{9.0 \times 10^{-6} \text{ C}} = 50.0 \text{ V}$$

$$26. (1) \left(0 - \frac{1}{2}mv^2 \right) = q(V_P - V_Q) \quad \dots(i)$$

$$\frac{1}{2}mv_1^2 - \frac{1}{2}mv^2 = -q(V_P - V_Q) \quad \dots(ii)$$

$$\text{From Eqs. (i) and (ii), } v_1 = \sqrt{2}v$$

27. (2) On connecting, the entire amount of charge will shift to the outer sphere. Heat generated is

$$U_i - U_f = \frac{q^2}{8\pi\epsilon_0 R_1} - \frac{q^2}{8\pi\epsilon_0 R_2}$$

$$= \frac{(20 \times 10^{-6}) \times 9 \times 10^9}{2} \left[\frac{1}{0.10} - \frac{1}{0.20} \right] = 9 \text{ J}$$

28. (2) $\vec{E} = a(y\hat{i} + x\hat{j})$

$$V_2 - V_1 = -\int (ay dx + ax dy)$$

Take $V_1 = C$ and $V_2 = V$. Then

$$V = -a \int (y dx + x dy) + C$$

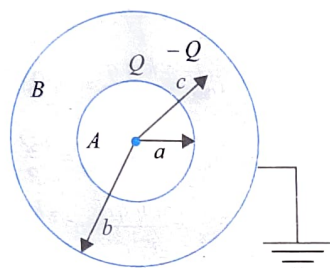
$$= -a \int d(yx) + C = -axy + C$$

29. (2) Charge $+Q$ will flow into earth.

$$V_c = \frac{kQ}{r} - \frac{kQ}{b}$$

$$V_A = \frac{kQ}{a} - \frac{kQ}{b}$$

Potential at B is zero.



30. (1) $E = \frac{V}{d}$, $F = eE = eV/d$

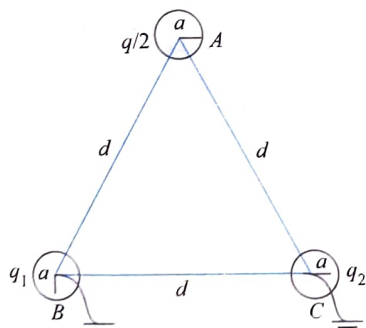
$$a = \frac{F}{m} = \frac{eV}{md}$$

$$d = \frac{1}{2}at^2 \text{ or } t = \sqrt{\frac{2d}{a}}$$

$$\text{or } t = \sqrt{\frac{2d md}{eV}} = \sqrt{\frac{2 md^2}{eV}}$$

31. (3) $\frac{kq_1}{a} + \frac{kq}{2d} = 0$ or $q_1 = \frac{-qa}{2d}$

$$\frac{kq_2}{a} + \frac{kq}{2d} + \frac{kq_1}{d} = 0 \text{ or } q_2 = \frac{-qa}{2d} \left(\frac{d-a}{d} \right)$$

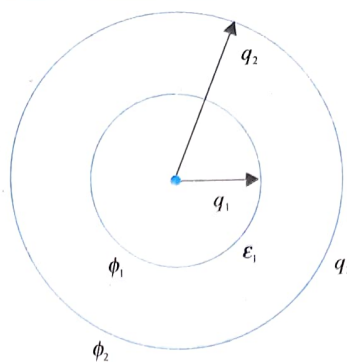


32. (1) $V_1 = \frac{kq_1}{a_1} + \frac{kq_2}{a_2}$

$$\text{Or } V_2 = \frac{kq_1}{a_2} + \frac{kq_2}{a_2}$$

Solve to get

$$q_1 = \frac{1}{4\pi\epsilon_0} \left[\frac{V_1 - V_2}{a_2 - a_1} \right] a_1 a_2$$

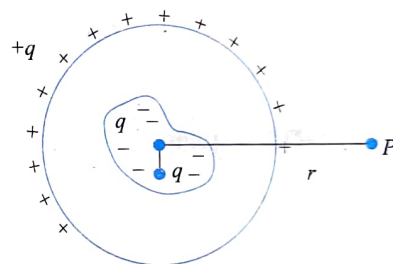


33. (1) Charge can be considered as located at a distance R from the center. Total charge is $(Q - 2Q + 3Q) = 2Q$

$$\text{Hence } V = \frac{1}{4\pi\epsilon_0} \frac{2Q}{R} = \frac{Q}{2\pi\epsilon_0 R}$$

34. (1) A negatively charge $-q$ will be induced on the inner surface of cavity and positive charge $+q$ will be induced on the outer surface, which will be uniformly distributed. There will be no field or potential at point P due to inner charges $+q$ and $-q$. Potential at P will only be due to outer $+q$ charge. So

$$V_P = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$



35. (1) If we throw the charged particle just right of the center of the tunnel, the particle will cross the tunnel. Hence, applying conservation of momentum between start point and center of tunnel we get

$$\Delta K + \Delta U = 0$$

$$\text{or } \left(0 + \frac{1}{2}mv^2 \right) + q(V_f - V_i) = 0$$

$$\text{or } V_f = \frac{V_s}{2} \left(3 - \frac{r^2}{R^2} \right) = \frac{\rho R^2}{6\epsilon_0} \left(3 - \frac{r^2}{R^2} \right)$$

$$\text{Hence } r = \frac{R}{2}$$

$$V_f = \frac{\rho R^2}{6\epsilon_0} \left(3 - \frac{R^2}{4R^2} \right) = \frac{11\rho R^2}{24\epsilon_0}; V_i = \frac{\rho R^2}{3\epsilon_0}$$

$$\frac{1}{2}mv^2 = 1 \left[\frac{11\rho R^2}{24\epsilon_0} - \frac{\rho R^2}{3\epsilon_0} \right] = \frac{\rho R^2}{\epsilon_0} \left[\frac{11}{24} - \frac{1}{3} \right] = \frac{\rho R^2}{8\epsilon_0}$$

$$\text{or } v = \left(\frac{\rho R^2}{4m\epsilon_0} \right)^{1/2}$$

Hence velocity should be slightly greater than V .

36. (1) Potential of outer sphere

$$V = \frac{kq_1}{3r} + \frac{kq_2}{3r} + \frac{kq_3}{3r} = 0$$

$$\text{or } q_1 + q_3 = -q_2$$

37. (3) Electrical potential energy of system

$$U = -2 \left[\frac{kq^2}{\sqrt{2}a} \right] = \frac{-\sqrt{2}kq^2}{a} \text{ (four pairs will cancel each other)}$$

If a decreases, U also decreases and if U decreases, the agent will do negative work.

$$38. (2) V = 10r = 10\sqrt{x^2 + y^2 + z^2}$$

$$E_x = -\frac{dv}{dx} = -\frac{10(2x)}{2\sqrt{x^2 + y^2 + z^2}}$$

$$= \frac{-10x}{\sqrt{x^2 + y^2 + z^2}} = \frac{-10 \times 3}{\sqrt{3^2 + 4^2 + 5^2}} = -3\sqrt{2}$$

Similarly,

$$E_y = -4\sqrt{2}$$

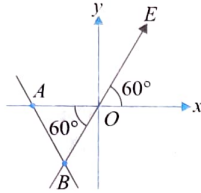
$$E_z = 5\sqrt{2}; \vec{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k}$$

$$39. (4) OB = OA \cos 60^\circ = 2 \times \frac{1}{2} = 1 \text{ m}$$

$$V_B - V_O = E(OB) = 100 \times 1 = 100 \text{ V}$$

$$V_A - V_O = 100 \text{ V} \quad [\because V_B = V_A]$$

$$V_O - V_A = -100 \text{ V}$$



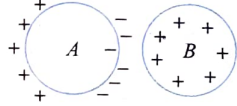
$$40. (4) \text{ Let } V_A \text{ be the potential at the origin and } V_B \text{ be the potential at } x = 2.0 \text{ m}$$

$$V_B - V_A = -\int E_x dx = -[\text{Area under } E_x - x \text{ curve}]$$

$$V_B - 10 = -\frac{1}{2} \times 2 \times (-20) = 20$$

$$V_B = 30 \text{ V}$$

41. (2) Charge on B will remain the same as it is not touched with any other body. Charge induced on A will decrease the potential of B (figure).



42. (1) Electric field produced due to charge Q will be zero at P , but potential produced by Q at P will not be zero.

43. (1) When S is open, spheres A and B have same, positive potential. When S is closed, potential of B will become zero. So the potential of B will decrease. This decrease of potential takes place due to flow of electrons from earth to inner sphere B . But magnitude of charge acquired by inner sphere will be less than Q .

44. (3) As in none of the options, the x -coordinate appears, so we have to find the locus in the yz plane.

$$U = \frac{kQ^2}{r_1} - \frac{kQ^2}{r_2} - \frac{kQ^2}{r_3} = 0$$

$$kQ^2 \left[\frac{1}{a} - \frac{1}{\sqrt{(a/2)^2 + y^2 + z^2}} - \frac{1}{\sqrt{(a/2)^2 + y^2 + z^2}} \right] = 0$$

$$\frac{1}{a} = \frac{2}{\sqrt{a^2/4 + y^2 + z^2}} \text{ or } y^2 + z^2 = 15a^2/4$$

45. (2) Potential energy of an electron at any point is $U = -eV$, where V is the potential at that point. V is negative maximum at point B , hence U is maximum for this point.

46. (4) Let $\sigma = Q/A$. For $0 < x < d$, $E_{\text{net}} = 4\sigma/\epsilon_0$ toward right, so potential decreases as we move from $x = 0$ to $x = d$.

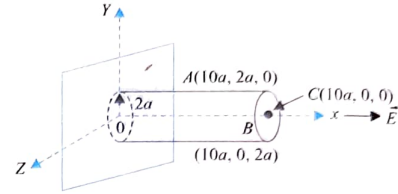
For $d < x < 2d$, $E_{\text{net}} = 6\sigma/\epsilon_0$ toward right, so potential decreases as we move from $x = d$ to $x = 2d$.

For $2d < x < 3d$, $E_{\text{net}} = 4\sigma/\epsilon_0$ toward left, so potential increases as we move from $x = 2d$ to $x = 3d$.

$$47. (3) U = -PE \cos \theta$$

As θ increases U increases

48. (2) The electric field will be as shown. A flat surface of cylinder of radius $2a$ and length $10a$ will be an equipotential surface.



Points A and B are located on a circle forming flat surface of the cylinder. The electric field is perpendicular at all the points of this plane surface. Thus, there will be no change in electrostatic PE in moving the charge $+Q$ on this plane surface.

49. (1) Negative of slope of potential versus x curve gives electric field. In this case, slope is negative, therefore electric field is positive. Thus field due to σ_2 is greater than σ_1 .

50. (1) Work done by electric field

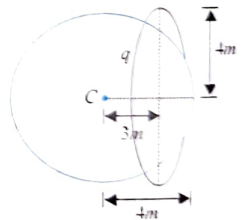
$$W_{AB} = q(V_A - V_B) = q \left[\frac{3Q}{8\pi\epsilon_0 R} - \frac{Q}{4\pi\epsilon_0 R} \right] = \frac{Qq}{8\pi\epsilon_0 R}$$

$$W_{BC} = q(V_B - V_C) = q \left[\frac{Q}{4\pi\epsilon_0 R} - \frac{Q}{8\pi\epsilon_0 R} \right] = \frac{Qq}{8\pi\epsilon_0 R}$$

$$\therefore \frac{W_{AB}}{W_{BC}} = 1$$

51. (3) $q = 1 \mu\text{C}$ is the charge on ring.

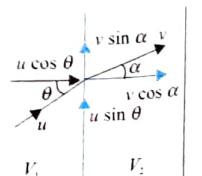
To find the potential of sphere, we find potential at the center of sphere, which will be same as the potential at any point on the surface of sphere. It is because, electric field inside the sphere will be zero and sphere will be an equipotential body. Due to charge q on the ring, charges will be induced on the sphere but net charge of sphere will remain zero. Hence potential at the center of sphere due to induced charges on sphere will be zero. Potential at the center of sphere will be only due to charge of ring.



$$\text{So } V_C = \frac{kq}{\sqrt{3^2 + 4^2}} = \frac{9 \times 10^9 \times 10^{-6}}{5}$$

$$= 1.8 \times 10^3 \text{ V} = 1.8 \text{ kV}$$

52. (1) As the electron is going from low potential to high potential, its potential energy decreases, which results in increase in KE. Hence speed definitely increases. Direction of motion may or may not change depending on the angle at which it enters the other region. If θ is zero, then direction of motion remains same.



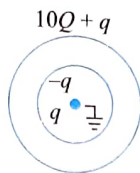
$v \sin \alpha = u \sin \theta$, but $v \cos \alpha > u \cos \theta$

$$\text{So } \alpha < \theta, \quad v > u$$

53. (1) Potential of sphere should be zero.

$$V = \frac{kq}{R} - \frac{kq}{2R} + \frac{k(10Q + q)}{3R} = 0$$

or $q = -4Q$



54. (1) $V_{(x,y)} - V_0 = -\int_0^x E_x dx - \int_0^y E_y dy$

or $V_{(x,y)} = V_0 - 4x - 5y$

55. (2) As E increases along x -axis, same potential drop will occur for lesser values of distance. Hence $x_2 < x_1$.

56. (4) $V_B - V_A = -\int_{30}^{-20} 10x dx - \int_{-20}^{30} (-20) dy$

or $V_B - 0 = 3500 \text{ V}$

57. (1) $\frac{q_1}{x-4} - \frac{q_2}{4} = 0$ $\frac{q_1}{x+7} - \frac{q_2}{7} = 0$

or $\frac{q_1}{q_2} = \frac{x-4}{4}$

or $\frac{q_1}{q_2} = \frac{x+7}{7}$

$\therefore \frac{x-4}{4} = \frac{x+7}{7}$

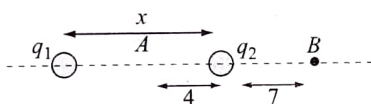
or $7x - 28 = 4x + 28$

or $3x = 56$

or $x = \frac{56}{3}$

$\therefore \frac{q_1}{q_2} = \frac{\frac{56}{3} - 4}{4} = \frac{11}{3}$

$|q_2| = +12 \mu\text{C}$ or $q_1 = 12 \times \frac{11}{3} = 44 \mu\text{C}$



58. (1) If we send the bead diametrically opposite, it will complete full circle as resultant of repulsive force between the charges and normal reaction between the ring and bead will provide force in tangential direction of ring.

Hence at point 2, $v_2 = 0$.

Given $\frac{1}{4\pi\epsilon_0} \frac{Q}{4a} = V$

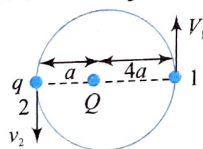
or $\frac{1}{4\pi\epsilon_0} \frac{Q}{a} = 4V$

Let velocity given at 1 be v_1 , then

$$\frac{1}{2}mv_1^2 + \frac{1}{4\pi\epsilon_0} \frac{Qq}{4a} = 0 + \frac{1}{4\pi\epsilon_0} \frac{Qq}{a}$$

or $\frac{1}{2}mv_1^2 + qV = q \cdot 4V$

or $\frac{1}{2}mv_1^2 = 3Vq$ or $v_1 = \sqrt{\frac{6Vq}{m}}$

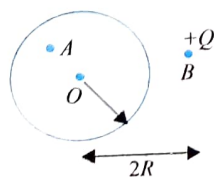


...(i)

59. (3) Since electric field inside the conducting shell is zero,

$$V_0 = V_A \text{ and } V_0 = \frac{Q}{8\pi\epsilon_0 R}$$

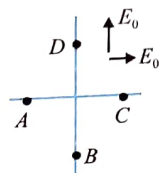
$\therefore V_A = \frac{Q}{8\pi\epsilon_0 R}$



60. (2) Since potential decreases along electric field as $E = -dV/dx$, $V_A > V_C$ and $V_B > V_D$. Also due to symmetry

$V_A = V_B$, $V_C = V_D$

$V_A = V_B > V_C = V_D$



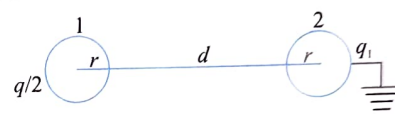
61. (3) The potential energy of the system can be found by computing the work done by external agent in assembling the system without changing the kinetic energy of the system. Let us first bring q_1 from infinity to the desired location, then in doing so we have to do work against external electric field which is equal to $W_1 = q_1 V_1$ (potential at infinity is considered as zero). Now, we will bring q_2 from infinity to the desired location, to do so, we have to do work against external electric field and against electric force of q_1 , so work done is

$$W_2 = q_2 V_2 + \frac{q_1 q_2}{4\pi\epsilon_0 r}$$

So total work done is

$$U = q_1 V_1 + q_2 V_2 + \frac{q_1 q_2}{4\pi\epsilon_0 r}$$

62. (3) Let third sphere be touched with '1'. Then $q/2$ charge will be transferred to '1'. Now '2' is earthed. Potential of '2' is zero.



$$\frac{kq}{2d} + \frac{kq_1}{r} = 0 \text{ or } q_1 = -\frac{qr}{2d}$$

63. (2) Due to the induced charges of sphere, its potential will remain zero because net induced charge is zero. Potential of sphere will be due to only eight discrete charges.

$$V = 8 \frac{kq}{(\sqrt{3}a/2)} = \frac{16kq}{\sqrt{3}a}$$

Multiple Correct Answers Type

1. (1), (4)

E is a vector quantity; V is a scalar quantity.

2. (1), (4)

When a negative charge moves from high potential to low potential, its potential energy increases.

$$W_{el} = q(V_1 - V_2)$$

As $V_1 > V_2$ and q is negative, W_{el} is negative.

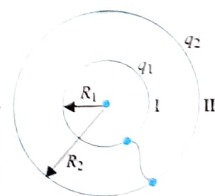
3. (1), (2), (3)

This question is based on the working principle of a generator.

$$V_1 - V_{II} = \left(\frac{q_1}{4\pi\epsilon_0 R_1} + \frac{q_2}{4\pi\epsilon_0 R_2} \right) - \left(\frac{q_1 + q_2}{4\pi\epsilon_0 R_2} \right)$$

$$V_1 - V_{II} = \frac{q_1}{4\pi\epsilon_0} \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$$

As two conductors are connected, transfer of charge takes place from one conductor to other till both acquire same potential, i.e., here $V_1 - V_{II} = 0$. Potential difference depends only on q_1 (charge of inner conductor), so $V_1 - V_{II} = 0$, where $q_1 = 0$, i.e., total charge is transferred to the outer conductor.



4. (1), (4)

From electric field line pattern, it is clear $|E_A| > |E_B|$ and electric potential decreases in the direction of field, hence $V_A < V_B$.

5. (2), (4)

Potential everywhere inside a charged conductor is same. Even if we touch A and B, nothing will happen because potential is same prior to touching.

6. (1), (4)

As no charge is present inside the conductor, potential at any point inside the conductor is same as that of the potential of conductor.

So potential of the conductor = potential at the center

$$= V_q + V_{\text{induced charges}}$$

$$\text{i.e., } V_{\text{conductor}} = \frac{q}{4\pi\epsilon_0(d+R)} + 0$$

As the total induced charge at conductor's surface is equal to zero, the potential at center due to the induced charges is zero. For point B

$$V_{\text{conductor}} = V_{\text{at point B}} = V_q + V_{\text{induced charges}}$$

$$V_{\text{induced charges}} = \frac{q}{4\pi\epsilon_0(d+R)} - \frac{q}{4\pi\epsilon_0 d} = \frac{-qR}{4\pi\epsilon_0 d(d+R)}$$

7. (1), (2), (3), (4)

Charge on a_1 is $(r_1 \theta) \lambda$

Charge on a_2 is $(r_2 \theta) \lambda$

Ratio of charges is r_1/r_2

$$E_1 \text{ (field produced by } a_1) = \frac{K[(r_1 \theta) \lambda]}{r_1^2} = \frac{K \theta \lambda}{r_1}$$

$$E_2 \text{ (field produced by } a_2) = \frac{K \theta \lambda}{r_2}$$

As $r_2 > r_1$, $E_1 > E_2$ i.e., net field at A is toward a_2 .

$$V_1 = \frac{K(r_1 \theta)}{r_1} = K \theta \lambda$$

$$V_2 = \frac{K(r_2 \theta)}{r_2} = K \theta \lambda$$

$$V_1 = V_2$$

8. (1), (3)

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{(R+d)}$$

R is radius, and d is distance from surface

$$100 = K \frac{Q}{(R+5) \times 10^{-2}} \quad \dots(i)$$

$$75 = K \frac{Q}{(R+10) \times 10^{-2}} \quad \dots(ii)$$

From (i) and (ii), $R = 10$ cm and $Q = \frac{50}{3} \times 10^{-10}$ C

Potential at surface is

$$V_0 = \frac{kQ}{R} = \frac{9 \times 10^9 \times 50}{10 \times 10^{-2} \times 3} \times 10^{-10} = 150 \text{ V}$$

Electric field on surface is $E_0 = \frac{kQ}{R^2} = 1500 \text{ Vm}^{-1}$

Potential at the center is $V_C = \frac{3}{2} V_0 = 225 \text{ V}$

9. (1), (3), (4)

$$0 \leq x \leq a; V_x = \left[-\int_0^x E_x dx \right] + V_{(0)} = 0$$

(as $E_x = 0$)

$$x \geq a; V = -\int_a^x E_x dx + V_{(a)}$$

$$= \left[-\int_a^x \frac{\sigma}{\epsilon_0} dx \right] + V_{(a)} = -\frac{\sigma}{\epsilon_0} (x-a) = \frac{-\sigma}{\epsilon_0} (x-a)$$

$$x \leq 0; V = -\int_a^x E_x dx + V_{(0)} \left[\because E_x = \frac{-\sigma}{\epsilon_0} \right]$$

$$= -\left(-\frac{\sigma}{\epsilon_0} x \right) + V_{(0)} = \frac{\sigma}{\epsilon_0} x$$

10. (1), (2), (3)

Equipotential surfaces will be inclined at 135° to the x -axis. If $a > b$, then while moving from B to A, we are going in the direction of field, i.e., from high to low potential. Similarly, options 1, 2, and 3 are correct.

11. (1), (3)

$$\vec{E} = 54\hat{i} + 74\hat{j}, E = 90 \text{ NC}^{-1}$$

$$90 = \frac{9 \times 10^9 Q}{r^2} \quad \dots(i)$$

$$V = 1800 = \frac{9 \times 10^9 Q}{r} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$r = 20 \text{ m}, q = 4 \mu\text{C}$$

$$\text{Now, } \frac{(9-x_0)\hat{i} + (4-y_0)\hat{j}}{20} = \frac{54\hat{i} + 72\hat{j}}{90}$$

$$\text{or } x_0 = -3 \text{ m}, y_0 = -12 \text{ m}$$

12. (1), (2)

Electric field lines will be from M to N, so potential of N will be less than that of M.

13. (2), (3), (4)

$$V_p = \frac{kq}{r} \text{ (independent of } x)$$

14. (1), (2), (3)

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r} = 600 \text{ V}$$

$$\text{And } E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} = 200 \text{ NC}^{-1}$$

$$\text{Hence, } \frac{V}{E} = r = \frac{600}{200} = 3 \text{ m}$$

$$\text{Now, } \frac{q}{4\pi\epsilon_0 r} = 600; \frac{q}{4\pi\epsilon_0 r'} = V'$$

$$\therefore \frac{V'}{600} = \frac{r}{r'} = \frac{3}{9}$$

$$\text{or } V' = 200 \text{ V}$$

$$W = q(V - V') = 10^{-6} (600 - 200) = 4 \times 10^{-4} \text{ J}$$

15. (3), (4)

$$\vec{r}_p = \hat{i} + 2\hat{j} + 4\hat{k}; \vec{r}_q = 3\hat{i} + 2\hat{j} + \hat{k}$$

$$|\vec{r}_q - \vec{r}_p| = \sqrt{[(2)^2 + (-3)^2]} = \sqrt{13}$$

$$\text{As } q = 2 \times 10^{-8} \text{ C,}$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{|\vec{r}_q - \vec{r}_p|} = \frac{9 \times 10^9 \times 2 \times 10^{-8}}{\sqrt{13}} = 49.9 \text{ V}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{|\vec{r}_q - \vec{r}_p|^2} \frac{(\vec{r}_q - \vec{r}_p)}{|\vec{r}_q - \vec{r}_p|} = \frac{1}{4\pi\epsilon_0} \frac{q}{|\vec{r}_q - \vec{r}_p|^3} (2\hat{i} - 3\hat{k})$$

(By filling a dielectric of dielectric constant K , electric field gets decreased as $E = E_0/K$ and $K > 1$.)

16. (1), (3), (4)

$$V(x) = 4 + 5x^2$$

$$V(x=1) = 9 \text{ V}$$

And $V(X = -2) = 24 \text{ V}$

Hence, $\Delta V = 15 \text{ V}$

$$E = -(\Delta V / \Delta x) = -10 \text{ x}$$

$\therefore E(\text{at } x = -1 \text{ m}) = 10 \text{ NC}^{-1}$

So, $F = qE = +10 \text{ N}$ (along positive x -axis).

17. (1), (2), (3)

If there is no external electric field, then the charge given to a conducting sphere gets uniformly distributed over its surface. Therefore, option (1) is correct. If an external electric field exists, then the charge gets distributed over the surface of the sphere in such a way that the electric field inside the sphere can become equal to zero. Hence, distribution of the charge on the surface of sphere will be non-uniform. Therefore, option (2) is correct. Obviously, option (4) is wrong.

Since electric field inside the conducting sphere is equal to zero, potential difference between two points in the sphere is equal to zero. It means, the potential is same at every point of the sphere. Therefore, option (3) is correct.

18. (2), (3), (4)

Charge on inner sphere can be supposed to be concentrated as a point charge at the center, hence electric field at a point in the region between the spheres at a distance r from the center is $(q / 4\pi\epsilon_0 r^2)$. Due to induction, equal and opposite charges will appear on the inner and outer surfaces of the outer sphere. Hence, net charge which can be supposed to be placed at the center is q and electric field due to it at a point outside the hollow sphere at a distance r from center is $(q / 4\pi\epsilon_0 r^2)$.

Potential of inner sphere is $\frac{q}{4\pi\epsilon_0} \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right)$

And potential of outer sphere is $\frac{q}{4\pi\epsilon_0 c}$

Potential of inner sphere with respect to the outer sphere

$$\frac{q}{4\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right)$$

19. (1), (2)

If the charge is given to a conducting sphere, then an electric field is established in the surrounding space. Magnitude of electric field is maximum just outside the sphere. This maximum electric field may be increased to the dielectric strength of the surrounding medium. Therefore, there is a limiting value of maximum charge which can be given to the conducting sphere. Hence, option (3) is wrong. Obviously, the conducting sphere cannot be charged to a potential greater than a certain value. Hence, option (1) is correct. It can be easily said that option (2) is also correct.

20. (1), (2), (3), (4)

Points A and B lie within the same metal hence $V_A = V_B$. The potential inside a hollow sphere is same as potential at the surface, hence $V_A = V_B = V_C = V_0$.

21. (1), (3)

$$\frac{1}{4\pi\epsilon_0} \frac{Q}{R} + \frac{1}{4\pi\epsilon_0} \frac{q}{(R_0 - vt)} = 0$$

where R_0 is the initial distance of the charged particle.

$$Q = \frac{Rq}{R_0 - vt} \text{ or } \frac{dQ}{dt} = i = \frac{Rqv}{(R_0 - vt)^2}$$

22. (2), (3)

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{x} = 600 \quad \dots(i)$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{x^2} = 200 \quad \dots(ii)$$

From Eq. (i) and Eq. (ii), $x = 3 \text{ m}$

From Eq. (i), $q = \frac{600 \times 3}{9 \times 10^9} = 2 \times 10^{-7} \text{ C}$

$$V' = \frac{1}{4\pi\epsilon_0} \frac{q}{x'} = 9 \times 10^9 \times \frac{2 \times 10^{-7}}{9} = 200 \text{ V}$$

$$W = q(V' - V) = 10^{-6} (200 - 600) = -4 \times 10^{-4} \text{ J}$$

23. (1), (3)

If the system is isolated (no external forces) p and E are conserved. Electrostatic forces are internal forces. So (1) is correct. To fix Y external forces must act on the system i.e., on Y . In that case p is not conserved. However these external forces do not work as there is no displacement of Y . E is conserved. So (3) is correct.

24. (2), (4)

Let us find potential difference between any two points:

$$V_2 - V_1 = - \int_{x_1}^{x_2} E_x dx - \int_{y_1}^{y_2} E_y dy$$

$$\Rightarrow V_2 - V_1 = + \int_{x_1}^{x_2} Ky dx - \int_{y_1}^{y_2} Kx dy$$

This can further be evaluated only if we know the dependance of x and y on each other or the path of integration. Hence field is nonconservative. To find the shape of lines of force:

$$\tan \theta = \frac{E_y}{E_x} \text{ or } \frac{dy}{dx} = \frac{E_y}{E_x}$$

$$\text{or } \frac{dy}{dx} = \frac{Kx}{-Ky} \text{ or } xdx + ydy = 0$$

$$\text{or } \frac{x^2}{2} + \frac{y^2}{2} = C \text{ or } x^2 + y^2 = 2C$$

This is the equation of circle.

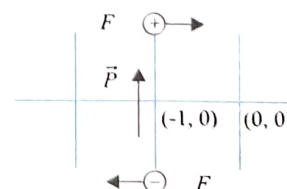
25. (1), (2), (3)

(1) is correct because potential of A and B is same. (2) is correct because charge is going from low potential to high potential. Electric potential energy increases and electrostatic forces do negative work. Similarly (3) is also correct.

26. (1), (3)

$$V(x) = 4 + 5x^2$$

Net force will be zero and torque not zero and rotation will be along clockwise direction.



27. (1), (3)

Distance AC is given by, $AC = 5 \text{ m}$

Potential at due to charge q at A is given by

$$V = \frac{kq}{AC} = \frac{9 \times 10^9 \times 1 \times 10^{-6}}{5} = 1.8 \times 10^3 = 1.8 \text{ kV}$$

Potential at B is given by

$$V_B = (V_B)_{\text{due to } q} + (V_B)_{\text{induced}}$$

$\{(V_B)_i = \text{Potential at } B \text{ due to induced charge}\}$

$$\text{Thus } 1.8 \times 10^3 = \frac{kq}{AB} + (V_B)_i = 2.25 \times 10^3 + (V_B)_i$$

$$(V_B)_i = -0.45 \text{ kV}$$

So (1) and (3) are correct.

Linked Comprehension Type

For Problems 1-4

1. (4), 2. (4), 3. (4), 4. (2)

Electric field at O.

$$\begin{aligned} E_O &= 2(2|\vec{E}_A| \cos 45^\circ) \\ &= 4 \frac{1}{4\pi\epsilon_0} \frac{q}{(a/\sqrt{2})^2} \times \frac{1}{\sqrt{2}} \\ &= \frac{\sqrt{2}q}{\pi\epsilon_0 a^2} \end{aligned}$$

Electric potential at O will be zero. Also $V_E = 0$, so work done in carrying a charge from O to E will be zero.

$$\begin{aligned} V_F &= 2 \left[\frac{1}{4\pi\epsilon_0} \frac{q}{(a^2 + a^2/4)^{1/2}} - \frac{1}{4\pi\epsilon_0} \frac{q}{a/2} \right] \\ &= \frac{q}{\pi\epsilon_0 a} \left[\frac{1}{\sqrt{5}} - 1 \right] \end{aligned}$$

$$\text{Now } W_{OF} = e(V_F - V_O)$$

For Problems 5-6

5. (3), 6. (2)

$$\begin{aligned} E_i = K_i + U_i &= \frac{1}{2}(0.0015 \text{ kg})(20.0 \text{ ms}^{-1})^2 \\ &+ \frac{k(2.00 \times 10^{-6} \text{ C})(8.00 \times 10^{-6} \text{ C})}{0.800 \text{ m}} \end{aligned}$$

$$= 0.48 \text{ J}$$

$$E_i = E_f = \frac{1}{2}mv_f^2 + \frac{kq_1q_2}{r_f}$$

$$v_f = \sqrt{\frac{2(0.48 \text{ J} - 0.36 \text{ J})}{0.0015 \text{ kg}}} = 4\sqrt{10} \text{ ms}^{-1}$$

At the closest point, the velocity is zero.

$$0.48 \text{ J} = \frac{kq_1q_2}{r}$$

$$\text{or } r = \frac{k(2.00 \times 10^{-6} \text{ C})(8.00 \times 10^{-6} \text{ C})}{0.48 \text{ J}} = 0.30 \text{ m}$$

For Problems 7-8

7. (1), 8. (3)

If q_1 and q_2 are the charges on the spheres of radii r and R , respectively, then by conservation of charge

$$q_1 + q_2 = Q \quad \dots(i)$$

$$\text{But } \sigma_1 = \sigma_2$$

$$\frac{q_1}{4\pi r^2} = \frac{q_2}{4\pi R^2}$$

$$\text{or } \frac{q_1}{q_2} = \frac{r^2}{R^2} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$q_1 = \frac{Qr^2}{r^2 + R^2} \text{ and } q_2 = \frac{QR^2}{r^2 + R^2}$$

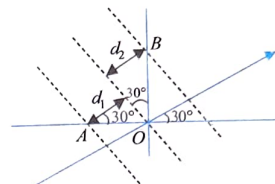
Hence, potential at common center is

$$\begin{aligned} V &= V_1 + V_2 \\ &= \frac{1}{4\pi\epsilon_0} \left[\frac{q_1}{r} + \frac{q_2}{R} \right] \\ &= \frac{1}{4\pi\epsilon_0} \left[\frac{Qr}{(R^2 + r^2)} + \frac{QR}{(R^2 + r^2)} \right] = \frac{1}{4\pi\epsilon_0} \frac{Q(R+r)}{(R^2 + r^2)} \end{aligned}$$

For Problems 9-10

9. (3), 10. (1)

$$d_1 = AO \cos 30^\circ = 2\sqrt{3}/2 = \sqrt{3} \text{ m}$$



$$d_2 = BO \sin 30^\circ = 4 \times \frac{1}{2} = 2 \text{ m}$$

$$V_A - V_O = E d_1 = 100\sqrt{3} \text{ V}$$

$$\text{or } V_O - V_A = -100\sqrt{3} \text{ V}$$

$$V_A - V_B = E(d_1 + d_2) = 100(\sqrt{3} + 2) \text{ V}$$

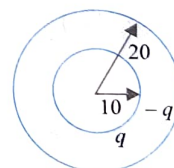
$$\text{or } V_B - V_A = -100(\sqrt{3} + 2) \text{ V}$$

For Problems 11-14

11. (2), 12. (1), 13. (3), 14. (4)

$$1.8 \times 10^6 = \frac{9 \times 10^9 \times q_1}{(0.1)^2} \text{ or } q_1 = 2 \times 10^{-6} \text{ C}$$

$$V = 9 \times 10^9 \times 2 \times 10^{-6} \left[\frac{1}{0.1} - \frac{1}{0.2} \right] = 9 \times 10^4 \text{ V}$$



$$E = \frac{kq_1}{r^2} = \frac{9 \times 10^9 \times 2 \times 10^{-6}}{(0.20)^2} = 4.5 \times 10^5 \text{ NC}^{-1}$$

Interaction energy is

$$U = \frac{kq}{0.2}(-q) = \frac{9 \times 10^9 \times (2 \times 10^{-6})^2}{0.2} = -0.18 \text{ J}$$

Self-energies are

$$U_1 = \frac{q_1^2}{8\pi\epsilon_0 r_1} = \frac{9 \times 10^9 \times (2 \times 10^{-6})^2}{2 \times 0.1} = 0.18 \text{ J}$$

$$U_2 = \frac{q_2^2}{8\pi\epsilon_0 r_2} = \frac{9 \times 10^9 \times (2 \times 10^{-6})^2}{2 \times 0.2} = 0.09 \text{ J}$$

Total energy is $U + U_1 + U_2 = 0.09 \text{ J}$

Both the charges will get neutralized and there will be no charge left on any sphere. So no energy will be left in the system. It means whole amount of energy will convert into heat.

For Problems 15–18

15. (1), 16. (2), 17. (4), 18. (1)

$$E_x = -\frac{\delta V}{\delta x} = -3 \text{ Vm}^{-1}; E_y = -\frac{\delta V}{\delta y} = -4 \text{ Vm}^{-1}$$

$$a_x = \frac{qE_x}{m} = -\frac{1 \times 10^{-6} \times 3}{0.1} = -3 \times 10^{-5} \text{ ms}^{-2}$$

$$a_y = \frac{qE_y}{m} = -\frac{1 \times 10^{-6} \times 4}{0.1} = -4 \times 10^{-5} \text{ ms}^{-2}$$

Time taken to cross the x-axis

Using $s = ut + \frac{1}{2}at^2$ we get

$$3.2 = \frac{1}{2} \times 4 \times 10^{-5} \times t^2$$

$$t = 400 \text{ s}$$

$$v_x = a_x t = -3 \times 10^{-5} \times 400 = 12 \times 10^{-3} \text{ ms}^{-1}$$

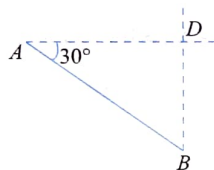
$$v_y = a_y t = -4 \times 10^{-5} \times 400 = 16 \times 10^{-3} \text{ ms}^{-1}$$

$$v = \sqrt{v_x^2 + v_y^2} = 20 \times 10^{-3} \text{ ms}^{-1}$$

For Problems 19–21

19. (1) $V_A - V_B = V_A - V_D = E AD$

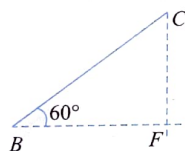
$$= E AB \cos 30^\circ = E 2\sqrt{3} \frac{\sqrt{3}}{2} = 3E$$



20. (4) $V_C - V_B = V_F - V_B = -E(BF)$

$$= -E(BC) \cos 60^\circ$$

$$= -E 4 \times \frac{1}{2} = -2E$$



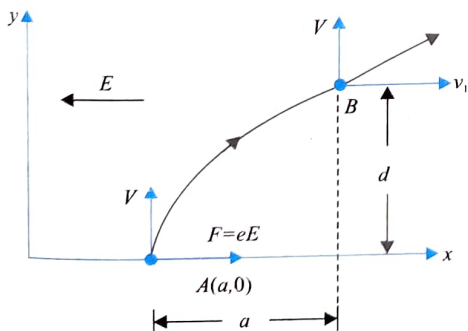
21. (2) $W_{el} = q(V_A - V_C)$

$$= q(V_A - V_B) + (V_B - V_C) = q(3E + 2E) = 5qE$$

For Problems 22–24

22. (1), 23. (2), 24. (3)

Electric field should be in negative x-direction because velocity along y-direction remains constant.



Acceleration of electron along x-axis is $a_x = eE/m$.

Time taken to go from A to B is $t = d/V$.

Along x-axis

$$a = \frac{1}{2}(a_x)t^2 = \frac{1}{2} \frac{eE}{m} \left(\frac{d}{V} \right)^2$$

or

$$E = \frac{2maV^2}{ed^2}$$

$$v_1 = a_x t = \frac{eE}{m} \frac{d}{V} = \frac{e}{m} \frac{d}{V} \frac{2maV^2}{ed^2} = \frac{2a}{d} V$$

$$\text{Velocity at B is } \sqrt{V^2 + v_1^2} = V \sqrt{1 + \left(\frac{2a}{d} \right)^2}$$

Rate of work done by field at B is

$$Fv_1 = eE \frac{2a}{d} V = \frac{2aeV}{d} \frac{2maV^2}{ed^2} = \frac{4ma^2V^3}{d^3}$$

For Problems 25–27

25. (3), 26. (3), 27. (1)

25. (3) $E = -\frac{dV}{dx} = -4x$

26. (3) $F = qE = 2.5 \times 10^{-6}(-4x) = -10^{-5}x$

$$W = \int_2^0 F dx = -10^{-5} \int_2^0 x dx$$

$$\frac{1}{2}mv^2 = 10^{-5} \left[\frac{x^2}{2} \right]_0^2 \quad \text{or } V = 2 \text{ ms}^{-1}$$

27. (1) $F = -10^{-5}x$

$$ma = -10^{-5}x \quad \text{or } 10 \times 10^{-6}a = -10^{-5}x$$

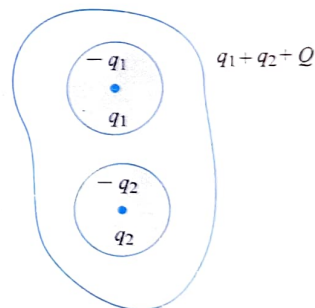
$$\text{or } a = -x$$

$$\omega^2 = 1, \quad \omega = 1 \text{ rad. } T = \frac{2\pi}{\omega} = \frac{2\pi}{1} = 2\pi \text{ s}$$

For Problems 28–32

28. (1), 29. (4), 30. (2), 31. (4), 32. (3)

The charge distribution on various surfaces is as shown in the following figure. $-q_1$ on the surface of cavity 1 will spread uniformly if q_1 is at the center, otherwise the distribution would be nonuniform. Same is the case with $-q_2$. The charge appearing on outer surface of the conductor is $q_1 + q_2 + Q$ which would be non-uniformly distributed as radius of curvature at various points of conductor's surface is different. The presence of q_1 and q_2 (and their location) has no effect on the distribution of charge on the outer surface. The presence of q will change the distribution of charge on outer surface, but still it remains nonuniform.



29. (4) Electric field intensity outside the cavity due to q_1 and $-q_1$ would be zero.

$$\vec{E}_{-q_1} + \vec{E}_{q_1} = 0$$

$$\vec{E}_{-q_1} = -\vec{E}_{q_1} = \frac{q_1}{4\pi\epsilon_0 r^2} \text{ toward center of cavity 1}$$

30. (2) $\vec{E}$ inside the conductor due to outside charges = 0

$$\vec{E}_q + \vec{E}_{q_1 + q_2 + Q} = 0$$

$$\text{or } \vec{E}_{q_1 + q_2 + Q} = -\vec{E}_q = \frac{q}{4\pi\epsilon_0 r^2} \text{ toward } q$$

31. (4) If q_2 is at point Q, then induced charge $-q_2$ would be non-uniformly distributed. So, we cannot determine E due to $-q_2$ at any inside point.

32. (3) Potential at point Q = potential of conductor + potential due to q_2 + potential due to $-q_2$

$$= V_0 + \frac{q_2}{4\pi\epsilon_0 r_2'} + \frac{-q_2}{4\pi\epsilon_0 r_2} = \frac{q_2}{4\pi\epsilon_0} \left(\frac{1}{r_2'} - \frac{1}{r_2} \right) + V_0$$

For Problems 33–36

33. (2), 34. (4), 35. (3), 36. (2)

Let the charge distribution be as shown in figure.

From Gauss's theorem, we know that facing surfaces of the conductor acquire equal and opposite charges. So

$$V_1 = V_3 \text{ and } V_2 = V_4$$

$$q_1 + q_3 - q_2 = +4Q$$

$$\text{Now, } q_2 - q_1 + q_4 - q_3 = -6Q$$

$$V_1 = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1}{R} + \frac{q_2 - q_1}{2R} + \frac{q_3 - q_2}{3R} + \frac{q_4 - q_3}{4R} \right]$$

$$V_2 = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1}{2R} + \frac{q_2 - q_1}{2R} + \frac{q_3 - q_2}{3R} + \frac{q_4 - q_3}{4R} \right]$$

$$V_3 = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1}{3R} + \frac{q_2 - q_1}{3R} + \frac{q_3 - q_2}{3R} + \frac{q_4 - q_3}{4R} \right]$$

$$V_4 = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1}{4R} + \frac{q_2 - q_1}{4R} + \frac{q_3 - q_2}{3R} + \frac{q_4 - q_3}{4R} \right]$$

$$\text{From } V_1 = V_3, q_1 = -\frac{q_2}{3}$$

$$\text{From } V_2 = V_4, q_2 = -\frac{q_3}{2}$$

$$\text{On solving Eq. (ii), } q_1 = \frac{2Q}{5}, q_2 = -\frac{6Q}{5}, \text{ and } q_3 = \frac{12Q}{5}$$

Substituting these values in Eq. (ii), we get $q_4 = -2Q$.

Charge on the inner surface of the third conductor is $-q_2 = 6Q/5$.

$$\text{Charge on the fourth conductor is } q_4 - q_3 = -2Q - \frac{12Q}{5} = \frac{-22Q}{5}$$

$$\text{Potential of conductor 1, } V_1 = \frac{-3Q}{40\pi\epsilon_0 R} = V_3$$

$$\text{Potential of conductor 2, } V_2 = \frac{Q}{8\pi\epsilon_0 R} = V_4$$

For Problems 37–39

37. (2), 38. (2), 39. (4)

$$q_1 + q_2 = 0$$

$$v_A = \frac{kq_1}{R} + \frac{kQ}{2R} + \frac{kq_2}{4R}$$

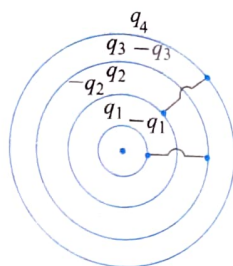
$$v_C = \frac{kq_1}{4R} + \frac{kQ}{4R} + \frac{kq_2}{4R}$$

$$v_A = v_C$$

$$q_1 = -\frac{Q}{3} \text{ and } q_2 = \frac{Q}{3}$$

$$v_A = k \left[\frac{-Q}{3R} + \frac{Q}{2R} + \frac{Q}{12R} \right] = \frac{Q}{16\pi\epsilon_0 R}$$

$$v_B = k \left[\frac{-Q}{6R} + \frac{Q}{2R} + \frac{Q}{12R} \right] = \frac{5Q}{48\pi\epsilon_0 R}$$



...(i)

...(ii)

For Problems 40–41

40. (4), 41. (1)

$$W = q(V_2 - V_1) = 9 \times 10^{-2} \left[\left(\frac{10}{0.3} - \frac{20}{0.5} \right) - \left(\frac{10}{0.5} - \frac{20}{0.3} \right) \right]$$

$$= 3.6 \text{ J}$$

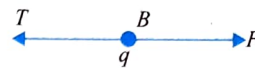
Since only conservative forces act on the system, potential energy changes to kinetic energy.

$$3.6 = \frac{1}{2}mv^2 \text{ or } v^2 = 72 \text{ or } v = 6\sqrt{2} \text{ ms}^{-1}$$

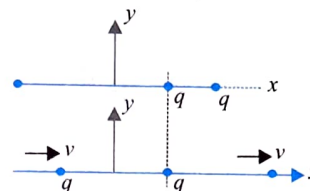
For Problems 42–44

42. (2), 43. (2), 44. (4)

$$T = F = \frac{kq^2}{(2a)^2} = \frac{9 \times 10^9 \times 10^{-12}}{4 \times 10^{-4}} = 22.5$$



When velocity is maximum, third charge will be at center of A and B .



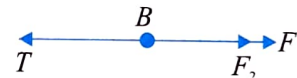
$$(KE + PE)_i = (KE + PE)_f$$

$$0 + \frac{kq^2}{a/2} + \frac{kq^2}{3a/2} = \frac{1}{2}(2m)v^2 + 2\frac{kq^2}{a}$$

$$\text{or } v = \sqrt{\frac{2kq^2}{3ma}} = \sqrt{\frac{2}{3} \times \frac{9 \times 10^9 \times 10^{-12}}{6 \times 10^{-3} \times 10^{-2}}} = 10 \text{ ms}^{-1}$$

When third charge is at center of A and B ,

net force due to third charge on A and B is zero.



So acceleration of system of A and B is zero. It means individual acceleration of A and B is also zero.

$$T = F_1 + F_2 = \frac{kq^2}{a^2} + \frac{kq^2}{(2a)^2} = 112.5 \text{ N}$$

Matrix Match Type

1. i. $\rightarrow$ b., c.; ii. $\rightarrow$ c.; iii. $\rightarrow$ b., c.; iv. $\rightarrow$ c.

i. Due to induced charge ($-q$) and also due to induced charge ($+q$) on outer surface, electric field inside the cavity is zero.

$$V_{\text{center}} = -\frac{kq}{a} + \frac{kq}{b} \neq 0$$

ii. Here induced charge on inner surface will not be uniformly distributed, so due to this charge electric field inside cavity is not zero. But induced charge on outer surface will be uniformly distributed, so electric field due to this charge inside the cavity will be zero.

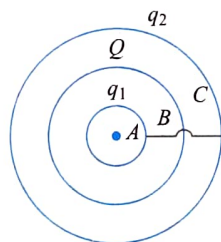
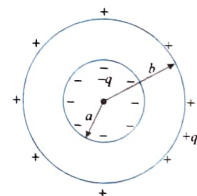
iii. No charge will be induced on the outer surface.

$$V_{\text{center}} = -\frac{kq}{a} \neq 0$$

iv. No charge will be induced on the outer surface.

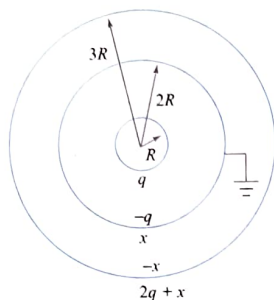
2. i. $\rightarrow$ a., d.; ii. $\rightarrow$ c., e.; iii. $\rightarrow$ c., e.

i. Due to q , charge will be induced on the conductor, such that net field due to q and induced charge becomes zero at any point inside the conductor. Since $E = 0$ everywhere inside the conductor, so potential is constant inside and same as that of surface of conductor.



- ii. Due to q , field and potential both will vary inside.
 iii. Due to q_2 , field and potential both will vary inside because inside charge system has nothing to do with outside system.
 3. i. $\rightarrow$ d.; ii. $\rightarrow$ c.; iii. $\rightarrow$ b.; iv. $\rightarrow$ a.

Charge induced at the surface of 1 is $q_1 = -q$. Any point located on the surface of sphere 2 will have zero potential.



$$V_p = 0 = V_{A, \text{out}} + V_{B, \text{surface}} + V_{C, \text{in}}$$

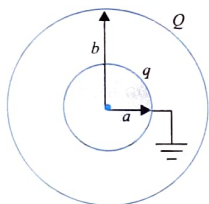
$$\text{or } 0 = k \frac{(q)}{2R} + k \frac{(-q+x)}{2R} + k \frac{(2q)}{3R}$$

$$\text{Hence, } x = -\frac{4}{3}q$$

Hence, charge appearing on surface 2 is $-(4/3)q$, charge appearing on surface 3 is $(4/3)q$, and charge appearing on surface 4 is $q_4 = 2q - (4/3)q = (2q/3)$.

4. i. $\rightarrow$ a.; ii. $\rightarrow$ c.; iii. $\rightarrow$ a.; iv. $\rightarrow$ a.

$$\frac{kq}{a} + \frac{kQ}{b} = 0 \quad \text{or} \quad q = -\frac{Qa}{b}$$



5. i. $\rightarrow$ a., b.; ii. $\rightarrow$ a., b.; iii. $\rightarrow$ a., b., d.; iv. $\rightarrow$ c., d.

In situation (i), (ii), and (iii), shells I and II are not at same potential. Hence, charge shall flow from Sphere I to Sphere II till both acquire same potential. If charge flows, the potential energy of system decreases and heat is produced.

In situations (i) and (ii), charges shall divide in some fixed ratio, but in situation (iii) complete charge shall be transferred to Shell II for potential of Shells I and II to be same.

In situation (iv), both the shells are at same potential, hence no charge flows through connecting wire.

6. i. $\rightarrow$ c.; ii. $\rightarrow$ d.; iii. $\rightarrow$ b.; iv. $\rightarrow$ b.

i. $U = -\vec{p} \cdot \vec{E}$. So, when $\theta = 180^\circ$, $U = pE$.

ii. Angular acceleration is maximum when torque is maximum.
 $\vec{\tau} = \vec{p} \times \vec{E}$ is maximum for $\theta = 90^\circ$.

iii. Angular momentum will not be conserved as there is no external force on the system but torque will act on the system.

iv. During movement of dipole, total energy remains conserved. Only KE cannot be conserved.

7. i. $\rightarrow$ a.; ii. $\rightarrow$ b.; iii. $\rightarrow$ d.; iv. $\rightarrow$ c.

$$\text{From question: } \frac{\partial V}{\partial x} = -(2xy + z^2)$$

$$\frac{\partial V}{\partial y} = -(2yz + x^2)$$

$$\frac{\partial V}{\partial z} = -(2zx + y^2)$$

$$V = x^2y + y^2z + z^2x + C$$

Using this, we can find out the works

Further, $E(x, y, z) = E(-x, -y, -z)$

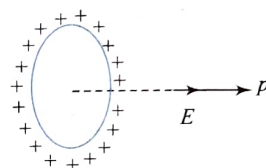
Direction of field at x, y, z is same as direction of field at $(-x, -y, -z)$ and direction of area vector is just opposite. That net flux will turn out to be zero.

8. i. $\rightarrow$ b.; ii. $\rightarrow$ c, d.; iii. $\rightarrow$ a, c.; iv. $\rightarrow$ b.

If $\lambda = \lambda_0$, charge is uniformly distributed and hence, field at center is zero. If $\lambda = \lambda_0 \cos \theta$ or $\lambda = \lambda_0 \sin \theta$, charge is positive in two quadrants and negative in the remaining two. Hence, potential at center is zero. Similarly, direction of electric field is also found by symmetry.

9. i. $\rightarrow$ b, c, d.; ii. $\rightarrow$ b, c, d.; iii. $\rightarrow$ c.; iv. $\rightarrow$ c.

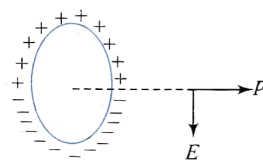
i. E is nonuniform, so $F \neq 0$.



E and p are in same direction, so $\tau = 0$ and $U = -pE < 0$.

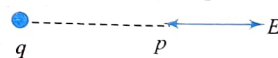
ii. Similar to (i).

iii. E is nonuniform, so $F \neq 0$, $\tau = pE \neq 0$



$$U = -pE \cos 90^\circ = 0$$

iv. E is nonuniform so $F \neq 0$, $\tau = pE \sin 180^\circ = 0$



$$U = -pE \cos 180^\circ = pE > 0$$

10. i. $\rightarrow$ d.; ii. $\rightarrow$ a.; iii. $\rightarrow$ c.; iv. $\rightarrow$ b.

$$\text{i. } E_x = \left(-\frac{dv}{dx} \right) \hat{i} = -1 \hat{i}$$

$$E_y = \left(-\frac{dv}{dx} \right) \hat{j} = \frac{1}{\sqrt{3}} \hat{j} \tan \theta = \frac{1}{\sqrt{3}} = 30^\circ$$

$$\text{ii. } E_x = \left(-\frac{dv}{dx} \right) \hat{i} = +1 \hat{i}$$

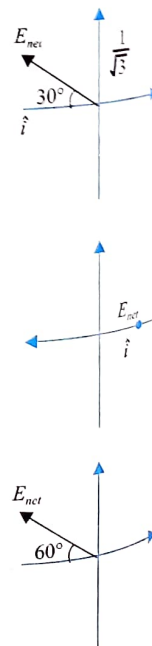
$$E_y = \left(-\frac{dv}{dx} \right) \hat{j} = 0 \quad \text{or} \quad \theta = 0^\circ$$

$$\text{iii. } E_x = \left(-\frac{dv}{dx} \right) \hat{i} = -1 \hat{i}$$

$$E_y = \left(-\frac{dv}{dx} \right) \hat{j} = +\sqrt{3} \hat{j} \quad \text{or} \quad \theta = 120^\circ$$

$$\text{iv. } E_x = \left(-\frac{dv}{dx} \right) \hat{i} = -\left(-\frac{1}{\sqrt{3}} \right) \hat{i} = \frac{1}{\sqrt{3}} \hat{i}$$

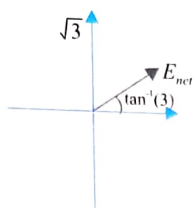
$$E_y = \left(-\frac{dv}{dy} \right) \hat{j} = -(-\sqrt{3}) \hat{j} = \sqrt{3} \hat{j}$$



Alternative method

$$\vec{E} = -\frac{\delta V}{\delta x} \hat{i} - \frac{\delta V}{\delta y} \hat{j}$$

- i. $\vec{E} = -\hat{i} + \frac{1}{\sqrt{3}}\hat{j}$ $\theta = 150^\circ$
 ii. $\vec{E} = -\hat{i} + 0\hat{j}$ $\theta = 0^\circ$
 iii. $\vec{E} = -\hat{i} + \sqrt{3}\hat{j}$ $\theta = 120^\circ$
 iv. $\vec{E} = \frac{1}{\sqrt{3}}\hat{i} + \sqrt{3}\hat{j}$ $\theta = \tan^{-1}(3)$



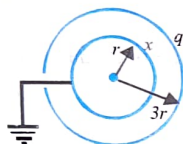
Numerical Value Type

1. (9) Let q be the charge on each drop, then $V_1 = kq/r$. Let R be radius of single drop. Then

$$\frac{4}{3}\pi R^3 = 27 \frac{4}{3}\pi r^3 \text{ or } R = 3r$$

$$V_2 = \frac{k(27q)}{R} = \frac{k27q}{3r} = 9V_1 \text{ or } \frac{V_2}{V_1} = 9$$

2. (3) Let charge appearing on inner sphere be x .



Now potential of inner sphere is zero, i.e.,

$$\frac{kq}{3r} + \frac{kx}{r} = 0 \text{ or } x = -q/3$$

Now charge that flows from the inner shell to the earth shall be $-x = q/3$.

3. (0) All the four corners will be at the same potential.

4. (0) The charge on the infinitesimal elements of arc that subtends an angle $d\theta$ at the center of the ring is

$$dQ = \lambda R d\theta = \lambda_0 \cos \frac{\theta}{2} R d\theta$$

Potential at the center of ring due to charge dQ is

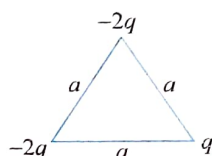
$$dV = \frac{1}{4\pi\epsilon_0} \frac{dQ}{R} = \frac{\lambda_0 \cos \frac{\theta}{2} R d\theta}{4\pi\epsilon_0 R}$$

$$\text{or } V = \int dV = \frac{\lambda_0}{4\pi\epsilon_0} \int_0^{2\pi} \cos \frac{\theta}{2} d\theta$$

$$= \frac{\lambda_0}{4\pi\epsilon_0} \left[\frac{\sin \frac{\theta}{2}}{\frac{1}{2}} \right]_0^{2\pi} = 0 \text{ V}$$

5. (0) $W = U_f - U_i$

$$\text{where } U_i = \frac{1}{4\pi\epsilon_0 a} \times [(-2q)(-2q) + (-2q)(q) + (q)(-2q)] = 0$$



Also, $U_f = 0$, so $W = 0$.

6. (6) Here the charge q_3 is attracted toward q_1 and q_2 both, so the net force on q_3 is toward the origin. As only force acting is conservative, we can use conservation of mechanical energy. Let v be the speed of particle at origin. From energy of conservation principle, we get

$$U_i + K_i = U + K$$

$$\text{or } \frac{1}{4\pi\epsilon_0} \left[\frac{q_3 q_2}{(r_{32})_i} + \frac{q_3 q_1}{(r_{31})_i} + \frac{q_2 q_1}{(r_{21})_i} \right] + 0 = \frac{1}{4\pi\epsilon_0} \left[\frac{q_3 q_2}{(r_{32})_f} + \frac{q_3 q_1}{(r_{31})_f} + \frac{q_2 q_1}{(r_{21})_f} \right] + \frac{1}{2} m v^2$$

Here, $(r_{21})_i = (r_{21})_f$. Substituting the proper values,

$$9 \times 10^9 \left[\frac{(-4)(2)}{5} + \frac{(-4)(2)}{5} \right] \times 10^{-12} = 9 \times 10^9 \left[\frac{(-4)(2)}{3} + \frac{(-4)(2)}{3} \right] \times 10^{-12} + \frac{1}{2} \times 10^{-3} v^2$$

$$\text{or } (9 \times 10^{-3}) \left(\frac{-16}{5} \right) = (9 \times 10^{-3}) \left(\frac{-16}{3} \right) + \frac{1}{2} \times 10^{-3} v^2$$

$$\text{or } 9 \times 10^{-3} (16) \left(\frac{2}{15} \right) = \frac{1}{2} \times 10^{-3} v^2$$

$$\text{or } v = 6.2 = 6 \text{ ms}^{-1}$$

7. (0) When shells are connected, charge on inner shell will be transferred to the outer shell.

$$V_i = \frac{KQ}{2R} + \frac{K(2Q)}{2R} = \frac{3KQ}{2R}$$

$$V_f = \frac{K(3Q)}{2R} \Rightarrow \Delta V = V_f - V_i = 0$$

8. (5) $E = 10 \text{ NC}^{-1}$, $S = 50 \text{ cm} = \frac{1}{2} \text{ m}$

$$\Delta V = ES = 10 \times \frac{1}{2} = 5 \text{ V}$$

9. (1) Decrease in KE = increase in PE

$$10 \text{ J} = (200 - 100) \times q_0$$

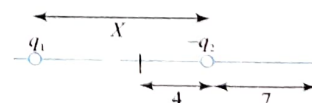
$$\text{or } 100 \times q_0 = 10 \text{ or } q_0 = \frac{10}{100} = 0.1 \text{ C}$$

10. (4) $\vec{E} = \frac{a\vec{r}}{r^2}$

$$V = \frac{-a}{r}$$

$$\Delta V = \frac{a}{r_2} - \frac{a}{r_1} = \frac{a}{5} - \frac{a}{7} = \frac{2a}{35} = 2 \times \frac{280}{35} = 16 \text{ V} = 4^2$$

11. (4)



$$\frac{q_1}{x-4} - \frac{q_2}{4} = 0 \quad \frac{q_1}{x+7} - \frac{q_2}{7} = 0$$

$$\text{or } \frac{q_1}{q_2} = \frac{x-4}{4} \quad \frac{q_1}{q_2} = \frac{x+7}{7}$$

$$\therefore \frac{x-4}{4} = \frac{x+7}{7}$$

$$\text{or } 7x - 28 = 4x + 28 \quad \text{or } 3x = 56 \quad \text{or } x = \frac{56}{3}$$

$$\text{or } \frac{q_1}{q_2} = \frac{\frac{56}{3} + 7}{7} = \frac{11}{3}$$

$$\text{or } |q_2| = +12 \mu\text{C}$$

$$\text{or } q_1 = 12 \times \frac{11}{3} = 44 \mu\text{C} = 11 \times 4 \mu\text{C}$$

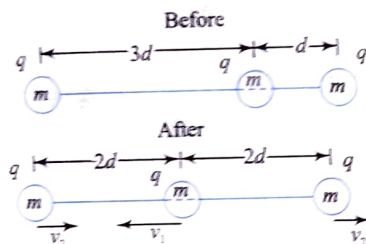
$$12. (2) \quad kq^2 \left(\frac{1}{r} - \frac{1}{2r} \right) = \frac{1}{2} m (V^2)$$

$$\text{or } \frac{kq^2}{2r} = \frac{1}{2} m V^2$$

$$\text{or } \frac{kq^2}{r} = \frac{1}{2} m (\sqrt{2}V)^2 = \frac{1}{2} m (V_{\max}^2)$$

$$\text{or } V_{\max} = V\sqrt{2} = (10\sqrt{2})\sqrt{2} = 20 \text{ ms}^{-1} = 2 \times 10 \text{ ms}^{-1}$$

13. (20)



Since the charge is evenly distributed, the charges can be treated as point charges. Looking at the "before" and "after" pictures and taking right as positive and left as negative, conservation of momentum gives $\sum \vec{P}_b = \sum \vec{P}_a$

$$0 = 2mv_2 - mv_1, \text{ which gives } v_2 = v_1/2$$

The maximum velocity of the middle charge will occur when the middle charge is at the middle. Beyond the middle point, there will be a net force to the right that will tend to slow the charge down. The situation will be the same for the middle charge even if the two end charges are moving. Applying the law of conservation of energy to the system and using the middle charge position of maximum velocity as the "after" position, one gets

$$E_b = E_a$$

$$PE_b = KE + PE_a$$

$$\text{or } \frac{kq^2}{3d} + \frac{kq^2}{d} = \frac{1}{2} mv_1^2 + \frac{1}{2} (2m)v_2^2 + \frac{2kq^2}{2d}$$

Now using the conservation of momentum result above for v_2 and simplifying, we get

$$\text{or } \frac{kq^2}{3d} = \frac{1}{2} mv_1^2 + m \frac{v_1^2}{4} = \frac{3mv_1^2}{4}$$

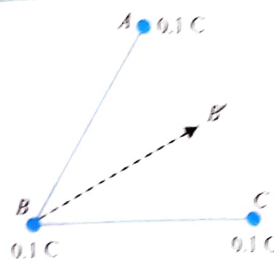
Finally solving for the maximum velocity

$$v_1 = \frac{2q}{3} \sqrt{\frac{k}{md}} = 20 \text{ ms}^{-1}$$

14. (50) As potential energy of two point charges separated by a distance r is given by $U = (q_1 q_2 / 4\pi\epsilon_0 r)$, the initial and final potential energies of the system will be

$$(U_s)_i = 3 \times \frac{1}{4\pi\epsilon_0} \frac{q^2}{r} = 3 \times 9 \times 10^9 \times \frac{(0.1)^2}{1} = 2.7 \times 10^8 \text{ J}$$

$$(U_s)_f = 2 \times \frac{1}{4\pi\epsilon_0} \frac{q^2}{(r/2)} + \frac{1}{4\pi\epsilon_0} \frac{q^2}{r} = 5 \times \frac{q^2}{4\pi\epsilon_0 r} = 4.5 \times 10^8 \text{ J}$$



So work done in changing the configuration of the system

$$W = (U_s)_f - (U_s)_i = 1.8 \times 10^8 \text{ J}$$

Now, as energy is supplied at the rate of 1 kW, i.e., 10^3 J/s , time required to do work W is given by

$$t = \frac{W}{P} = \frac{1.8 \times 10^8}{10^3} = 1.8 \times 10^5 \text{ s} = 50 \text{ h}$$

15. (700) Beta particles (i.e., electrons) escaped in t seconds.

$$\eta = \frac{40}{100} \times 5 \times 10^{10} t = 2 \times 10^{10} t$$

Therefore, the deficiency of electrons from a conductor results in positive charge. As 40% beta particles escape from the source, positive charge gained by source (metal sphere),

$$q = ne = 2 \times 10^{10} t \times 1.6 \times 10^{-19} \text{ C} = 3.2 \times 10^{-9} t \text{ C}$$

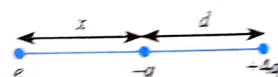
Due to this charge, the potential acquired by sphere is

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{R}$$

Substituting given values, we get

$$2 = 9.0 \times 10^9 \times \frac{3.2 \times 10^{-9} t}{10^{-2}} \quad \text{or } t = \frac{1}{1440} \text{ s} = 700 \mu\text{s}$$

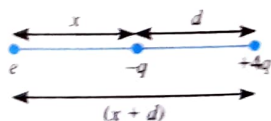
16. (2) For (1): Let the electron be held at a distance x from $-q$ charge



$$\text{For equilibrium } \frac{q(-e)}{4\pi\epsilon_0 x^2} = \frac{(-e)(-4q)}{4\pi\epsilon_0 (x+d)^2}$$

We can find value of x for which $F_{\text{net}} = 0$ which means that electron will be in equilibrium.

$$\text{For (2): For equilibrium } \frac{(-e)(-q)}{4\pi\epsilon_0 x^2} = \frac{(-e)4q}{4\pi\epsilon_0 (x+d)^2}$$



We can find value of x for which $F_{\text{net}} = 0$ which means that electron will be in equilibrium.

In case (3) and (4) the electron will not remain at rest, since it experiences a net non-zero force.

17. (0) $A = (2, 2)$ and $B = (4, 1)$; $W_{A \rightarrow B} = q(V_B - V_A)$

$$= q \int_A^B dV = - \int_A^B q \vec{E} \cdot d\vec{r}$$

$$= q \int_A^B (y \hat{i} + x \hat{j}) \cdot (dx \hat{i} + dy \hat{j}) = q \int_A^B (y dx + x dy)$$

$$= -q \int_{(2,2)}^{(4,1)} d(xy) = -q [xy]_{(2,2)}^{(4,1)} = -q [4 - 4] = 0$$

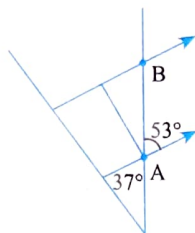
18. (0) Potential due to dipole at the centre of the circle is zero.

$$\text{Potentials due to charge on circle } V_1 = \frac{K \cdot (-3 \times 10^{-6})}{6 \times 10^{-2}}$$

$$\text{Potential due to arc } V_2 = \frac{K \cdot (2 \times 10^{-6})}{4 \times 10^{-2}}$$

$$\text{Net potential} = V_1 + V_2 = 0$$

19. (3)



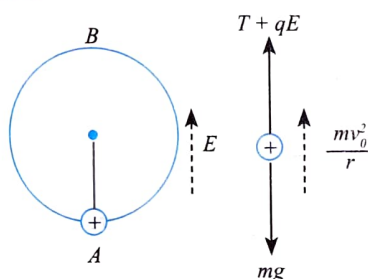
$$E = \frac{\sigma}{2\epsilon_0} = 1 \text{ N/C}$$

$$\therefore V_B - V_A = -\int \vec{E} \cdot d\vec{l} = \left(\frac{\sigma}{2\epsilon_0} \right) (5 \cos 53^\circ) = 3 \text{ V}$$

20. (6) Weight mg ($= 0.5 \times 10 = 5$ newton) of sphere acts vertically downward and electric force qE ($= 110 \times 10^{-6} \times 10^5 = 11$ newton) vertically upward.

Since upward force qE is greater than downward force mg , therefore critical condition corresponds to the tension in the thread to be zero at lowest point A of the circle instead of highest point B .

Let velocity at lowest point be v_0 . Considering free body diagram of sphere at A .



$$T + qE - mg = \frac{mv_0^2}{r} \text{ where, } T = 0 \Rightarrow v_0 = \frac{6}{\sqrt{5}} \text{ ms}^{-1}$$

But corresponding velocity at highest point is to be calculated. As sphere moves from A to B , work is done on it by electric field. Due to this work, both kinetic and gravitational potential energy of sphere increases. Hence, according to law of conservation of energy, minimum required velocity v at highest point is given by,

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_0^2 + qE \cdot 2r - mg \cdot 2r \Rightarrow v = 6 \text{ ms}^{-1}$$

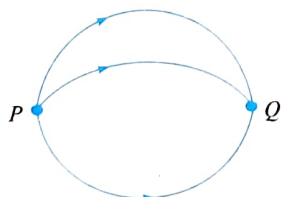
Archives

JEE Main

1. (3) Work done, $W = (V_Q - V_P) \times q$

The potential difference between the points P and Q depends on the initial and final positions and not on the path. As electrostatic force is a conservative force.

If the loop is completed, $V_Q - V_P = 0$. No net work is done as the initial and final potentials are the same. Both the statements are true but statement 2 is not the explanation for statement 1.



2. (4) $W = QdV = Q(V_q - V_p) = -100 \times (1.6 \times 10^{-19}) \times (-4 - 10)$
 $= +100 \times 1.6 \times 10^{-19} \times 14$
 $= +2.24 \times 10^{-16} \text{ J}$

3. (4) The electrostatic potential, $f = ar^2 + b$

$$\text{Electric field, } E = \frac{-d\phi}{dr} = -2ar$$

According to Gauss's theorem,

$$\phi = \oint \vec{E} \cdot d\vec{S} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

$$\Rightarrow \oint E \cdot ds \cos 0 = \frac{q_{\text{enclosed}}}{\epsilon_0} \Rightarrow E \oint ds = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

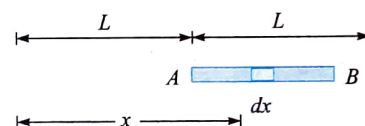
$$\text{or } -2ar(4\pi r^2) = \frac{q_{\text{enclosed}}}{\epsilon_0} \Rightarrow q_{\text{enclosed}} = -8\epsilon_0 a\pi r^3$$

Charge density inside the ball,

$$\rho_{\text{inside}} = -\frac{q_{\text{enclosed}}}{\frac{4}{3}\pi r^3}$$

$$\Rightarrow \rho_{\text{inside}} = -\frac{-8\epsilon_0 a\pi r^3}{\frac{4}{3}\pi r^3} = -6a\epsilon_0$$

4. (3) Let us take a small element dx at a distance x from O .



$$\text{Potential at } O \text{ due to the element, } dV = \frac{1}{4\pi\epsilon_0} \frac{dQ}{x}$$

$$\text{Charge on the element, } dQ = \frac{Q}{L} dx$$

$$\Rightarrow dV = \frac{1}{4\pi\epsilon_0} \frac{dQ}{x} = \left(\frac{1}{4\pi\epsilon_0} \frac{Q}{L} \right) \frac{dx}{x}$$

$$\text{Hence, } V = \int dV = \frac{Q}{4\pi\epsilon_0 L} \int_L^{2L} \frac{dx}{x}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{Q}{L} [\ln x]_L^{2L} = \frac{Q \ln 2}{4\pi\epsilon_0 L}$$

5. (None) $\vec{E} = 30x^2 \hat{i}$

$$dV = -\int E \cdot dx$$

$$\int_{V_0}^{V_A} dV = -\int_0^2 30x^2 dx$$

$$V_A - V_0 = -80 \text{ volt}$$

(Units given in question are incorrect)

6. (3, 4) Electric potential at the surface,

$$V_0 = k \frac{Q}{R}$$

...(i)

Electric potential inside,

$$V_I = \frac{kQ}{2R^3} (3R^2 - r^2)$$

$$\text{At } R = 0, V = \frac{3}{2} V_0 \text{ Hence } R_1 = 0$$

$$\text{Case II: } \frac{5kQ}{4R} = kQ \frac{(3R^2 - R_2^2)}{2R^3}$$

$$\Rightarrow R_2 = \frac{R}{\sqrt{2}}$$

$$\text{Case III: } \frac{3kQ}{4R} = \frac{kQ}{R_3} \Rightarrow R_3 = \frac{4R}{3}$$

$$\text{Case IV: } \frac{1kQ}{4R} = \frac{kQ}{R_4}$$

$$\Rightarrow R_4 = 4R$$

$$\Rightarrow R_4 > 2R \text{ and } R_2 < (R_4 - R_3)$$

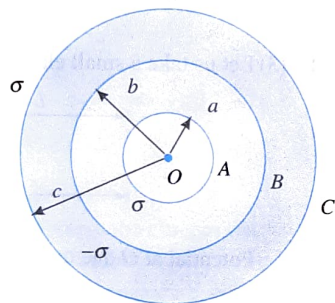
$$7. (3) V_B = V_{A, \text{out}} + V_{B, \text{surface}} + V_{C, \text{in}}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{Q_A}{b} + \frac{1}{4\pi\epsilon_0} \frac{Q_B}{b} + \frac{1}{4\pi\epsilon_0} \frac{Q_C}{c}$$

$$= \frac{1}{4\pi\epsilon_0} \left[\frac{\sigma \cdot 4\pi a^2}{b} + \frac{(-\sigma \cdot 4\pi b^2)}{b} + \frac{\sigma \cdot 4\pi c^2}{c} \right]$$

$$\Rightarrow V_B = \frac{\sigma}{\epsilon_0} \left(\frac{a^2}{b} - b + c \right)$$

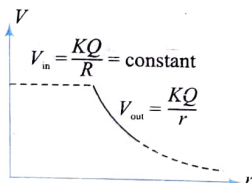
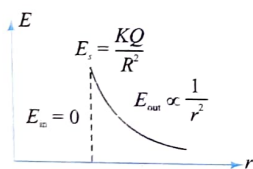
$$= \frac{\sigma}{\epsilon_0} \left(\frac{a^2 - b^2}{b} + c \right)$$



JEE Advanced

Single Correct Answer Type

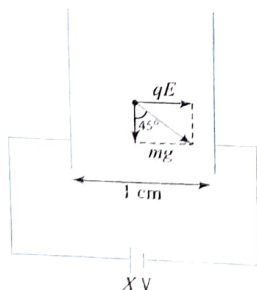
1. (4)



$$2. (3) \quad mg = qE$$

$$1.67 \times 10^{-27} \times 10 = 1.6 \times 10^{-19} \times \frac{X}{0.01}$$

$$\text{or } X = \frac{1.67}{1.6} \times 10^{-9} \text{ V} = 1 \times 10^{-9} \text{ V}$$



Multiple Correct Answers Type

1. (1), (2), (3), (4)

From conservation of charge:

$$Q_A + Q_B = 2Q \quad \dots(i)$$

Potential of both should be the same:

$$\frac{KQ_A}{R_A} = \frac{KQ_B}{R_B} \quad \dots(ii)$$



From Eqs. (i) and (ii)

$$\frac{Q_A}{R_A} = \frac{Q_B}{R_B} \quad \dots(iii)$$

Solving Eqs. (i) and (iii), we get

$$Q_A = \frac{2QR_A}{R_A + R_B} \text{ and } Q_B = \frac{2QR_B}{R_A + R_B} \text{ or } Q_A > Q_B$$

$$\frac{\sigma_A}{\sigma_B} = \frac{Q_A / 4\pi R_A^2}{Q_B / 4\pi R_B^2} = \frac{R_B}{R_A}$$

using Eq. (iii)

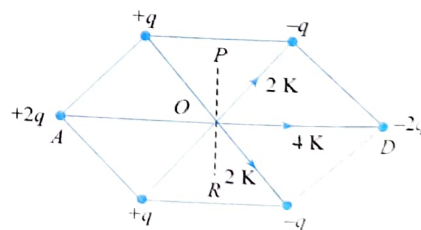
$$\sigma_B > \sigma_A \quad (\text{as } R_A > R_B)$$

$$\text{On surface, } E_A = \frac{\sigma_A}{\epsilon_0} \text{ and } E_B = \frac{\sigma_B}{\epsilon_0}$$

$$\text{or } E_A < E_B \quad (\because \sigma_A < \sigma_B)$$

2. (1), (2), (3)

$$E_0 = 6 \text{ K (along OD)}$$



$$V_0 = 0$$

Potential on line PR is zero.

Chapter 4

Concept Application Exercises

Exercise 4.1

1. (a) $U = Q^2 / 2C$

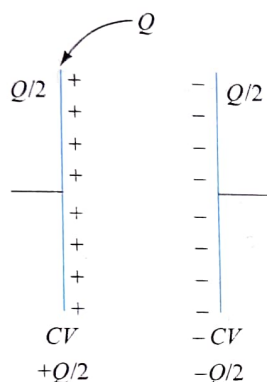
$$Q = \sqrt{2UC} = \sqrt{2(25.0 \text{ J})(5.00 \times 10^{-9} \text{ F})} = 5.00 \times 10^{-4} \text{ C}$$

The number of electrons N that must be removed from one plate and added to the other is

$$N = Q/e = (5.00 \times 10^{-4} \text{ C}) / (1.6 \times 10^{-19} \text{ C}) = 3.125 \times 10^{15} \text{ electrons}$$

(b) To double U while keeping Q constant, decrease C by a factor of 2. $C = \epsilon_0 A/d$, half the plate area or double the plate separation.

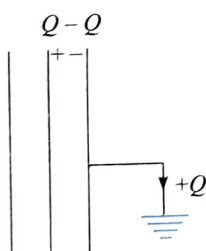
2. Positive plate is given a charge Q ; the charge distribution on different surface are shown in figure.



Hence, new charge on capacitor is $\left(CV + \frac{Q}{2}\right)$. The potential difference between the plates will be

$$V' = \frac{CV + \frac{Q}{2}}{C} = V + \frac{Q}{2C}$$

On connecting with earth, the charge on central plate will shift to its right face. $-Q$ charge will be induced on the left face of the earthed plate. Thus, $+Q$ charge will flow to the earth.



As $q = CV$, C decreases in both the cases.

- In the first case, V remains the same; hence, q decreases, due to which force of attraction will decrease.
- But in the second case, q will remain the same; hence, force of attraction remains the same. So more work will be done in the second case.

- True
- True
- True

From the given graphs, find the voltages, V_A and V_B , on capacitors A and B corresponding to charge Q on each of the capacitors. Clearly,

$$V_A = \frac{Q}{C_A} \text{ and } V_B = \frac{Q}{C_B}$$

or $\frac{V_B}{V_A} = \frac{Q/C_B}{Q/C_A} = \frac{C_A}{C_B}$

Since $V_B > V_A$, $C_A > C_B$, i.e., the capacitor A has the higher capacitance.

7. Since slope of the graph is $q/V = C$, and the line A has a larger slope than the line B , it represents the capacitor of larger capacitance. Further, as $C \propto A$ and the plate area of C_2 is double that of C_1 , C_2 has the larger capacitance. Hence, the line A (with larger slope) represents capacitor C_2 , and the line B represents capacitor C_1 .

8. (a) $Q = CV_0$

(b) They must have equal potential difference, and their combined charge must add up to the original charge. Therefore,

$$V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \text{ and also } Q_1 + Q_2 = CV_0$$

$$C_1 = C \text{ and } C_2 = \frac{C}{2},$$

So $\frac{Q_1}{C} = \frac{Q_2}{(C/2)}$

or $Q_2 = \frac{Q_1}{2}$

or $Q = \frac{3}{2}Q_1$ or $Q_1 = \frac{2}{3}Q$

So $V = \frac{Q_1}{C_1} = \frac{\frac{2}{3}Q}{\frac{C}{2}} = \frac{2}{3}V_0$

(c) $U = \frac{1}{2} \left(\frac{Q_1^2}{C_1} + \frac{Q_2^2}{C_2} \right) = \frac{1}{2} \left[\frac{(\frac{2}{3}Q)^2}{C} + \frac{(\frac{1}{3}Q)^2}{\frac{C}{2}} \right]$
 $= \frac{1}{3} \frac{Q^2}{C} = \frac{1}{3} CV_0^2$

(d) The original U was $U = \frac{1}{2} CV_0^2$ or $\Delta U = \frac{-1}{6} CV_0^2$.

(e) Thermal energy of capacitor, wires, etc., and electromagnetic radiation.

9. (a) $U_0 = \frac{Q^2}{2C} = \frac{xQ^2}{2\epsilon_0 A}$

(b) Increase the separation by dx .

$$U = \frac{(x+dx)Q^2}{2\epsilon_0 A} = U_0(1+dx/x)$$

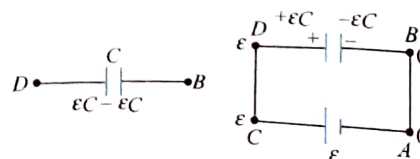
The change is then $(Q^2/2\epsilon_0 A)dx$.

(c) The work done in increasing the separation is given by

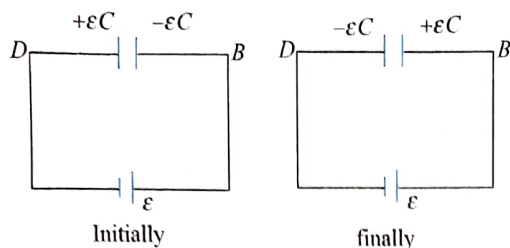
$$dW = U - U_0 = \frac{dxQ^2}{2\epsilon_0 A} = Fdx \text{ or } F = \frac{Q^2}{2\epsilon_0 A}$$

(d) The reason for the difference is that E is the field due to both plates. The force is QE if E is the field due to one plate and Q is the charge on the other plate.

10. Since the initial and final charges on the capacitor are same before and after connection, no charge will flow in the circuit; so heat loss is 0.



11.



From figure, net charge flow through battery is

$$Q_{\text{final}} - Q_{\text{initial}} = \epsilon C - (-\epsilon C) = 2\epsilon C$$

 Thus, work done by battery (W) is $Q \times V = 2\epsilon C \times \epsilon = 2\epsilon^2 C$ or heat produced is $2\epsilon^2 C$.

12. Capacitance of sphere

$$C = 4\pi\epsilon_0 R = \frac{0.5 \times 10^{-3}}{9 \times 10^9} = \frac{1}{18} \times 10^{-12} \text{ F}$$

Rate to escape of charge from surface

$$= \frac{80}{100} \times 6.25 \times 10^{10} \times 1.6 \times 10^{-19} = 8 \times 10^{-9} \text{ C/s}$$

 therefore $q = (8 \times 10^9)t$ and

$$q = CV \Rightarrow 8 \times 10^9 t = \frac{1}{18} \times 10^{-12} \times 1$$

$$\Rightarrow t = \frac{10^{-12}}{8 \times 10^9 \times 18} = \frac{10^{-3}}{144} = 6.95 \mu\text{s}$$

 13. Energy losses $\Delta U = \frac{1}{2} CV_0^2 - \frac{1}{2} CV^2$

Fractional energy loss

$$\frac{\Delta U}{U_0} = \frac{\frac{1}{2} CV_0^2 - \frac{1}{2} CV^2}{\frac{1}{2} CV_0^2} = \frac{V_0^2 - V^2}{V_0^2}$$

$$= \frac{(100)^2 - (80)^2}{(100)^2} = \frac{20 \times 180}{(100)^2} = \frac{9}{25}$$

14. As the plates were moved very slowly, there will not loss of energy in the form of heat.

 (a) The charge in capacitor, $Q_0 = C_0 V_0$

$$\text{Initial energy stored, } U_i = \frac{1}{2} C_0 V_0^2$$

 After the separation is made half new capacitance, $C = 2C_0$

The charge will remain constant,

$$\text{Final energy stored, } U_f = \frac{Q_0^2}{2(C)} = \frac{C_0^2 V_0^2}{2(2C_0)} = \frac{1}{4} C_0 V_0^2$$

$$\text{Work done by the external agent, } W = \Delta U = U_f - U_i = -\frac{1}{4} C_0 V_0^2$$

The two plates attract each other. The external agent applies force that is opposite to the displacement. Hence, work done is negative,

(b) This time the potential difference across the capacitor remains constant.

$$\text{Initial energy stored, } U_i = \frac{1}{2} C_0 V_0^2$$

$$\text{Final energy stored, } U_f = \frac{1}{2} (2C_0) V_0^2$$

$$\text{Change in potential energy, } \Delta U = \frac{1}{2} C_0 V_0^2$$

The additional charge supplied by battery,

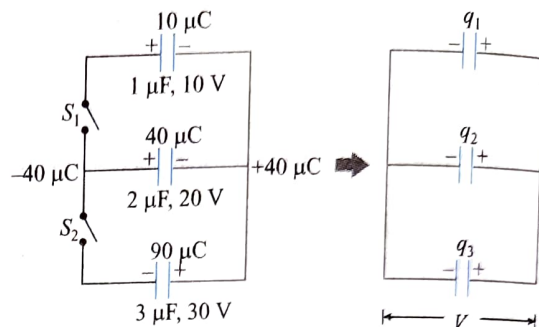
$$\Delta Q = 2C_0 V_0 - C_0 V_0 = C_0 V_0$$

 Work done by the battery, $W_{\text{battery}} = \Delta Q V_0 = C_0 V_0^2$

$$W_{\text{external}} + W_{\text{battery}} = \Delta U$$

$$\therefore W_{\text{external}} = \frac{1}{2} C_0 V_0^2 - C_0 V_0^2 = -\frac{1}{2} C_0 V_0^2$$

15. The situation is shown in figure


 Total charge on all three capacitors = $(90 - 40 - 10) \mu\text{C} = 40 \mu\text{C}$

 Total capacity = $(1 + 2 + 3) \mu\text{F} = 6 \mu\text{F}$

(a) Common potential,

$$V = \frac{\text{Total charge}}{\text{Total capacity}} = \frac{40 \mu\text{C}}{6 \mu\text{F}} = \frac{20}{3} \text{ volt}$$

$$(b) \quad q_1 = C_1 V = (1 \mu\text{F}) \left(\frac{20}{3} \right) = \frac{20}{3} \mu\text{C}$$

$$q_2 = C_2 V = (2 \mu\text{F}) \left(\frac{20}{3} \right) = \frac{40}{3} \mu\text{C}$$

$$q_3 = C_3 V = (3 \mu\text{F}) \left(\frac{20}{3} \right) = 20 \mu\text{C}$$

Exercise 4.2

 1. Since E inside the conductor is zero, the induced electric field $\vec{E}_{\text{ind}}$ exactly nullifies the external electric field; $\vec{E}_{\text{ext}} = \vec{E}_0$. Outside the plate the equal and opposite surface charges contribute no net field. Hence, outside the plate, the net field is equal to the applied (external) field E_0 .

 Since, $\vec{E}_{\text{outside}} = \frac{\sigma}{\epsilon_0} \hat{n}$ and $\vec{E}_{\text{outside}} = E_0$, we can write, $\sigma = \epsilon_0 E_0$

 2. (i) The charge q remains constant

$$\text{Hence, } \frac{U}{U_0} = \frac{C_0}{C} = \frac{1}{\epsilon_r}$$

$$\text{Or } U = \frac{U_0}{\epsilon_r} = \frac{10^{-3}}{100} = 10^{-5} \text{ J}$$

 (ii) Since $U < U_0$, $\Delta U = -U_0 \left(1 - \frac{1}{\epsilon_r} \right)$; this extra energy goes to work done by the external agent who introduces the dielectric slab into the space between capacitor plates.

$$3. (i) \quad \frac{U}{U_0} = \frac{\left(\frac{1}{2} C V^2 \right)}{\left(\frac{1}{2} C_0 V^2 \right)} = \frac{C}{C_0} = \epsilon_r$$

$$\text{Or } U = U_0 \epsilon_r = (2 \times 10^{-4}) (150) = 0.03 \text{ J}$$

The excess energy $\Delta U = U_0(\epsilon_r - 1) > 0$. This extra energy is drawn from the supply.

(ii) $\Delta q = (\Delta C)V = (\epsilon_r - 1)C_0 = 149 \times 4 \times 10^{-8} = 5.96 \times 10^{-6} \text{ C}$

(iii) The charge induced in dielectric, $q_{\text{induced}} = q_f \left(1 - \frac{1}{\epsilon_r}\right) = \frac{149}{150} q_f$,

$$\Rightarrow \frac{q_{\text{induced}}}{q_f} = \frac{149}{150}$$

4. Let the charge on the capacitor be Q . The surface charge density should be $\sigma = \frac{Q}{A}$ and the electric field $E = \frac{Q}{KA\epsilon_0}$.

This electric field should not exceed the dielectric strength

$$E_{\text{break}} = 1.9 \times 10^7 \text{ V/m}$$

$\therefore$ if the maximum charge which can be given is Q then

$$\frac{Q}{KA\epsilon_0} = E_{\text{break}}$$

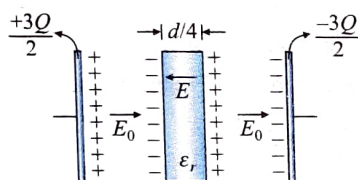
$$Q = E_{\text{break}} \times KA\epsilon_0 = 1.9 \times 10^7 \times (5.0) \times (10^{-2}) \times (8.85 \times 10^{-12})$$

$$= 8.4 \times 10^{-6} \text{ C.}$$

5. (a) The charge density in the inner side of conductors is given as

$$\sigma = \frac{\left[\frac{Q}{A} - \frac{(-2Q)}{2A}\right]}{A} = \frac{3Q}{2A}$$

(b) The charge density on the dielectric slab is



$$\sigma' = \sigma \left(1 - \frac{1}{\epsilon_r}\right) = \sigma \left(1 - \frac{1}{2}\right) = \frac{\sigma}{2} = \frac{3Q}{4A}$$

(c) The electric field in the dielectric is

$$E = \frac{E_0}{\epsilon_r} = \frac{\left(\frac{\sigma}{\epsilon_r}\right)}{\epsilon_r} = \frac{3Q}{2\epsilon_0\epsilon_r A} = \frac{3Q}{4\epsilon_0 A}$$

(d) The potential difference between the conductor is

$$V = E_0 \left(d - \frac{d}{4}\right) + E \frac{d}{4} = \frac{3Q}{2\epsilon_0 A} \cdot \frac{3d}{4} + \frac{3Q}{4\epsilon_0 A} \cdot \frac{d}{4} = \frac{21Qd}{16\epsilon_0 A}$$

(e) The capacitance is

$$C = \frac{\left(\frac{3Q}{2}\right)}{V} = \frac{\left(\frac{3Q}{2}\right)}{\left(\frac{21Qd}{16\epsilon_0 A}\right)} = \frac{8\epsilon_0 A}{7d}$$

6. (a) (i) In case of a capacitor as $E = (V/d)$, the potential difference between the plates before the introduction of metal plate

$$V = E \times d = 3 \times 10^4 \times 0.05 = 1.5 \text{ KV}$$

(ii) Now as after charging battery is removed, capacitor is isolated so $q = \text{constant}$. If C' and V' are the capacity and potential after the introduction of plate

$$q = CV = C'V' \text{ i.e., } V' = \frac{C}{C'} V$$

And as $C = \frac{\epsilon_0 A}{d}$

and $C' = \frac{\epsilon_0 A}{(d-t) + (t/K)}, V' = \frac{(d-t) + (t/K)}{d} \times V$

So in case of metal plate as

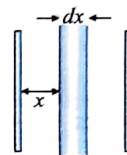
$$K = \infty, V_M = \left[\frac{d-t}{d}\right] \times V = \left[\frac{0.05 - 0.01}{0.05}\right] \times 1.5 = 1.2 \text{ KV}$$

(b) And if instead of metal plate, dielectric with $K = 2$ is introduced

$$V_D = \frac{(d-t) + (t/K)}{d} \times V$$

$$= \left[\frac{(0.05 - 0.01) + (0.01/2)}{0.05}\right] \times 1.5 = 1.35 \text{ KV}$$

7. (a) The capacitance of the system can be thought of as the series combination of elementary capacitance dC



$$\frac{1}{C_{\text{eq}}} = \int \frac{1}{dC} = \int \frac{1}{\left(\frac{\epsilon_0 \epsilon_r A}{dx}\right)} = \frac{d}{\epsilon_0 A} \int_0^d \frac{dx}{\epsilon_r}$$

$$= \frac{1}{\epsilon_0 A} \int_0^d \frac{dx}{a + bx} = \frac{1}{\epsilon_0 A b} \ln \left(\frac{a + bd}{a} \right)$$

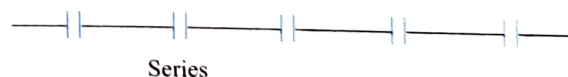
Or $C_{\text{eq}} = \frac{\epsilon_0 A b}{\ln \left(\frac{a + bd}{a} \right)}$

(b) $V = \int E dx = \int \frac{Q}{\epsilon_0 A \epsilon_r} dx = \frac{Q}{\epsilon_0 A} \int_0^x \frac{dx}{\epsilon_r}$

$$= \frac{Q}{\epsilon_0 A} \int_0^x \frac{dx}{a + bx} = \frac{Q}{\epsilon_0 A b} \ln \left(\frac{a + bx}{a} \right)$$

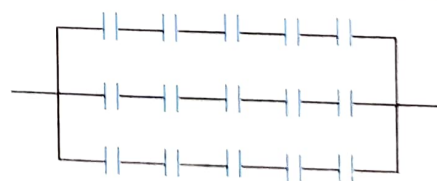
Exercise 4.3

1. (a) Five capacitors in series



Series

(b) Three rows in parallel with each row containing five capacitors



Series and parallel common grouping

2. In series, potentials of all capacitors will be added, so net potential will be NV .

3. Capacitors 4 and 5 are short-circuited. So, they are useless. '1', '2', and '3' will be in parallel and then 6 will be in series with them.

$$C_{\text{eq}} = 3C/4$$

4. $V = 10 \text{ V}$

$$C_{\text{eq}} = C_1 + C_2 \text{ [} C_1 \text{ and } C_2 \text{ are in parallel]}$$

$$= 6 + 5 = 11 \mu\text{F}$$

$$Q = CV = 11 \times 10 = 110 \mu\text{C}$$

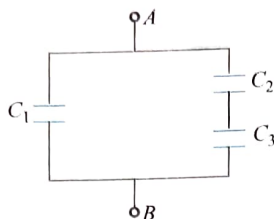
5. It is equivalent to

$$C = C_1 + \frac{C_2 C_3}{C_2 + C_3}$$

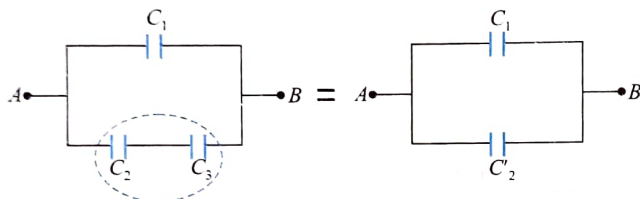
$$= \frac{A_1 K_1 \epsilon_0}{d_1 + d_2} + \frac{\frac{A_2 K_2 \epsilon_0}{d_1} \cdot \frac{A_2 K_3 \epsilon_0}{d_2}}{\frac{A_2 K_2 \epsilon_0}{d_1} + \frac{A_2 K_3 \epsilon_0}{d_2}}$$

$$= \frac{A_1 K_1 \epsilon_0}{d_1 + d_2} + \frac{A_2^2 K_2 K_3 \epsilon_0^2}{A_2 K_2 \epsilon_0 d_2 + A_2 K_3 \epsilon_0 d_1}$$

$$= \frac{A_1 K_1 \epsilon_0}{d_1 + d_2} + \frac{A_2 K_2 K_3 \epsilon_0}{K_2 d_2 + K_3 d_1}$$



6. The given system can be redrawn as shown in figure.



$$\text{Here } C_1 = \frac{\epsilon_0 \epsilon_r (A/2)}{d} = \frac{\epsilon_0 A}{2d}$$

$$C_2 = \frac{\epsilon_0 \epsilon_r (A/2)}{(d/2)} = \frac{2\epsilon_0 A}{d}$$

$$\text{and } C_3 = \frac{\epsilon_0 \epsilon_r (A/2)}{(d/2)} = \frac{3\epsilon_0 A}{d}$$

The equivalent capacitance across A and B,

$$C_{AB} = C_1 + \frac{C_2 C_3}{C_2 + C_3} = \frac{17\epsilon_0 A}{10d}$$

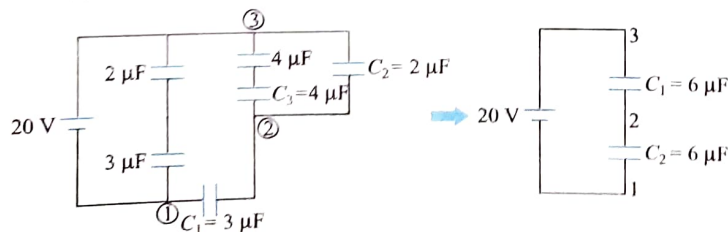
$$7. \frac{V_1}{V_{1.5}} = \frac{1.5}{1}$$

$$\therefore V_1 = \left(\frac{1.5}{1.5+1} \right) (30) = 18 \text{ V}$$

$$\frac{V_{2.5}}{V_{0.5}} = \frac{0.5}{2.5} = \frac{1}{5} \therefore V_{2.5} = \left(\frac{1}{1+5} \right) (30) = 5 \text{ V}$$

$$\text{Now, } |V_{ab}| = V_1 - V_{2.5} = 13 \text{ V}$$

8. (a) Simple circuit is as shown below.


 (b) Hence equivalent capacitance $C_{\text{eq}} \Rightarrow \frac{6 \times 6}{6+6} = 3 \mu\text{F}$.

 (c) Upper network and lower network both have same capacitance $= 6 \mu\text{F}$

$$V_1 - V_2 = \frac{20}{2} = 10 \text{ V} \Rightarrow V_{C_1} = 10 \text{ V}$$

$$q_{C_1} = (C_1)(V_{C_1}) = 30 \mu\text{C}$$

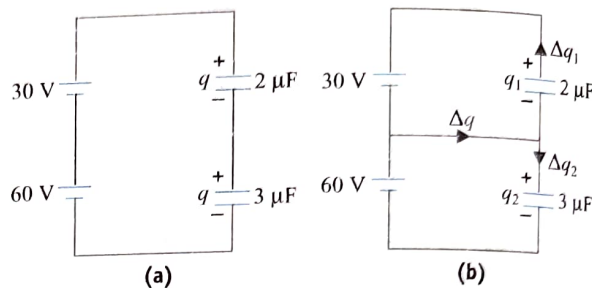
$$(d) V_{C_2} = 10 \text{ V}, \therefore q_{C_2} = (C_2)(V_{C_2}) = 20 \mu\text{C}$$

$$(e) V_{C_3} = 5 \text{ V}, \therefore q_{C_3} = (C_3)(V_{C_3}) = 20 \mu\text{C}$$

 9. When switch is opened, $2 \mu\text{F}$ and $3 \mu\text{F}$ capacitors are in series. So,

$$C_{\text{eq}} = \frac{2 \times 3}{5} = \frac{6}{5} \mu\text{F}$$

$$\text{Hence, charge on each capacitor, } q = CV = \frac{6}{5} \times 90 = 108 \mu\text{C}$$


 When switch S is closed, let q_1 and q_2 be charge on the two capacitors as Fig. (b). So,

$$q_1 = 2 \times 30 = 60 \mu\text{C}$$

$$q_2 = 3 \times 60 = 180 \mu\text{C}$$

 From figure it is clear that, after closing the switch the charge flowing out of $2 \mu\text{F}$ capacitor,

$$\Delta q_1 = 108 - 60 = 48 \mu\text{C}$$

 And the charge flown into $3 \mu\text{F}$ capacitor, $\Delta q_2 = 180 - 108 = 72 \mu\text{C}$

So the net charge flown through the switch,

$$\Delta q = \Delta q_1 + \Delta q_2 = 48 + 72 = 120 \mu\text{C}$$

 10. Charge across $5 \mu\text{F}$ capacitor $10 \mu\text{C}$.

 Potential difference across $5 \mu\text{F}$ capacitor is

$$\frac{q}{C} = \frac{10 \times 10^{-6}}{5 \times 10^{-6}} = 2 \text{ volt}$$

 As $3, 4$, and $5 \mu\text{F}$ capacitors are in parallel, so potential difference across each of them equals to 2 V .

 Charge on $3 \mu\text{F}$ capacitor $= CV = 3 \times 10^{-6} \times 2 = 6 \mu\text{C}$

 Charge on $4 \mu\text{F}$ capacitor $4 \times 10^{-6} \times 2 = 8 \mu\text{C}$

Total charge flowing in upper branch of circuit is

$$10 \mu\text{C} + 6 \mu\text{C} + 8 \mu\text{C} = 24 \mu\text{C}$$

 Potential difference across C_2 is $\frac{24 \mu\text{C}}{4 \mu\text{F}} = 6 \text{ V}$

 Total potential difference across AB is $= 6 + 2 = 8 \text{ V}$.

Equivalent capacitance of lower branch of circuit is

$$\frac{6 \times 3}{6+3} = 2 \mu\text{F}$$

 So, charge flowing is $= 2 \times 10^{-6} \times 8 = 16 \mu\text{C}$

Therefore, potential difference between A and C is

$$\frac{16 \mu\text{C}}{3 \mu\text{F}} = \frac{16}{3} = 5.33 \text{ V}$$

11. Before closing the switch,

$$C_{\text{eq}} = \frac{2C \times C}{2C + C} = \frac{2}{3} C = \frac{2}{3} \times 2 = \frac{4}{3} \mu\text{F}$$

$$\therefore q_i = C_{\text{eq}} V = \frac{4}{3} \times 30 = 40 \mu\text{C}$$

 The energy stored is $U_i = \frac{1}{2} C_{\text{eq}} V^2 = 6 \times 10^{-4} \text{ J}$

 after closing the switch $C'_{\text{eq}} = C$ { $\because$ Left capacitors are short circuited }.

$$V = CV = 2 \times 30 = 60 \mu\text{C}$$

$$\text{and } U_f = \frac{1}{2} C_{eq} V^2 = \frac{1}{2} C V^2 = 9 \times 10^{-4} \text{ J}$$

$$(a) \Delta q = q_f - q_i = 60 - 40 = 20 \mu\text{C}$$

$$(b) \text{ and } (c) W_{\text{cell}} = \Delta U + \Delta H \text{ or } V \Delta q = U_f - U_i + \Delta H$$

$$\Delta U = 6 \times 10^{-4} - 3 \times 10^{-4} = 3 \times 10^{-4} \text{ J} = 0.3 \text{ mJ}$$

$$W_{\text{cell}} = V \Delta q = 30 \times 20 \times 10^{-6} = 6 \times 10^{-4} = 0.6 \text{ mJ}$$

$$\text{Heat generated } \Delta H = W_{\text{battery}} - \Delta U$$

$$= 0.6 - 0.3 = 0.3 \text{ mJ}$$

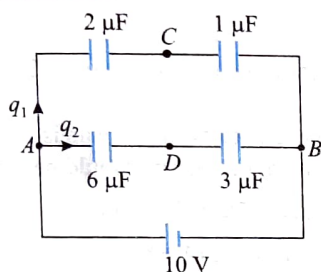
$$(d) \text{ The charge flow through switch} = \text{charge supplied by battery} + \text{charges flowing for neutralization of leftward capacitors}$$

$$= 20 + 20 + 20 = 60 \mu\text{C}$$

Charges on each leftward capacitors are $20 \mu\text{C}$ before short circuiting.

Exercise 4.4

1. Given circuit is balanced Wheatstone bridge. Hence $V_C = V_D = 0 \text{ V}$.
Now the circuit reduces to



$$\text{Hence } V_{AC} = 10 \left(\frac{1}{2+1} \right) = \frac{10}{3} \text{ V}$$

$$\text{and } V_{AD} = 10 \left(\frac{3}{6+3} \right) = \frac{10}{3} \text{ V}$$

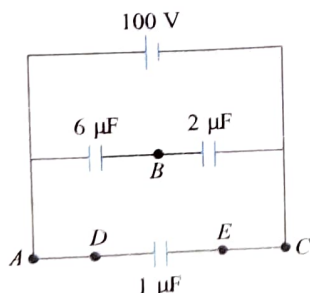
$$\text{It means } V_A = \frac{10}{3} \text{ V}$$

$$\text{As } V_A - V_B = 10 \text{ V}$$

$$\frac{10}{3} - V_B = 10 \Rightarrow V_B = -\frac{20}{3} \text{ V}$$

$$\text{Charge on } 2 \mu\text{F capacitor } q_{C_2} = 2 \times \frac{10}{3} = \frac{20}{3} \mu\text{C}$$

2. Both $3 \mu\text{F}$ capacitors will be in parallel, and similarly upper two $1 \mu\text{F}$ capacitors will be in parallel. The circuit may be redrawn as follows:



$$(a) V_A - V_B = \frac{100 \times 2}{6+2} = 25 \text{ V}$$

$$(b) V_A - V_C = \frac{100 \times 6}{6+2} = 75 \text{ V}$$

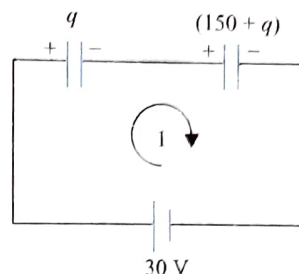
$$(c) V_D - V_E = 100 \text{ V}$$

$$(d) C_{eq} = \frac{6 \times 2}{6+2} + 1 = \frac{5}{2} \mu\text{F}$$

$$U = \frac{1}{2} C_{eq} V^2 = \frac{1}{2} \left(\frac{5}{2} \times 10^{-6} \right) \times (100)^2$$

$$= 125 \times 10^{-4} \text{ J}$$

3. The initial charge on the larger capacitor is



$$Q = C \Delta V = 10 \mu\text{F} (15 \text{ V}) = 150 \mu\text{C}$$

An additional charge is pushed through the 30 V battery, giving the smaller capacitor charge q and the larger charge $(150 + q)$.

In close loop (1)

$$30 - \frac{q}{5} - \frac{(150+q)}{10} = 0 \text{ or } q = 50 \mu\text{C}$$

Potential difference across C_1

$$V_1 = \frac{q}{C_1} = \frac{50}{5} = 10 \text{ V} \text{ and } V_2 = \frac{200}{10} = 20 \text{ V}$$

4. (a) Since a switch is open for long time. The capacitor is in series. Hence equivalent capacity is $C_{eq} = C/2$. Initial charge on each capacitor is $q_0 = C\varepsilon/2$. When switch is closed, final charge on capacitor C_1 , is $q_1 = C\varepsilon$ and final charge in C_2 , is $q_2 = 0$. Hence, when the switch is closed, charge flow through battery is

$$\Delta q = \left(C\varepsilon - \frac{C\varepsilon}{2} \right) = \frac{C\varepsilon}{2}$$

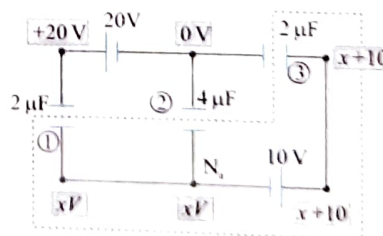
$$(b) \text{ Work done by battery is } \Delta q \cdot \varepsilon = \frac{C\varepsilon}{2} \times \varepsilon = \frac{C\varepsilon^2}{2}$$

(c) Change in energy stored is

$$\frac{1}{2} C \varepsilon^2 - \frac{1}{2} \times \left(\frac{C}{2} \right) \times \varepsilon^2 = \frac{1}{4} C \varepsilon^2$$

$$(d) \text{ Heat developed is } W_{\text{battery}} - \Delta U = \frac{C\varepsilon^2}{2} - \frac{C\varepsilon^2}{4} = \frac{C\varepsilon^2}{4}$$

5. Figure shows an isolated system.



For isolated system, net charge is zero.

$$2(x-20) + 4x + (x+10)2 = 0$$

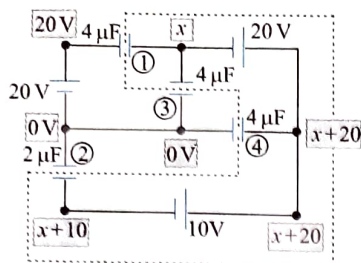
$$\text{or } 8x - 20 \text{ or } x = 2.5 \text{ V}$$

Thus, charge on capacitor (1) is $35 \mu\text{C}$

Thus, charge on capacitor (2) is $10 \mu\text{C}$

Thus, charge on capacitor (3) is $25 \mu\text{C}$

6. The figure shows an isolated system that includes all the capacitor plates isolated from other parts of circuit. We can conclude net sum of charge should be zero.



$$2(x+10) + 4(x+20) = 6(0-x) + 4(x-20) = 0$$

$$\text{or } 4x + 20 = 0 \text{ or } x = -5V$$

$$\text{Hence } q_1 = 4[20 - (-5)] = 100 \mu\text{C}$$

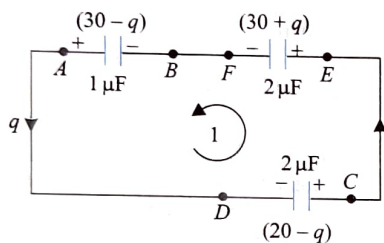
$$q_2 = 2[(-5) + 10 - 0] = 10 \mu\text{C}$$

$$q_3 = 6[0 - (-5)] = 30 \mu\text{C}$$

$$q_4 = 4[(-5) + 20 - 0] = 60 \mu\text{C}$$

7. Let charge flow be q .

Now applying Kirchhoff's voltage law, we get



$$\frac{(20-q)}{2} - \frac{(30+q)}{2} + \frac{(30-q)}{1} = 0$$

$$\text{or } -2q = -25 \text{ or } q = 12.5 \mu\text{C}$$

$$\text{Charge on } C_1 \text{ is } q_1 = 30 - 12.5 = 17.5 \mu\text{C}$$

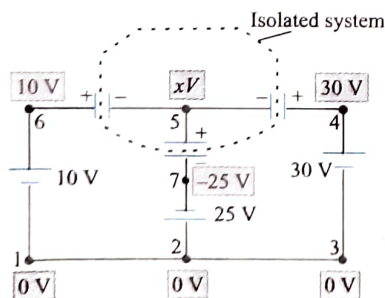
$$\text{Charge on } C_2 \text{ is } q_2 = 30 + 12.5 = 42.5 \mu\text{C}$$

$$\text{Charge on } C_3 \text{ is } q_3 = 20 - 12.5 = 7.5 \mu\text{C}$$

8. Let potential at 1 is 0, so at 4 it is 30 V, at 6 it is 10 V, and at point 7 potential is -25 V. Let us assume potential at point 5 be xV . Let us take isolated system as shown in the figure. (Total charge of all the plates connected to '5' must be same as before, i.e., 0.)

$$\therefore 1(x-10) + 2(x-30) + 2(x+25) = 0$$

$$\text{or } 5x = 20 \text{ or } x = 4$$



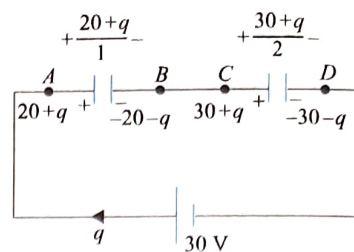
Final charges:

$$\text{On } C_1, Q_1 \text{ is } (30 - 4)2 = 52 \mu\text{C}$$

$$\text{On } C_2, Q_2 \text{ is } 2(4 - (-25)) = 58 \mu\text{C}$$

$$\text{On } C_3, Q_3 \text{ is } 1(10 - 4) = 6 \mu\text{C}$$

9. Now applying Kirchhoff's voltage law



$$\text{or } \frac{-(20+q)}{1} - \frac{30+q}{2} + 30 = 0$$

$$\text{or } -40 - 2q - 30 - q = -60$$

$$\text{or } 3q = -10$$

$$\text{Charge flow is } -10/3 \mu\text{C}$$

$$\text{Charge on capacitor of capacitance } 1 \mu\text{F is } 20 + q = \frac{50}{3} \mu\text{C}$$

$$\text{Charge on capacitor of capacitance } 2 \mu\text{F is } 30 + q = \frac{80}{3} \mu\text{C}$$

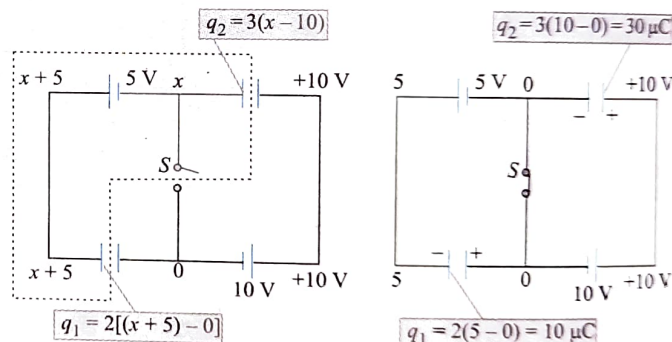
10. When switch is opened, let us consider an isolated system as shown. In this isolated system, net charge should be zero.

$$3(x-10) + 2(x+5)2 = 0 \text{ or } x = 4V$$

$$\text{Hence charge on } C_1 \text{ is } q_1 = 2(4+5) = 18 \mu\text{C}$$

$$\text{And charge on } C_2 \text{ is } q_2 = 3(4-10) = -18 \mu\text{C, or left plate is negative.}$$

$$\text{Hence, charge on } C_2 \text{ is } q_2 = 18 \mu\text{C (right plate is positive)}$$

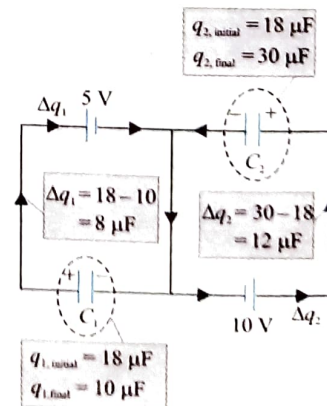


When switch is open

When switch is closed

When switch is closed, the final charge on the capacitors is shown in the figure. Note that $\Delta q_1 = 8 \mu\text{C}$ is charging battery; hence, work done by 5 V battery will be negative.

$$W_{SV} = -(8 \mu\text{C})(5V) = -40 \mu\text{J}$$



Δq_2 is moving from positive terminal of 10 V battery, hence work done by 10 V battery is $(120 \mu\text{C})(10 \text{ V}) = 120 \mu\text{J}$.

Net work done by batteries is

$$W_{\text{battery}} = -40 + 120 = 80 \mu\text{J}$$

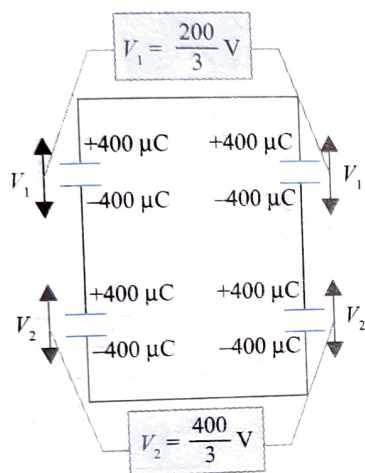
Change in potential energy of the capacitors

$$\Delta U = U_f - U_i$$

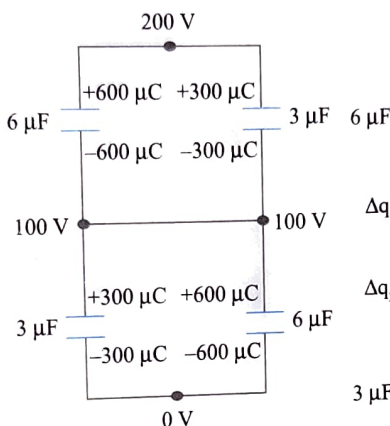
$$= \left(\frac{1}{2} 3[10]^2 + \frac{1}{2} 2[5]^2 \right) - \left(\frac{1}{2} 3[6]^2 + \frac{1}{2} 2[9]^2 \right) = 40 \mu\text{J}$$

Heat developed in the circuit is

$$H = W_{\text{battery}} - \Delta U = 80 - 40 = 40 \mu\text{J}$$



Before closing the switch



When switch is closed

When switch is closed, the potentials of a and b will be equal to 100 V. Thus, initial charge and final charge distribution on all the plates is shown in figure. Figure shows charge transfer on plates connected by switch. Charge flowing through switch is $300 \mu\text{C}$.

Exercise 4.5

1. (a) $q_1 = 80 \mu\text{C}$, $q_2 = 20 \mu\text{C}$

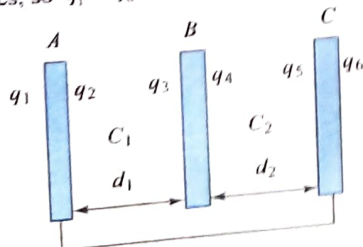
$$q_3 = -20 \mu\text{C}$$
, $q_4 = 80 \mu\text{C}$

(b) $q_1 = 0$, $q_2 = -60 \mu\text{C}$

$$q_3 = 60 \mu\text{C}$$
, $q_4 = 0$

(c) $q_1 = q_2 = q_3 = q_4 = 0$

2. Since charges on outer surfaces will be half of the total charge of all the three plates, so $q_1 = q_6 = 60/2 = 30 \mu\text{C}$.



$$q_3 = -q_2, q_4 = -q_5$$

$$\Rightarrow V_{AB} = V_{CB}$$

$$\text{or } \frac{q_2}{C_1} = \frac{q_5}{C_2}$$

11. When switch S is open, for isolated system 1,

$$6(x - 200) + 3(x - 0) = 0$$

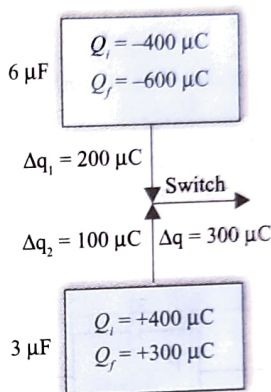
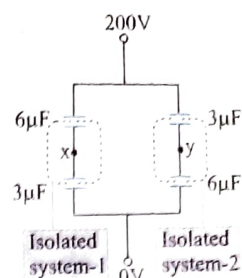
$$\text{or } x = \frac{400}{3} \text{ V}$$

Similarly for isolated system 2

$$3(y - 200) + 6(y - 0) = 0$$

$$\text{or } y = \frac{200}{3} \text{ V}$$

$$\text{Thus } V_a - V_b = x - y = \frac{400}{3} - \frac{200}{3} = \frac{200}{3} \text{ V}$$



$$q_2 d_1 = q_5 d_2 \Rightarrow q_2 2d_2 = q_5 d_2$$

$$q_5 = 2q_2 \Rightarrow q_4 = 2q_3$$

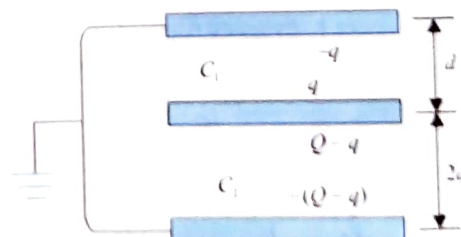
$$\text{And } q_3 + q_4 = 60$$

$$q_3 = 20 \mu\text{C} \Rightarrow q_4 = 40 \mu\text{C}$$

$$\text{And } q_2 = -20 \mu\text{C} \Rightarrow q_5 = -40 \mu\text{C}$$

$$3. C = \frac{\epsilon_0 A}{d} \Rightarrow C \propto \frac{1}{d}$$

$$\frac{C_1}{C_2} = \frac{2d}{d} = 2$$



Potential of middle plate

$$V = \frac{q}{C_1} = \frac{Q - q}{C_2}$$

$$\text{or } \frac{q}{Q - q} = \frac{C_1}{C_2} = 2 \quad \text{or } q = \frac{2Q}{3}$$

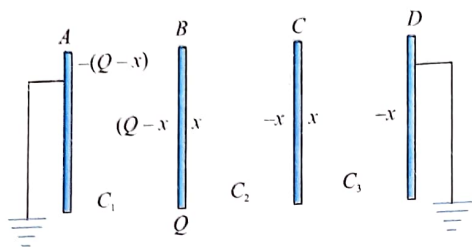
So the induced charge on upper plate is $-2Q/3$ and on lower plate is $-Q/3$.

4. Moving from plate *A* to *D*

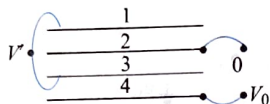
$$0 + \frac{(Q-x)}{C} - \frac{x}{C} - \frac{x}{C} = 0 \quad \text{or} \quad x = \frac{Q}{3}$$

Hence, potential difference between *B* and *C*

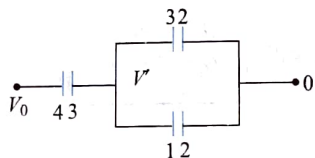
$$V_{BC} = \frac{x}{C} = \frac{Q}{3C} = \frac{Qd}{3\epsilon_0 S}$$



5. (a)



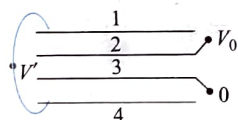
Equivalent circuit



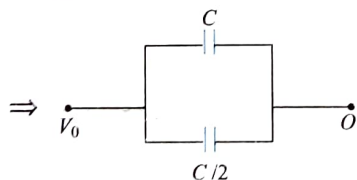
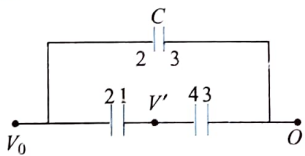
$$\Rightarrow V_0 \quad C \quad 2C \quad 0$$

$$C_{eq} = \frac{2C}{3} = \frac{2\epsilon_0 A}{3d}$$

(b)

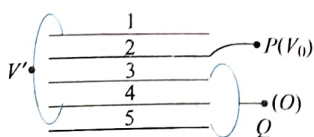


Equivalent circuit

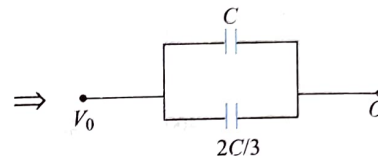
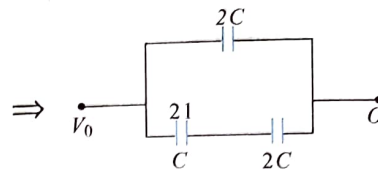
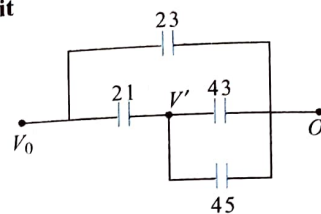


$$C_{eq} = \frac{3}{2}C = \frac{3\epsilon_0 A}{2d}$$

(c)

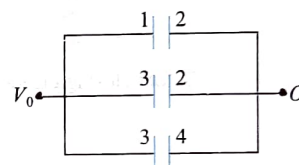
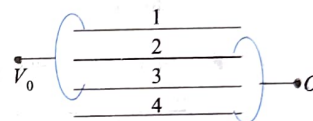


Equivalent circuit



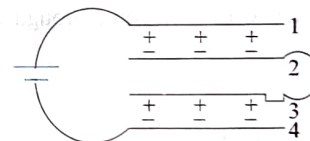
$$C_{eq} = \frac{5C}{2} = \frac{5\epsilon_0 A}{3d}$$

(d)



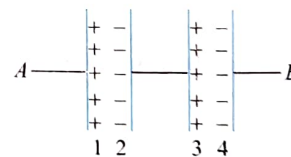
$$C_{eq} = 3C = \frac{3\epsilon_0 A}{d}$$

6.



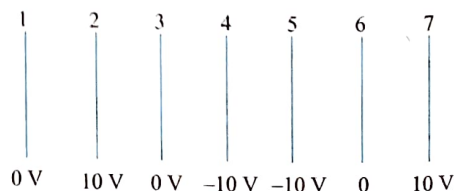
Plates can be rearranged as shown in figure.

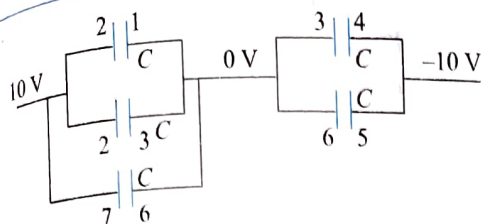
Plates 1 and 2 and plates 3 and 4 form two capacitors, which are in series between *A* and *B*. Plates 2 and 3 do not form any capacitor as they are at the same potential.



$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{\left(\frac{\epsilon_0 A}{d}\right) \times \left(\frac{\epsilon_0 A}{3d}\right)}{\frac{\epsilon_0 A}{d} + \frac{\epsilon_0 A}{3d}} = \frac{\epsilon_0 A}{4d}$$

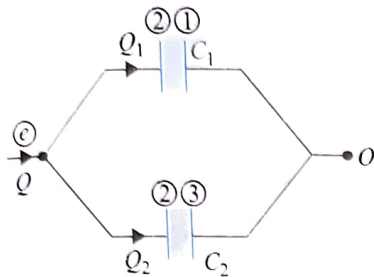
7.





$$C_{eq} = \frac{3C \times 2C}{3C + 2C} = \frac{6C}{5} = \frac{6\epsilon_0 A}{5l}$$

8. Equivalent circuit is two capacitors in parallel



$$C_1 = \frac{\epsilon_0 \epsilon_1 A}{d} = \frac{2\epsilon_0 A}{d} \quad \dots(i)$$

$$C_2 = \frac{\epsilon_0 \epsilon_2 A}{d} = \frac{3\epsilon_0 A}{2d} \quad \dots(ii)$$

C_1 and C_2 are in parallel.

$$C_{eq} = C_1 + C_2 = \frac{7\epsilon_0 A}{2d}$$

Charge drawn through the battery is

$$Q = C_{eq} \epsilon = \frac{7\epsilon_0 A \epsilon}{2d}$$

Charge in C_1 is $Q_1 = C_1 \epsilon = \frac{2\epsilon_0 A \epsilon}{d}$

And charge in C_2 is $Q_2 = C_2 \epsilon = \frac{3\epsilon_0 A \epsilon}{2d}$

The charge on outer surfaces of plates will be zero.

Hence, charge on plate (1) is $q_1 = -\frac{2\epsilon_0 A \epsilon}{d}$

Charge on plate (2) is $q_2 = \frac{2\epsilon_0 A \epsilon}{d} + \frac{3\epsilon_0 A \epsilon}{2d} = \frac{7\epsilon_0 A \epsilon}{2d}$

Charge on plate (3) is $q_3 = -\frac{3\epsilon_0 A \epsilon}{2d}$

Energy stored in system is $U = \frac{1}{2} C_{eq} \epsilon^2 = \frac{7\epsilon_0 A \epsilon^2}{4d}$

9. The charge appearing on outer most surface should be zero,

$$q_1 = q_6 = 0$$

and $V_{AB} = V$ with $V_A > V_B$

$$|q_2| = |q_3| = CV = \left(\frac{\epsilon_0 A}{d} \right) V$$

$$q_2 = +\frac{\epsilon_0 AV}{d} \text{ and } q_3 = -\frac{\epsilon_0 AV}{d}$$

Similarly, $V_{BC} = V$ with $V_C > V_B$

$$|q_4| = |q_5| = CV = \left(\frac{\epsilon_0 A}{2d} \right) V$$

$$\text{With } q_5 = +\frac{\epsilon_0 AV}{2d} \text{ and } q_4 = -\frac{\epsilon_0 AV}{2d}$$

Exercises

Single Correct Answer Type

1. (4) $C_{eq} = C + 2C + 3C + \dots + nC$

$$= \frac{(n+1)n}{2} C$$

2. (1) As battery is disconnected, so charge will remain the same. It is given that final potential is the same. So final capacitance should be equal to initial capacitance, i.e.,

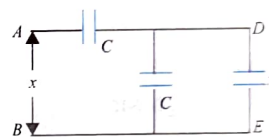
$$\frac{\epsilon_0 A}{d} = \frac{\epsilon_0 A}{(1.6+d) - t(1-1/K)}$$

$$\text{or } K = 5$$

3. (3) There are two capacitors in parallel.

$$U = \frac{1}{2} CV^2, C = \frac{2\epsilon_0 A}{d}$$

4. (2) Let capacitance between A and B be x . If we remove the first two capacitors, then capacitance across D and E will also be x . So the circuit can be redrawn as follows:



$$x = \frac{C(C+x)}{C+(C+x)}$$

$$\text{Solve to get } x = \frac{\sqrt{5}-1}{2} C$$

5. (4) $C_{eq} = \frac{\epsilon_0}{\frac{d}{K_1} + \frac{d}{K_2} + \frac{d}{K_3}}$

$$\text{Here, } K_1 = K_3 = 1, K_2 = \epsilon/\epsilon_0$$

6. (2) $V - 0 = \frac{Q}{C} = \frac{Q(b-a)}{K 4\pi\epsilon_0 ab}$

$$\text{or } V = \frac{9 \times 10^9 \times 2.5 \times 10^{-6} (0.13 - 0.12)}{32 \times 0.13 \times 0.12} = 450 \text{ V}$$

7. (4) $W = U_2 - U_1 = \frac{q^2}{2} \left[\frac{1}{C_2} - \frac{1}{C_1} \right]$

$$C_1 = \frac{\epsilon_0 A}{d}, C_2 = \frac{C_1}{2} = \frac{\epsilon_0 A}{2d}$$

$$q = C_1 V = \frac{\epsilon_0 AV}{d}$$

$$\text{Solve to get } W = \frac{1}{2} \frac{\epsilon_0 AV^2}{d}$$

Alternatively:

$$W = Fd = \frac{Q^2}{2A\epsilon_0} d = \frac{C_1^2 V^2}{2\epsilon_0 A} d = \frac{1}{2} \frac{\epsilon_0 AV^2}{d}$$

8. (4) $V = Ed$. As d increases, V also increases. Note that E remains the same.

9. (3) $V_A - \frac{q}{C_1} - E - \frac{q}{C_2} = V_B$ or $V_A - V_B - E = q \left(\frac{1}{C_1} + \frac{1}{C_2} \right)$
or $q = -\frac{40}{3} \mu\text{C}$

10. (2) Capacitors B and C are in parallel, then A is in series.

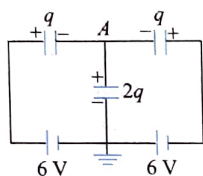
$$C_{\text{eq}} = \frac{2 \times (3+4)}{2+(3+4)} = \frac{14}{9} \mu\text{F}$$

$$Q = C_{\text{eq}} V = \frac{14}{9} (7-6) = \frac{14}{9} \mu\text{C}$$

Q will be divided between B and C . So charge on B is

$$q = \frac{3 \times (14/9)}{3+4} = \frac{2}{3} \mu\text{C}$$

11. (1) $6 = \frac{q}{2} + \frac{2q}{4}$ or $q = 6 \mu\text{C}$



12. (3) Initial charge on C_1 is $Q_1 = C_1 V = 110 \mu\text{C}$

Let x charge flow through wires.

$$\frac{Q_1 - x}{C_1} = \frac{x}{C_{\text{eq}}}$$

where $C_{\text{eq}} = \frac{C_2 \times C_3}{C_2 + C_3}$.

Solve to get $x = 60 \mu\text{C}$.

13. (3) At junction A , Q_1 will be divided into Q_2 and Q_3 . Hence, $Q_1 = Q_2 + Q_3$. C_2 and C_3 are in parallel, so potential on them will be the same, i.e., $V_2 = V_3$.

V will be divided into V_1 and V_2 (or V_3).

Hence, $V = V_1 + V_2$ or $V = V_1 + V_3$

14. (1) $C_1 = C$, $V_1 = V_0$, $C_2 = KC$, $V_2 = 0$, and $V_{\text{common}} = V$

We know that

$$V_{\text{common}} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

$$\therefore V = \frac{CV_0 + KC \times 0}{C + KC}$$

or $K = \frac{V_0}{V} - 1$

15. (1) $V_1 = \frac{C \times 100}{3C + C} = 25 \text{ V}$, $V_2 = 100 - 25 = 75 \text{ V}$

16. (4) Start with C_3 and C_4 in parallel, then C_2 in series, then C_5 in parallel, then C_1 in series, and finally C_6 in parallel.

17. (3) $W = \frac{1}{2} C [(2V)^2 - V^2] = \frac{3}{2} CV^2$

$$W' = \frac{1}{2} C [(2V)^2 - (2V)^2] = 6CV^2$$

$$= \frac{6 \times 2 W}{3} = 4W$$

18. (2) $A = 90 \text{ cm}^2$, $d = 2 \text{ mm}$

$$E = \frac{V}{d}, V = 400 \text{ V}$$

$$u = \frac{1}{2} \epsilon_0 E^2 = \frac{1}{2} \epsilon_0 \left(\frac{V}{d} \right)^2$$

$$= \frac{1}{2} (8.85 \times 10^{-12}) \left(\frac{400}{2 \times 10^{-3}} \right)^2 = 0.177 \text{ Jm}^{-3}$$

19. (1) 22 V get will divide into series combination of C_3 and C_4 .

20. (3) $V_1 - V_2$ will be divided between C_1 and C_2 in series. Potential difference across C_1 is

$$V_1 - V_2 = \frac{C_2 (V_1 - V_2)}{C_1 + C_2}$$

or $V_D = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$

21. (3) $U = \frac{1}{2} CV^2$

Potential of each capacitor now is $V/2$.

$$U' = \frac{1}{2} C \left(\frac{V}{2} \right)^2 = \frac{U}{4}$$

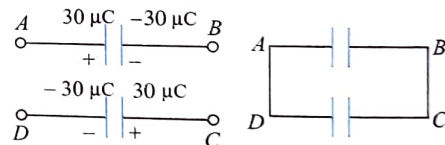
22. (1) Given $\frac{\epsilon_0 A}{d} = 10$

$$C' = \frac{\epsilon_0 A/2}{d} + \frac{4\epsilon_0 (A/2)}{d} = 10 \left[\frac{1}{2} + 2 \right] = 25 \mu\text{F}$$

23. (2) The whole amount of energy stored in capacitor will convert into heat.

$$\text{Heat} = \frac{1}{2} CV^2 = \frac{1}{2} \times 2 \times 10^{-6} \times (100)^2 = 0.01 \text{ J}$$

24. (4) $V = \frac{Q_1 + Q_2}{C_1 + C_2} = 0$, where $Q_1 = 30 \mu\text{C}$, $Q_2 = -30 \mu\text{C}$



Final potential difference is zero. Final energy is zero. Charge flow is $30 \mu\text{C}$ from A to d .

25. (2) Potential of larger capacitor after the first charging is

$$V_1 = \frac{C_0 V_0}{(C + C_0)}$$

After second charging, potential is

$$V_2 = \frac{C_0 V_1}{(C + C_0)} = \left(\frac{C_0}{C + C_0} \right)^2 V_0$$

After n^{th} charging, potential is

$$V_n = \left(\frac{C_0}{C + C_0} \right)^n V_0$$

But $V_n = V$

So $C = C_0 \left[\left(\frac{V_0}{V} \right)^{1/n} - 1 \right]$

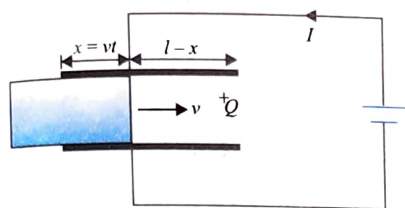
26. (3) $0 - \frac{Q}{C} + 10 - \frac{Q}{2C} = 0$

$$Q = \frac{20C}{3} = \frac{20 \times 6}{3} = 40 \mu\text{C}$$

$$(2) C = \frac{K\epsilon_0 v t b}{d} + \frac{\epsilon_0 (l - vt)b}{d}; q = CV$$

$$I = \frac{dq}{dt} = V \frac{dC}{dt} = V \left[\frac{K\epsilon_0 bv}{d} - \frac{\epsilon_0 bv}{d} \right] = \text{constant}$$

So till the dielectric is fully inserted, a constant current will flow. When the dielectric is fully inside, C will be constant, so $I = 0$. And when dielectric starts coming out, again a constant current will flow but in opposite direction.



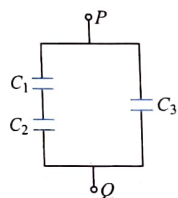
$$(2) 2000 \times 0.04 = \frac{1}{2} \times 40 \times 10^{-6} V^2$$

$$\text{or } V^2 = 4 \times 10^6 \text{ or } V = 2000 \text{ V}$$

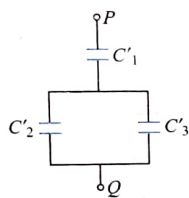
(1) We can make equivalent circuit of given system in two ways as in Fig. (a) and (b). But Fig. (b) is wrong and Fig. (a) is correct.

$$\text{Here } C_1 = \frac{\epsilon_0 A / 2}{t / 2} = \frac{\epsilon_0 A}{t}$$

$$C_2 = \frac{K\epsilon_0 A / 2}{t / 2} = \frac{K\epsilon_0 A}{t} \text{ and } C_3 = \frac{\epsilon_0 A / 2}{t} = \frac{\epsilon_0 A}{2t}$$



(a) (Right)



(b) (Wrong)

$$\text{Now, } C_{eq} = \frac{C_1 C_2}{C_1 + C_2} + C_3$$

$$= \frac{\epsilon_0 A}{t} \left[\frac{3K + 1}{2(K + 1)} \right]$$

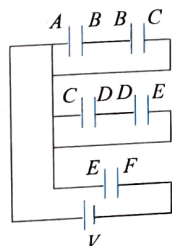
(3) After closing S_1 , charge on C_1 is $q = 6 \times 20 = 120 \mu\text{C}$. Now, S_1 is opened. On closing S_2 , charge q will be distributed between C_1 and C_2 according to their capacitances. So charge on C_2 is

$$q_2 = \frac{C_2 q}{C_1 + C_2} = \frac{3 \times 120}{3 + 6} = 40 \mu\text{C}$$

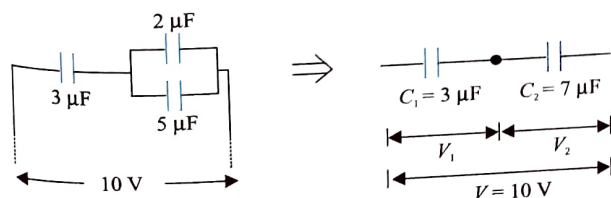
$$(3) V_A = V_C \text{ and } V_C = V_E$$

$$\text{Net energy stored is } U = \frac{1}{2} CV^2$$

$$C = \frac{\epsilon_0 A}{d} \quad (\text{For } EF)$$



(3) Potential difference across upper branch is 6 V.



$$V_2 = V \left(\frac{C_1}{C_1 + C_2} \right) = 10 \left[\frac{3}{3 + 7} \right] = 3 \text{ V}$$

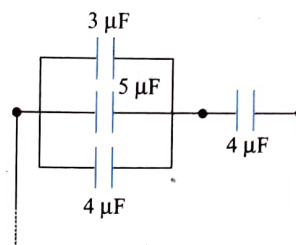
Hence, potential difference across $5 \mu\text{F}$ capacitor is 3 V.

Thus, charge in $5 \mu\text{F}$ capacitor is

$$Q_{5\mu\text{F}} = 5 \times 3 = 15 \mu\text{C}$$

33. (1) Potential difference across 5F capacitor is $120/5 = 24 \text{ V}$.

Hence, potential difference across all three capacitors connected in parallel is $V_1 = 24 \text{ V}$.

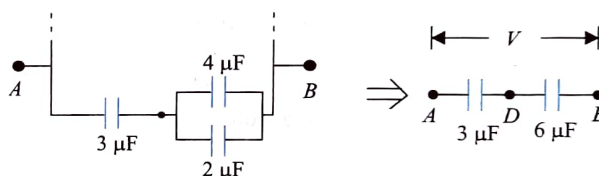


$$V_1 = V \left(\frac{C_2}{C_1 + C_2} \right)$$

$$\text{or } V = V_1 \left(\frac{C_1 + C_2}{C_2} \right)$$

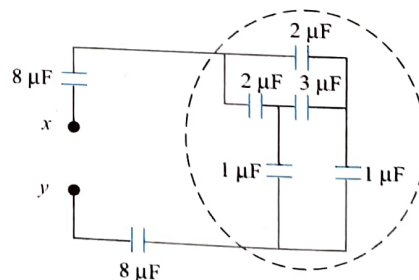
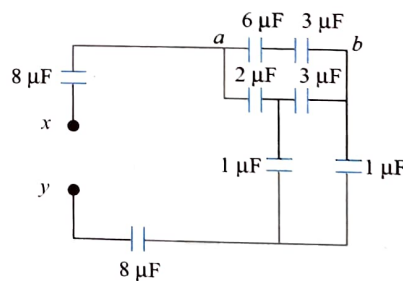
$$= 24 \left[\frac{12 + 4}{4} \right] = 96 \text{ V}$$

$$\therefore V_{ab} = V = 96 \text{ V}$$

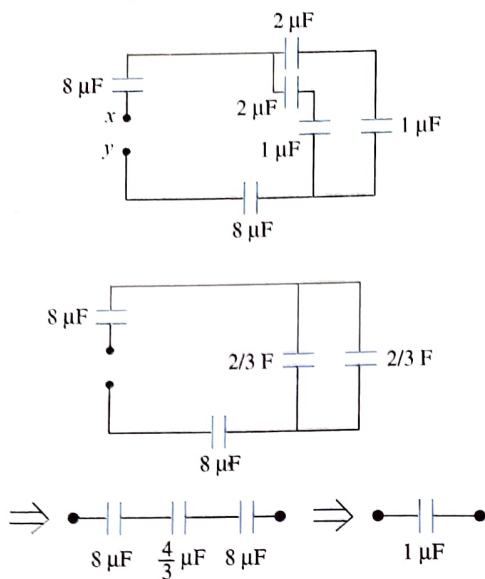


$$V_{AD} = V \left(\frac{6}{30 + 6} \right) = 96 \left[\frac{6}{30 + 6} \right] = 16 \text{ V}$$

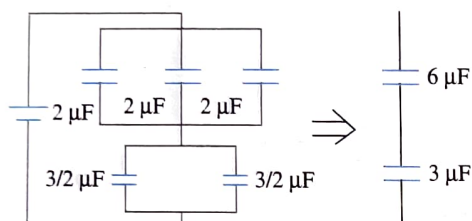
34. (2)



Dotted circuit makes balanced Wheatstone bridge. The circuit can be simplified as follows:



35. (1) The circuit can be simplified as follows:



Hence $C_{eq} = 2 \mu\text{F}$. Thus, charge supplied by battery is $Q = 2 \times 20 = 40 \mu\text{C}$.

Charge on required capacitor is $20 \mu\text{C}$

36. (4) Figure (a) can be simplified as follows:

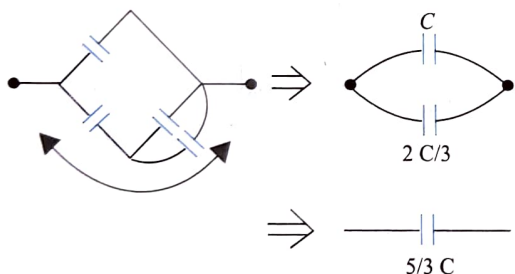


Figure (b) can be simplified as follows:

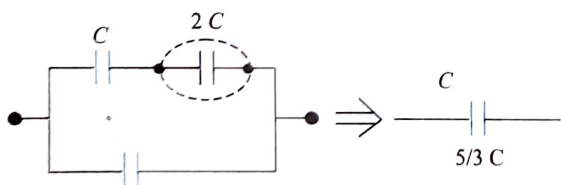
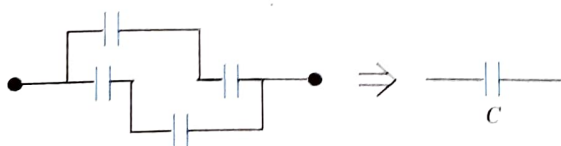


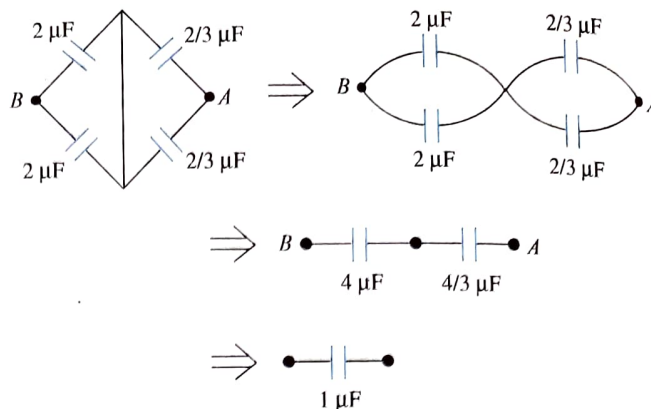
Figure (c) is balanced Wheatstone bridge, which can be simplified as follows



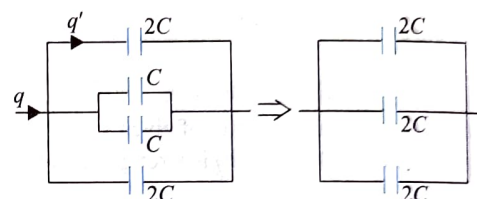
Hence ratio is

$$\frac{5}{3}C : \frac{5}{3}C : C = 5 : 5 : 3$$

37. (3) The redrawn circuit is as follows:



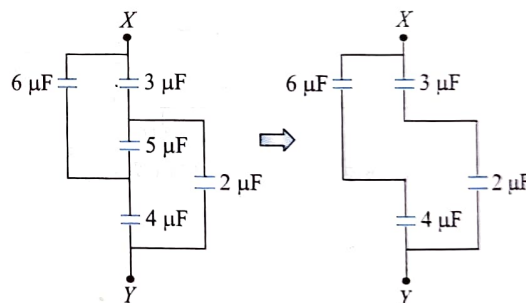
38. (2) Equivalent circuit may be drawn as follows:



Here $C_{eq} = 6C = 6 \mu\text{F}$.

Charge supplied by battery is $q = 6 \times 6 = 36 \mu\text{C}$. Hence, charge on each branch will be $12 \mu\text{C}$, and so charge on required capacitor is $6 \mu\text{C}$.

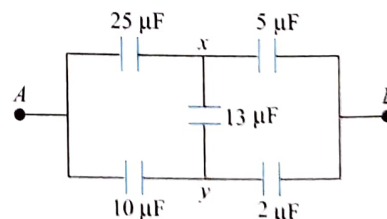
39. (3) Equivalent circuit may be drawn as follows:



As the structure is balanced Wheatstone bridge, so $5 \mu\text{F}$ capacitor will draw no charge from battery and may be removed. Now the circuit is a simple circuit.

$$C_{eq} = \frac{4 \times 6}{(6 + 4)} + \frac{3 \times 2}{(3 + 2)} = \frac{18}{5} \mu\text{F}$$

40. (1) Circuit can be redrawn as follows:



Now, $V_x = V_y$. Hence

$$\frac{25 \mu\text{F}}{5 \mu\text{F}} = \frac{7 \mu\text{F}}{2 \mu\text{F}}$$

$$C_{eq} = \left(\frac{5 \times 25}{5 + 25} \right) + \left(\frac{10 \times 2}{10 + 2} \right) = \frac{35}{6} = \frac{35}{6} \mu\text{F}$$

41. (2) Initially, when the switch is closed on position 1, the capacitor C is connected in series with batteries E_1 and E_2 . From KVL, we have

$$\frac{Q_i}{C} - E_2 + E_1 = 0$$

or $Q_i = (E_2 - E_1)C$... (i)
Depending upon the sign of $(E_2 - E_1)$, charge Q_i on the left plate may be positive (if $E_2 > E_1$), or negative (if $E_2 < E_1$); charge on right plate would be equal and opposite. When the switch is moved to position 2, the left plate (earlier having charge $+Q_i$) will now have charge

$$Q_f = -E_1 C$$
 ... (ii)

The net charge flow through the circuit is

$$\Delta Q = Q_f - Q_i = [-E_1 - (E_2 - E_1)]C = -E_2 C$$

We can say that a net positive charge equal to $E_2 C$ is pulled by the battery of emf E_1 from the left plate of the capacitor, which flows through battery E_1 and is transferred to the right plate of the capacitor. Work done by battery E_1 in the process of charge transfer is

$$\Delta W = E_1 E_2 C$$
 ... (iii)

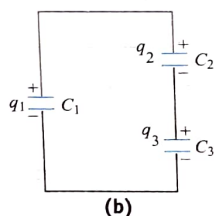
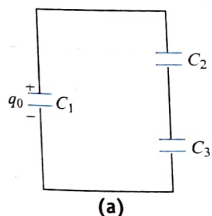
A part of this work changes the energy of the capacitor.

$$\begin{aligned} \Delta W_c &= \frac{Q_f^2}{2C} - \frac{Q_i^2}{2C} = \frac{1}{2} E_1^2 C - \frac{1}{2} (E_2 - E_1)^2 C \\ &= \frac{1}{2} (2E_1 E_2 - E_2^2) C \end{aligned}$$

and the remaining part is lost as Joule heat:

$$H = \Delta W - \Delta W_c = \frac{1}{2} E_2^2 C$$

42. (4) Initial situation after the reconnection is shown in Fig. (a) and the final situation in Fig. (b). The charge transferred by C_1 is $q_0 - q_1$ and the capacitors C_2 and C_3 are in series, so $q_2 = q_3$. Other way to solve the questions is to equalize the potentials across C_1 and series combination of C_2 and C_3 .



$$43. (4) V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

$$Q_1 + Q_2 = Q$$

where $C_1 = \epsilon_0 A/a$ and $C_2 = \epsilon_0 A/b$.
Solve to find

$$Q_2 = \frac{Qa}{a+b}$$

$$44. (2) q_i = CV_i = 16 \times 5 = 80 \mu C, q_f = CV_f = 16 \times 10 = 160 \mu C$$

$$\text{Given } \frac{dq}{dt} = 40t \Rightarrow \int_{q_i}^{q_f} dq = \int_0^{10} 40t dt$$

$$\text{or } q_f - q_i = \frac{40t^2}{2}$$

$$\text{or } 160 - 80 = 20t^2 \text{ or } T = 2 \text{ s}$$

45. (3) p and q are in parallel and then r in series.

46. (3) The charge on capacitor before dielectric is

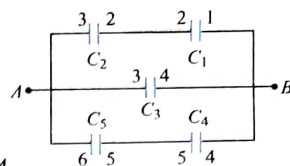
$$q_i = CV = 50 \times 10^{-9} C$$

Final charge on capacitor after dielectric is

$$\begin{aligned} q_2 &= (KC)V = 5 \times 50 \times 10^{-9} \\ &= 250 \times 10^{-9} C \end{aligned}$$

Charge flow from battery, $\Delta q = q_2 - q_1 = 200 \text{ nC}$

47. (1) Given plates can be rearranged as shown:



$$C_1 = \frac{\epsilon_0 A}{d}; C_2 = \frac{\epsilon_0 A}{2d}$$

$$C_3 = \frac{\epsilon_0 A}{3d}; C_4 = \frac{\epsilon_0 A}{2d}; C_5 = \frac{\epsilon_0 A}{d}$$

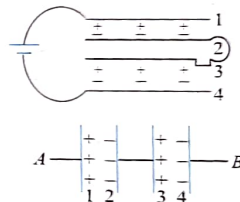
C_1 and C_2 are in series and its effective capacity is

$$\begin{aligned} \frac{\frac{\epsilon_0 A}{d} \times \frac{\epsilon_0 A}{2d}}{\frac{\epsilon_0 A}{d} + \frac{\epsilon_0 A}{2d}} &= \frac{\epsilon_0 A}{3d} \end{aligned}$$

Effective capacitance of C_4 and $C_5 = \frac{\epsilon_0 A}{3d}$

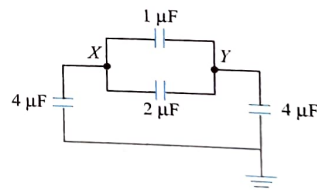
$$C_{AB} = \frac{\epsilon_0 A}{3d} + \frac{\epsilon_0 A}{3d} + \frac{\epsilon_0 A}{3d} = \frac{\epsilon_0 A}{d}$$

48. (2) Plates can be rearranged as shown in figure. Plates 1 and 2 and plates 3 and 4 form two capacitors which are in series between A and B. Plates 2 and 3 do not form any capacitor as they are at same potential.



$$\begin{aligned} \text{So, } C_{eq} &= \frac{C_1 C_2}{C_1 + C_2} = \frac{\left(\frac{\epsilon_0 A}{d}\right) \times \left(\frac{\epsilon_0 A}{3d}\right)}{\frac{\epsilon_0 A}{d} + \frac{\epsilon_0 A}{3d}} \\ &= \frac{\epsilon_0 A}{4d} \end{aligned}$$

49. (4) Circuit is redrawn as shown in figure.



In the circuit, capacitors $4 \mu F$ and $4 \mu F$ are connected in series. Combination of this is in parallel with $1 \mu F$ and $2 \mu F$ capacitors.

50. (1) The potential of point B is $+10 \text{ V}$, therefore no potential difference exists across $12 \mu F$ capacitor, hence $q_{12} = 0$. Charge on the $4 \mu F$ capacitor is $q_4 = (4)(10) = 40 \mu C$.

51. (1) Electrical force is balanced by the weight of the mass

$$mg = qE, \text{ where } E = \text{electric field}$$

$$\text{or } mg = q \frac{V}{d} \text{ (where } d = \text{separation at the plates)}$$

$$= q \frac{V}{2} \left(\therefore \frac{\epsilon_0 A}{d} = C \frac{1}{d} = \frac{C}{\epsilon_0 A} \right)$$

$$\text{or } V = \frac{2mg\epsilon_0 A}{qC}$$

$$= \frac{10^{-6} \times 10 \times 2.2 \times 10^{-12} \times 10^{-2}}{10^{-5} \times 4 \times 10^{-8}}$$

$$= \frac{10^{-6} \times 9.8 \times 8.88 \times 10^{-12} \times 100 \times 10^{-4}}{10^{-8} \times 0.04 \times 10^{-6}}$$

$$= 43 \text{ mV}$$

52. (4) Initial total energy = $\frac{1}{2} CV^2 + \frac{1}{2} CV^2 = CV^2$

When the switch is open, dielectric is introduced. Then capacitance, $C = KC = 3C$.
Energy stored in C is

$$\frac{1}{2} 3CV^2 = \frac{3}{2} CV^2$$

Since the switch is open, charge will be the same in B so energy is $B = \frac{1}{2} \frac{C}{3} V^2$.

So total final energy = $\frac{3}{2} CV^2 + \frac{1}{6} CV^2$

$$= \frac{9CV^2 + 1CV^2}{6} = \frac{10}{6} CV^2$$

So required ratio = $\frac{CV^2}{\frac{10}{6} CV^2} = \frac{3}{5} = 3 : 5$

53. (2) Initially at switch position Q , by conservation of charge, the voltage V across the two capacitors is thus given by

$$q = q_1 + q_2 = C_1 V + C_2 V = (C_1 + C_2) V$$

or $18 = (3 + 6) V$
or $V = (18/9) = 2 \text{ V}$

54. (1) $\frac{q_1}{1} = \frac{q_2}{2} = \frac{q_3}{3} = K$
 $q_1 + q_2 + q_3 = 6$
Solving, we get
 $K = 1$
or $q_3 = 3 \mu\text{C}$

55. (4) Charge on positive plate of A is $C_1 V_1$.

Charge on negative plate of B is $-C_2 V_2$. When d plate of capacitor C is connected with the plate of A , then the total charge of (d, a) plate system will be $C_1 V_1$ (conservation of charge).

Similarly on (c, s) plates, total charge will be $-C_2 V_2$.

The total charge on (b, g) plate system will be $+C_2 V_2 - C_1 V_1$.

56. (3) Circuit is redrawn as shown in figure. Equivalent capacity of combination is

$$C_{eq} = \frac{3 \times 6}{9} = 2 \mu\text{F}$$

Hence, charge

$$q = C_{eq} V = 2 \mu\text{F} \times 30 \text{ V} = 60 \mu\text{C}$$

57. (4) The capacitance of capacitor C is $\epsilon_0 A/x$. Charge on capacitor

$$q = CV = \frac{\epsilon_0 AV}{x}; U = \frac{1}{2} qV = \frac{1}{2} \frac{\epsilon_0 AV^2}{x}$$

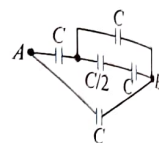
So, $\frac{dU}{dt} = \frac{1}{2} \epsilon_0 AV^2 \left(-\frac{1}{x^2} \right) \frac{dx}{dt}$

$$= \frac{1}{2} \epsilon_0 AV^2 v \left(-\frac{1}{x^2} \right)$$

From the earlier expression,

$$\frac{dU}{dt} \propto \frac{1}{x^2}$$

58. (2) $C_{eq} = \frac{\frac{4C}{3} \times C}{\frac{3}{3}} + C = \frac{11C}{7}$



59. (1) The total charge on each plate will have to be same. They should also carry opposite charges.

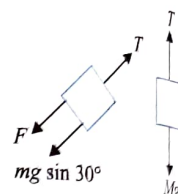
60. (1) For equilibrium, we have

$$T = Mg$$

And $T = F + \sin 30^\circ$

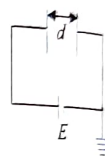
or $Mg = F + \frac{mg}{2}$

or $m = \frac{m}{2} + \frac{F}{g} = \frac{m}{2} + \frac{E^2 \epsilon_0 A (k-1)}{2lgd}$



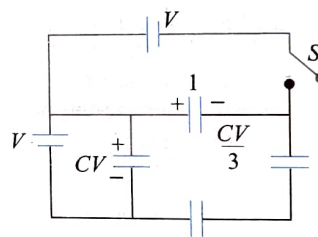
61. (2) The circuit can be redrawn as shown in figure. Charge on capacitor is

$$Q = CE = \frac{\epsilon_0 AE}{d}$$

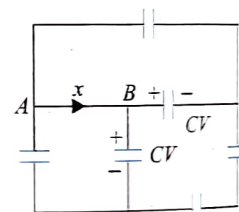


Options (1), (3), and (4) are incorrect.

62. (4) Charge flowing from A to B :



Open switch



Switch closed

x = change in charge on capacitor 1

$$= CV - \frac{CV}{3} = \frac{2CV}{3}$$

63. (2) Electric field between the capacitor plates:

$$E = \frac{Q+x}{2A\epsilon_0} + \frac{x}{2A\epsilon_0} = \frac{1}{2A\epsilon_0} [Q+2x]$$

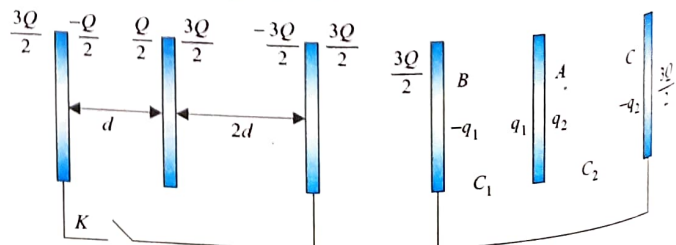
Potential difference = $Ed = \frac{d}{2A\epsilon_0} [Q+2x] = \epsilon$

$$\epsilon = \frac{Q+2x}{2C} - x = \frac{Q}{2} - C\epsilon$$

64. (1) This is the case of a balanced Wheatstone bridge. The middle five capacitors will have no charge and will be useless.

65. (1) Before closing the switch

After closing the switch



$$q_1 + q_2 = 2Q$$

$$V_{AB} = V_{AC} \text{ or } \frac{q_1}{C_1} = \frac{q_2}{C_2}$$

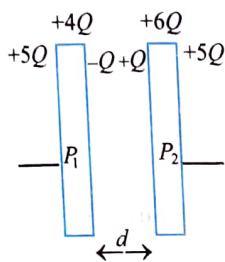
$$\frac{q_1}{q_2} = \frac{C_1}{C_2} = 2 \text{ or } q_1 = 2q_2$$

$$\text{Solving, we get } q_1 = \frac{4Q}{3}, q_2 = \frac{2Q}{3}$$

Charge flown through K is

$$-\frac{Q}{2} - (-q_1) = -\frac{Q}{2} + \frac{4Q}{3} = 5Q/6$$

67. (2) Potential difference between the plates is

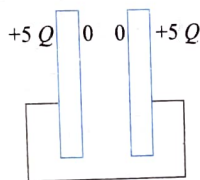


$$V_{P1} - V_{P2} = \frac{-Q}{(\epsilon_0 A/d)} = \frac{-Qd}{\epsilon_0 A}$$

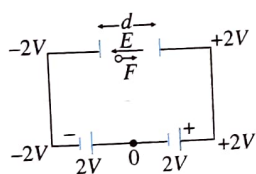
67. (4) Final charges are as shown. Energy stored in the capacitor will be zero finally.

$$\text{Heat produced} = \Delta H = U_i - U_f$$

$$= \frac{1}{2} \frac{Q^2}{\epsilon_0 A/d} - 0$$



68. (3) Circuit (3) represents these equipotential lines



69. (3) After connection the charge of capacitor $Q' = CV = C \frac{2Q_0}{C} = 2Q_0$

The final charge in inner plate of A will be $2Q_0$. Hence charge supplied by battery

$$\Delta Q = \left(2Q_0 - \left(-\frac{Q_0}{2} \right) \right) = \frac{5Q_0}{2}$$

Hence work done by battery

$$W = (\Delta Q)V = \frac{5Q_0}{2} \times \frac{2Q_0}{C} = \frac{5Q_0^2}{C}$$

70. (1) Let q_1 charge flows through S_1 and q_2 through S_2 . For capacitor of capacitance $2C$,

$$Q_0 + q_1 = 2CV = Q_0 \text{ or } q_1 = 0$$

For capacitor of capacitance C ,

$$\frac{Q_0}{3} + q_2 = CV = \frac{Q_0}{2} \text{ or } q_2 = \frac{Q_0}{6}$$

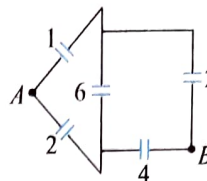
71. (2) Potential drops across C_1 and C_2 are 6 V and 4 V, respectively. Since they are in series, same charge flows through them and

$$\frac{C_1}{C_2} = \frac{V_2}{V_1} = \frac{2}{3}$$

72. (1) Dielectric constant on relative permittivity

$$K = \frac{E_{\text{ext}}}{E_{\text{net}}} = \frac{5}{3}$$

73. (1) The figure can be redrawn as follows:

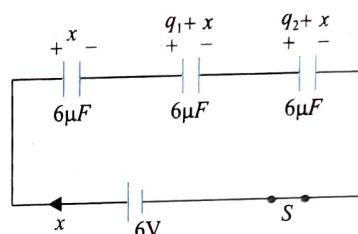


This is a balanced Wheatstone bridge. $6 \mu\text{F}$ is useless.

Find $C_{eq} = 2 \mu\text{F}$

74. (2) Let charge x flow through circuit on closing switch. Then final charge on each capacitor is as shown.

$$\frac{x}{6} + \frac{q_1 + x}{6} + \frac{q_2 + x}{6} = 6$$



$$\text{or } 3x + q_1 + q_2 = 36 \text{ or } x = 6 \mu\text{C}$$

75. (3) Case I: $C_{eq} = 2C$, $U_1 = \frac{1}{2} 2CE^2 = CE^2$

$$\text{Case II: } C_{eq} = \frac{8C}{3}, U_2 = \frac{1}{2} \frac{8C}{3} E^2 = \frac{4}{3} CE^2$$

Case III: The upper $2C$ will be short-circuited.

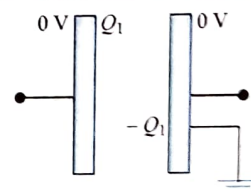
$$U_3 = \frac{1}{2} CE^2 + \frac{1}{2} 2CE^2 = \frac{3}{2} CE^2$$

$$U_1 : U_2 : U_3 = 6 : 8 : 9$$

Multiple Correct Answers Type

1. (2), (3), (4)

As the switch is closed, the charge on the outer surface of the second plate becomes zero. From the concept in electrostatics that electric field inside the bulk of the material of conductor is zero, we can find the charges on various faces. So it is clear that $Q_1 + Q_2$ charge goes from the second plate to the earth. Charge on capacitor is Q_1 and hence its potential is Q_1/C .



2. (2), (3), (4)

As the dielectric slab is pulled out, the equivalent capacity of the system decreases, and hence charge supplied by the battery decreases as potential of the system remains constant. It means charging of battery takes place and a positive charge flows from a to b . As the two capacitors are connected in series, so charge on both capacitors remains the same at all instants. From energy conservation law,

$$U_i + W_{\text{ext}} = U_f + \text{work done on battery} + \Delta H$$

As dielectric slab is attracted by the plates of capacitors, to pull it out, F has to perform some work, i.e., $W_{\text{ext}}(F) > 0$.

3. (1), (2), (4)

As electric field is conservative field hence $W_{\text{ele}} \Rightarrow -\Delta U = -W_{\text{ext}}$. Hence option (1) is correct. As $-\int \vec{F}_{\text{elec}} \cdot d\vec{x} \Rightarrow -W_{\text{ele}} \Rightarrow W_{\text{ext}}$. Hence option (2) is correct.

$$C = \frac{\epsilon_0 A}{d_1}, C' = \frac{\epsilon_0 A}{d_2}$$

$$\text{Extra charge flown} = Q' - Q = (C' - C)V$$

$$= \epsilon_0 A V \left[\frac{1}{d_2} - \frac{1}{d_1} \right]$$

Work done by battery is

$$W_b = V \times \text{charge flown} = \epsilon_0 A V^2 \left[\frac{1}{d_2} - \frac{1}{d_1} \right]$$

Change in potential energy of capacitor is

$$\Delta U = \frac{1}{2} (C' - C) V^2 = \frac{1}{2} \epsilon_0 A V^2 \left[\frac{1}{d_2} - \frac{1}{d_1} \right]$$

It means $W_{\text{battery}} = 2\Delta U$. Hence option (4) is correct.

4. (1), (3), (4)

This system can be considered as two capacitors in parallel, with

$$C_1 = \frac{\epsilon_0 A}{2d} \text{ and } C_2 = \frac{K\epsilon_0 A}{2d}$$

C_1 is due to upper half of two plates, while C_2 is due to lower half.

As potential difference across two capacitors is the same, charges would be different as capacitors are different.

5. (1), (2), (3)

$$\text{Effective capacitance is } C = \frac{4 \times 5}{9} = \frac{20}{9} \mu\text{F}$$

$$\text{Charge on } C_1 \text{ is } Q_1 = CV = \frac{20}{9} \times 9 = 20 \mu\text{C}$$

$$\text{Charge on } C_2 \text{ is } Q_2 \left(\frac{C_2}{C_2 + C_3} \right) Q_1 = \frac{2}{3} \times 20 = 8 \mu\text{C}$$

$$\text{Charge on } C_3 \text{ is } Q_3 \left(\frac{C_3}{C_2 + C_3} \right) Q_1 = \frac{3}{3} \times 20 = 12 \mu\text{C}$$

$$\text{Since, } \frac{Q_2}{C_2} = \frac{Q_3}{C_3}$$

$$\text{and } Q_1 = Q_2 + Q_3 = Q_2 + \frac{C_3}{C_2} Q_2 = Q_2 \left(\frac{C_2 + C_3}{C_2} \right)$$

Therefore,

$$Q_2 = \left(\frac{C_2}{C_2 + C_3} \right) Q_1$$

$$Q_3 = \left(\frac{C_3}{C_2 + C_3} \right) Q_1$$

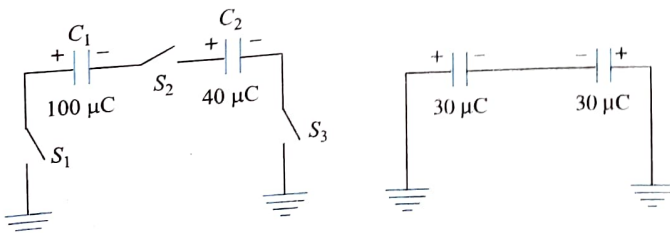
Thus, options (1), (2), and (3) are correct.

6. (1), (3)

When switches are open, charges on the capacitor are shown.

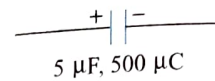
When switches are closed, capacitors become parallel; charges are shown on two capacitors. Comparing two diagrams, it is clear that 70 μC of charge will pass through switches S_1 and S_3 .

Hence, choices (1) and (3) are correct and choices (2) and (4) are wrong.



7. (2), (3)

Initial condition just after the connection of battery:



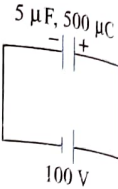
Condition after a long time:

It means battery supplies 1000 μC charge from its positive terminal and an equal and opposite charge enters from its negative terminal, i.e., charge flow through battery is 1000 μC . Work done by the battery is

$$W_{\text{battery}} = 100 \text{ V} \times 10^3 \mu\text{C} = 0.1 \text{ J}$$

From energy conservation law, $U_i + W_{\text{battery}} = U_f + \Delta H$

$$U_i = U_f \text{ so } \Delta H = 0.1 \text{ J}$$



8. (1), (3), (4)

$$8 - \frac{q_1}{2} - 2 - \frac{q_3}{2} = 3 \quad \text{or} \quad 6 - 3 = \frac{q_1}{2} + \frac{q_3}{2}$$

$$\text{or } q_1 + q_3 = 6 \quad \dots(i)$$

$$8 - \frac{q_1}{2} + \frac{q_2}{2} = 2 \quad \text{or} \quad \frac{q_1}{2} - \frac{q_2}{2} = 6$$

$$\text{or } q_1 - q_2 = 12 \quad \dots(ii)$$

$$\text{Also, } q_1 + q_2 = q_3 \quad \dots(iii)$$

Solving (i), (ii), and (iii), we get $q_1 = 6 \mu\text{C}$, $q_2 = -6 \mu\text{C}$, and $q_3 = 0$.

Hence, choices (1), (2), and (3) are correct. Choice (4) is wrong.

9. (1), (3), (4)

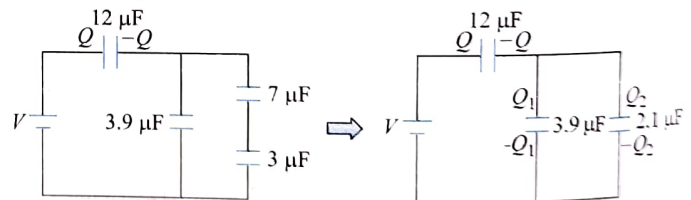
Capacitors 1 μF , 2 μF , and 3 μF are in parallel, and their total capacitance is 6 μF . Thus, we have three capacitors in series, each of capacitance 6 μF across the 12 V power supply. So potential drop across each is $12/3 = 4 \text{ V}$. So options (3) is correct. Charge on 2 μF capacitor is $CV = 2 \times 4 = 8 \mu\text{C}$. So option (1) is correct.

Charge on 6 μF capacitor is $6 \times 10^{-6} \times 4 = 24 \mu\text{C}$. So option (2) is wrong.

Thus, correct options are (1), (3), and (4).

10. (1), (2), (4)

Equivalent capacitance of network is $C_{\text{eq}} = 4 \mu\text{F}$.



$$\text{Charge } Q = C_{\text{eq}} V = 4 \text{ V } \mu\text{C}$$

The charge on the 7 μF or 3 μF capacitor is

$$Q_2 = (7 \mu\text{F})(6 \text{ V}) = 42 \mu\text{C}$$

$$\frac{Q_2}{2.1 \mu\text{F}} = \frac{Q_1}{3.9 \mu\text{F}} \quad \text{or} \quad Q_1 = (42 \mu\text{C}) \left(\frac{3.9 \mu\text{F}}{2.1 \mu\text{F}} \right) = 78 \mu\text{C}$$

$$Q = Q_1 + Q_2 = (78 \mu\text{C} + 42 \mu\text{C}) = 120 \mu\text{C} = 4 \text{ V } \mu\text{C}$$

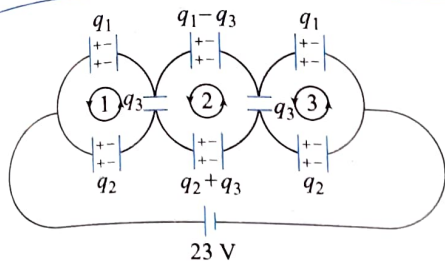
Emf of the battery is $V = 30 \text{ V}$

The potential drop across 12 μF capacitor is

$$\frac{Q}{12 \mu\text{F}} = \frac{120 \mu\text{C}}{12 \mu\text{F}} = 10 \text{ V}$$

11. (1), (2)

The distribution of charge is shown in the figure, in compliance with the point rule. Applying loop rules for loops A, B, and D, we get



$$-\frac{q_2}{5} + \frac{q_3}{0.75} + \frac{q_1}{15} = 0$$

$$q_1 - 3q_2 + 20q_3 = 0 \quad \dots(i)$$

$$-\frac{q_2 + q_3}{15} - \frac{q_3}{0.75} + \frac{q_1 - q_3}{5} - \frac{q_3}{0.75} = 0$$

$$3q_1 - q_2 - 44q_3 = 0 \quad \dots(ii)$$

$$23 - \frac{q_2}{5} - \frac{q_2 + q_3}{15} - \frac{q_2}{5} = 0$$

$$345 = 7q_2 + q_3 \quad \dots(iii)$$

Solving for q_1, q_2, q_3 , we get

$$q_1 = \frac{19 \times 345}{92}, q_2 = \frac{13 \times 345}{92}, q_3 = \frac{345}{92}$$

Potential difference between A and B is $\frac{q_3}{0.75} = \frac{345}{92} \times \frac{4}{3} = 5 \text{ V}$

Potential difference between E and F is also 5 V but in the opposite direction.

1. (1), (4)

When capacitors are connected in parallel, initial capacitance is $C = 2\epsilon_0 A/d$. After the distance between the plates is changed, the capacitance becomes

$$C = \frac{\epsilon_0 A}{d+a} + \frac{\epsilon_0 A}{d-a} = \frac{2\epsilon_0 A}{d - (a^2/d)}$$

which is greater than initial one. Hence, option (1) is correct and option (2) is incorrect. When capacitors are connected in series, then

$$\frac{1}{C} = \frac{2d}{\epsilon_0 A}$$

After the distance between the plates is changed,

$$\frac{1}{C} = \frac{d+a}{\epsilon_0 A} + \frac{d-a}{\epsilon_0 A} = \frac{2d}{\epsilon_0 A}$$

That is, the capacitance remains unchanged. Hence, option (4) is correct and option (3) is incorrect.

13. (1), (4)

Let q_1 and q_2 be the instantaneous charges on capacitors. Since they are in parallel, then

$$\frac{q_1}{C_1} = \frac{q_2}{C_2} \text{ and } q_1 + q_2 = Q$$

$$C_1 = \frac{\epsilon_0 A}{d_0 + vt}, C_2 = \frac{\epsilon_0 A}{d_0 - vt}$$

$$\text{So } \frac{q_1}{C_1} = \frac{C_2}{C_1} = \frac{d_0 - vt}{d_0 + vt} \text{ or } q_2 \left(\frac{d_0 - vt}{d_0 + vt} \right) + q_2 = 0$$

$$\text{So } q_2 = \frac{Q(d_0 + vt)}{2d_0} \text{ and } q_1 = \frac{Q(d_0 - vt)}{2d_0}$$

Hence, option (1) is correct and option (2) is incorrect.

$$i = \frac{-dq_1}{dt} \text{ or } \frac{dq_2}{dt} \text{ or } i = \frac{Qv}{2d_0}$$

which does not depend on time. So option (4) is correct and option (3) is incorrect.

14. (1), (3)

$$\text{Initial stored energy } U_i = \frac{1}{2} \frac{\epsilon_0 AV_1^2}{x}$$

$$\text{Final stored energy } U_f = \frac{1}{2} \frac{\epsilon_0 AV_0^2}{2(x+dx)}$$

$$\text{So } \Delta U = U_f - U_i = \frac{1}{2} \epsilon_0 AV_0^2 \left[\frac{1}{x+dx} - \frac{1}{x} \right]$$

$$= \frac{1}{2} \epsilon_0 AV_0^2 \left[\frac{x - x - dx}{x(x+dx)} \right]$$

$$= -\frac{1}{2} \frac{\epsilon_0 AV_0^2}{x^2} dx = -\frac{1}{2} \frac{\epsilon_0 AV_0^2}{x} \cdot \frac{dx}{x} = -\frac{U dx}{x}$$

So, option (1) is correct and option (2) is incorrect.

$$F = -\frac{dU}{dx} = -\frac{U}{x}$$

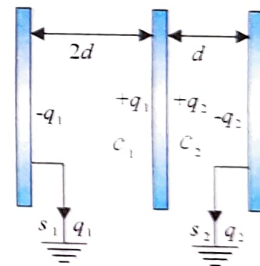
$$= \frac{1}{2} \frac{\epsilon_0 A}{x} \cdot \frac{V_0^2}{x} = -\frac{1}{2} \left(\frac{\epsilon_0 A}{x} V_0 \right) \frac{V_0}{x} = -\frac{1}{2} qE$$

So magnitude of attractive force is $1/2 qE$.

So, option (3) is correct and option (4) is incorrect.

15. (1), (2), (3)

When either A or C is earthed (but not both together), a parallel plate capacitor is formed with B, with $\pm Q$ charges on the inner surfaces. (The other plate, which is not earthed, plays no role.) Hence, charge of amount $+Q$ flows to the earth. When both are earthed together, A and C effectively become connected. The plates now form two capacitors in parallel, with capacitances in the ratio 1 : 2, and hence share charge Q in the same ratio.



$$C_1 = \frac{\epsilon_0 A}{2d}, C_2 = \frac{\epsilon_0 A}{d} \quad \frac{C_1}{C_2} = \frac{1}{2}$$

Now we can write

$$\frac{q_1}{C_1} = \frac{C_2}{C_1} = \frac{1}{2}$$

because potential difference of both the capacitors is same and $q_1 + q_2 = Q$. Solving these, we get $q_1 = Q/3, q_2 = 2Q/3$.

16. (2), (3)

$$C = \frac{Q}{V} \quad \text{Also, } C = \frac{\epsilon_0 A}{d}$$

As d increases, C will decrease. Therefore, V should increase, as Q remains the same.

$$E = \frac{V}{d} = \frac{Q}{\epsilon_0 A} \text{ will be constant.}$$

Energy, $U = \frac{Q^2}{2C}$. As C decreases, U will increase.

17. (1), (4)

Suppose field in air gap is E , then in dielectric it is E/K .

For dielectric medium

$$dV = -\frac{E}{K} dx \text{ or } V = -\frac{E}{K} x$$

For air gap medium, $V = -Ex$.

18. (2), (4)

Plate 2 acquires a net positive charge and 3 acquires a net negative charge.

Due to induction, plate 1 acquires negative charge and plate 4 positive charge. 1 and 4 are equipotential (since they are joined).

Due to symmetry

$$V_{1,2} = V_{3,4}$$

$$\text{and } V_{2,3} = 2V_{2,1} = 2V_{4,3}$$

because charge capacitor 2-3 is double.

$$\text{Electric field} = \frac{\text{potential}}{d}$$

$$E_1 : E_2 : E_3 = 1 : 2 : 1 = \frac{1}{2} : 1 : \frac{1}{2}$$

and from plate 1 to 4

Potential first increases, then decreases, and again increases. Since we are going first in direction opposite of E and then in direction of E .

19. (1), (3), (4)

Here effective dielectric constant of conductor can be taken as infinity.

20. (2), (4)

As battery is disconnected, hence $Q = \text{constant}$, plates of capacitor are moved apart,

Hence C decreases $\left(C \propto \frac{1}{d}\right)$.

$$\text{Now, } V = \frac{Q}{C} \text{ and } U = \frac{Q^2}{2C}$$

$$V = \text{constant, } C = KC_0 \text{ i.e., } C > C_0$$

$$Q = CV, \text{ i.e., } Q > Q_0$$

$$U = \frac{1}{2} CV^2, \text{ i.e., } U < U_0$$

21. (1), (2), (3)

Being a conductor, each plate has the same potential at each point. But because $E = -(\Delta V/\Delta x)$, hence E is highest where the plates are closest to each other. Hence, surface charge density is also higher at the closer end.

22. (1), (3)

Capacities in different arms are

$$2F, 1F, \frac{1}{2}F, \frac{1}{4}F, \dots$$

$$C_{\text{eff}} = 2 + 1 + \frac{1}{2} + \frac{1}{4} + \dots = 2 \left(1 + \frac{1}{2} + \frac{1}{4} + \dots\right) = 4F$$

As PD across each row is same, thus the charge will be maximum across the capacitor in first row.

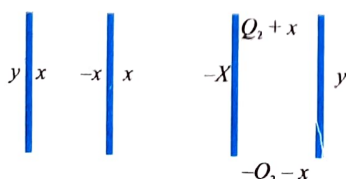
23. (1), (2)

$$xa + xb + (Q_2 + x)c = 0$$

$$x = \frac{-Q_2 c}{a + b + c}$$

$$Q_1 = y + x + y - Q_2 - x$$

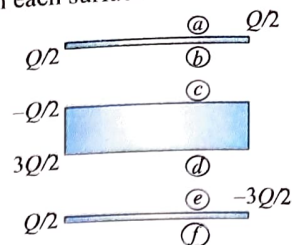
$$y = \frac{Q_1 + Q_2}{2}$$



$$V = \frac{Q_2 ca}{(a + b + c) S \epsilon_0}$$

24. (1), (2)

Initial charges on each surface

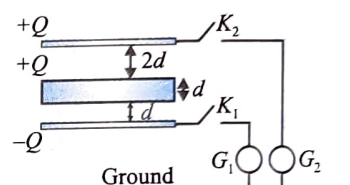


Charge on outer surfaces

$$q_a = q_b = \frac{Q + Q - Q}{2} = \frac{Q}{2}$$

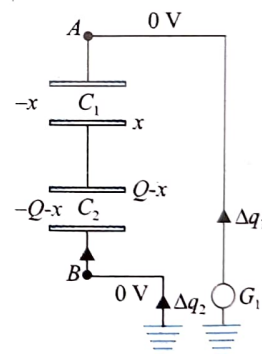
$$\text{and } q_b = \frac{Q - (Q - Q)}{2} = \frac{Q}{2}$$

$$C_1 = C = \frac{\epsilon_0 A}{2d} \text{ or } C_2 = 2C = \frac{\epsilon_0 A}{d}$$

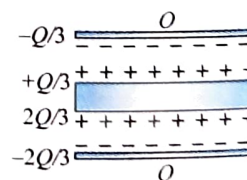


When both switches are closed the circuit can be redrawn as shown. Moving from A to B

$$0 + \frac{x}{C} - \frac{(Q - x)}{2C} = 0 \text{ or } x = \frac{Q}{3}$$



Final charges on each plate will be

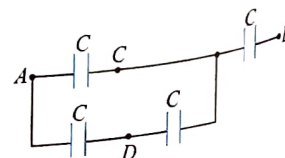


Hence charge moving from top plate to ground is $Q - \left(-\frac{Q}{3}\right)$
 $= \frac{4Q}{3}$ and from bottom plate to ground is $-Q - \left(-\frac{2Q}{3}\right) = -\frac{Q}{3}$

25. (2), (3), (4)

$$C = \frac{A \epsilon_0}{d}$$

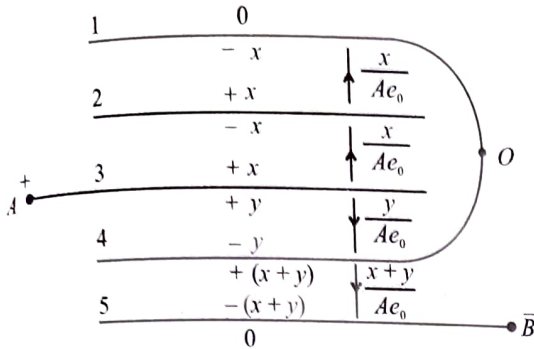
$$\frac{1}{C_{\text{eq}}} = \frac{1}{C} + \frac{2}{3C} = \frac{5}{3C}$$



$$\text{or } C_{eq} = \frac{3C}{5} = \frac{3A\epsilon_0}{5d}$$

Alternative method

$$C = \frac{Q}{V} = \frac{x+y}{V_{AB}}$$



$$C_{eq} = \frac{Q}{V} = \frac{x+y}{V_{AB}}$$

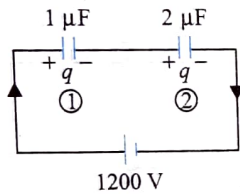
Potential of 1 and 4 is same

$$\frac{y}{A\epsilon_0} = \frac{2x}{A\epsilon_0} \quad \text{or } y = 2x$$

$$V = \left(\frac{2y+x}{A\epsilon_0} \right) d, \quad C_{eq} = \frac{(x+2x)A\epsilon_0}{(5x)d} = \frac{3A\epsilon_0}{5d}$$

Linked Comprehension Type

1. (2), 2. (4)

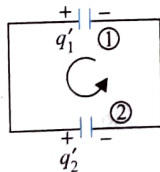


$$1200 - \frac{q}{1} - \frac{q}{2} = 0 \quad \text{or } q = 800 \mu\text{C}$$

Charge on each capacitor is 800 μC.

$$V_1 = \frac{q}{C_1} = \frac{800}{1} = 800 \text{ V}$$

$$V_2 = \frac{q}{C_2} = \frac{800}{2} = 400 \text{ V}$$



...(i)

$$(i) \quad q'_1 + q'_2 = 1600$$

$$\text{Also, } \frac{q'_1}{1} - \frac{q'_2}{2} = 0$$

$$\text{or } q'_2 = 2q'_1$$

$$\text{From Eq. (i), } 3q'_1 = 1600 \quad \text{or } q'_1 = \frac{1600}{3} \mu\text{C}$$

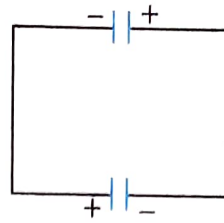
$$3q'_2 = \frac{3200}{3} \mu\text{C}$$

$$V = \frac{q_1 + q_2}{C_1 + C_2} = \frac{1600}{3} \text{ V}$$

$$(ii) \quad q'_1 + q'_2 = 800 - 800 = 0$$

$$\text{Also, } \frac{q'_1}{1} - \frac{q'_2}{2} = 0$$

...(ii)



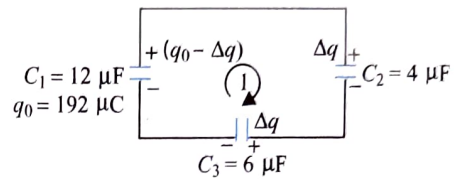
$$\text{From Eq. (ii), } q'_1 = q'_2 = 0.$$

Therefore, potential difference across each capacitor is zero.

3. (3), 4. (2), 5. (1)

In the close loop,

$$\frac{q_0 - \Delta q}{C_1} - \frac{\Delta q}{C_2} - \frac{\Delta q}{C_2} = 0$$



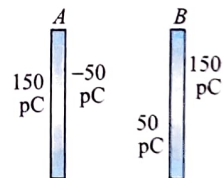
$$\frac{q_0}{C_1} = \Delta q \left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right]$$

$$\Delta q = \frac{q_0}{C_1 \left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right]} = \frac{192}{[1+3+2]} = 32 \mu\text{C}$$

$$\text{Potential difference across } C_2 = \frac{\Delta q}{C_2} = 8 \text{ V}$$

$$\text{Potential difference across } C_1 = \frac{(192-32)}{12} = \frac{160}{12} = \frac{40}{3} \text{ V}$$

6. (2), 7. (1)



$$C = \frac{\epsilon_0 A}{d} = \frac{8.8 \times 10^{-12} \times 5 \times 10^{-3}}{8.8 \times 10^{-3}} = 5 \times 10^{-12} \text{ F}$$

Charge on the plate after connection with the battery is

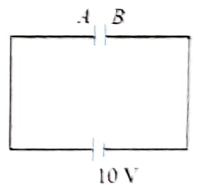
$$q = CV = 5 \times 10^{-12} \times 10 = 50 \text{ pC}$$

Charge supplied by the battery is

$$\Delta q = (50 + 50) \text{ pC} = 100 \text{ pC}$$

Energy supplied by the battery is

$$U_{\text{battery}} = \Delta q V = 100 \times 10 = 1000 \text{ pJ} = 10^{-9} \text{ J}$$



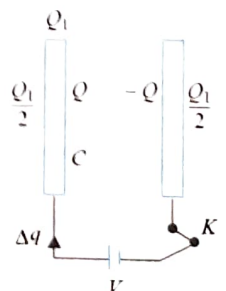
8. (4), 9. (4)

$$Q = CV$$

$$\text{Charge flow is } \Delta q = \left(Q + \frac{Q_1}{2} \right) - Q_1$$

$$Q - \frac{Q_1}{2} = CV - \frac{Q_1}{2}$$

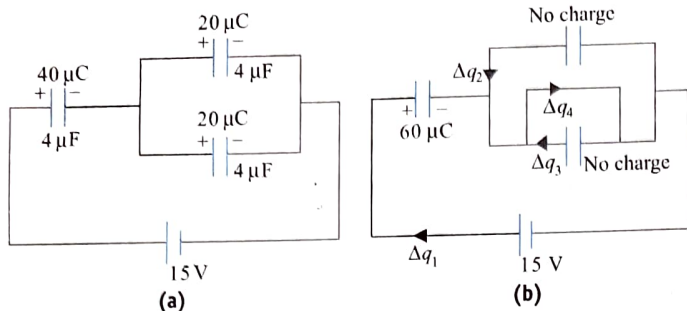
and charge on the inner surface of the left plate is $Q = CV$



10. (1), 11. (2)

Figure (a) corresponds to when switch is open.

Figure (b) corresponds to when switch is closed.



Various charges flowing after closing the switch are as follows:

$$\Delta q_1 = 60 - 40 = 20 \mu\text{C}$$

$$\Delta q_2 = 20 - 0 = 20 \mu\text{C}$$

$$\Delta q_3 = 20 - 0 = 20 \mu\text{C}$$

$$\Delta q_4 = \Delta q_1 + \Delta q_2 + \Delta q_3 = 60 \mu\text{C}$$

12. (3), 13. (1), 14. (4), 15. (2), 16. (3)

Potential difference across C_4 is V_4 . As discussed above

$$V_4 = V_1$$

$$V_4 = 17.5 \text{ V}$$

Consider any branch, say, XAY.

Since C_1 and C_2 are in series, charge on C_2 is also $35 \mu\text{C}$.

Using $Q = CV$ for C_2

$$C_2 = C = \frac{35}{V_2} = \frac{35}{12.5} = 2.8 \mu\text{F}$$

Equivalent capacitance of branch XAY is the series equivalent of

$$C_1 = 2 \mu\text{F} \text{ and } C_2 = 2.8 \mu\text{F}, \text{ i.e., } \frac{2 \times 1.8}{(2 + 2.8)} = 1.17 \mu\text{F}$$

Equivalent capacitance of branch XBY is the series equivalent of $C_3 = C = 2.8 \mu\text{F}$ and $C_4 = 2 \mu\text{F}$, i.e., $1.17 \mu\text{F}$. These branches are in parallel between X and Y. Hence, equivalent capacitance between X and Y is $1.17 + 1.17 = 2.34 \mu\text{F}$.

Capacitor C_5 is connected in the part of circuit between X and A either in series or parallel with $C_1 = 2 \mu\text{F}$. Let the equivalent capacitance of C_1 and C_5 , i.e., the equivalent capacitance between X and A be C_1' .

Capacitor C_6 is connected between A and B. Obviously, the circuit then becomes a Wheatstone bridge. Further, since equivalent capacitance between X and Y is independent of the value of C_6 , it implies that the bridge is in the balanced condition and potentials at A and B are now equal, so that

$$\frac{C_1'}{C_3} = \frac{C_2}{C_4}$$

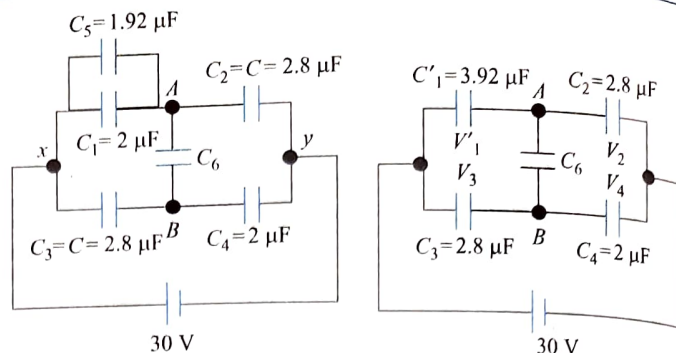
$$\text{or } \frac{C_1'}{2.8} = \frac{2.8}{2} \quad (C_2 = 2.8 \mu\text{F}, \text{ as determined earlier})$$

$$\text{or } C_1' = 3.92 \mu\text{F}$$

C_1' is the equivalent capacitance of $C_1 = 2 \mu\text{F}$ and C_5 . Since $C_1' > C_1$, we can conclude that C_1 and C_5 could not be connected in series. We know that series equivalent capacitance is less than each individual capacitance. Hence, C_1 and C_5 are in parallel, so

$$C_1' = C_1 + C_5 \text{ or } C_5 = C_1' - C_1 = 3.92 - 2 = 1.92 \mu\text{F}$$

$$V_1 + V_2 = 30 \quad \dots(\text{iv})$$



Since the bridge is in a balanced condition, A and B are at same potential and C_6 has zero charge. C_1' and C_2 are in series. Therefore, charge on C_1' and C_2 has the same value, so that potential difference will be in the inverse ratio of capacitance

$$[Q = CV \text{ or } V = \frac{Q}{C} \text{ hence for same } Q, V \propto \frac{1}{C}]$$

$$\frac{V_1'}{V_2} = \frac{2.8}{3.92} \approx 0.71 \quad \text{or } V_1' = 0.71 V_2$$

$$\text{From Eq. (iv), } 0.71 V_2 + V_2 = 30 \quad \text{or } V_2 = 17.5 \text{ V}$$

$$\text{Hence } V_1' = 30 - V_2 = 12.5 \text{ V}$$

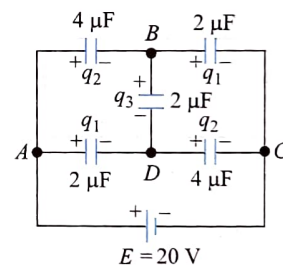
Since C_1' is parallel equivalent of C_1 and C_5 , so potential difference across each is $V_1' = 12.5 \text{ V}$.

Hence, charge on C_5 is

$$C_5 V_1' = 1.92 \times 12.5 = 24 \mu\text{C}$$

17. (2), 18. (1)

If $2 \mu\text{F}$ were not present between B and D, then potential drop across upper $4 \mu\text{F}$ will be less than potential drop across lower $2 \mu\text{F}$, i.e.,



$$V_A - V_B < V_A - V_D \Rightarrow V_B > V_D$$

Now if $2 \mu\text{F}$ is connected between B and D, charge will flow from B to D and finally $V_B > V_D$.

$V_A = 20 \text{ V}$. At junction B:

$$q_2 = q_1 + q_3$$

$$\text{or } (V_A - V_B)4 = (V_B - V_D)2 + (V_B - V_C)2$$

$$\text{or } 4(V_A - V_B) + 2(V_D - V_B) = 2V_B$$

At junction D:

$$q_2 = q_1 + q_3$$

$$\text{or } 4(V_D - V_C) = 2(V_A - V_D) + 2(V_B - V_D)$$

$$\text{or } 2(V_A - V_D) + 2(V_B - V_D) = 4V_D$$

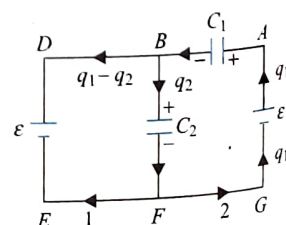
19. (2), 20. (4)

Charge on capacitors C_1 and C_2 before closing the switch S (figure) is

$$q_0 = \left(\frac{C_1 C_2}{C_1 + C_2} \right) \epsilon$$

After closing S, charge in C_2 (final charge) is

$$q_2 = C_2 \epsilon$$



In loop ABDEFGA, $\varepsilon - \frac{q_1}{\varepsilon} - \varepsilon = 0$ or $q_1 = 0$.

Final charge on C_1 , $q_1 = 0$. To make final charge in C_1 , the charge q_0 will flow toward battery.

Hence, charge flown toward direction 2 is

$$\Delta q_2 = -\left(\frac{C_1 C_2}{C_1 + C_2}\right) \varepsilon$$

To make charge on capacitor C_2 to final value q_2 , the charge flow into capacitor is

$$\Delta q = C_2 \varepsilon - \frac{C_1 C_2}{C_1 + C_2} \varepsilon = C_2 \varepsilon \left[1 - \frac{C_1}{C_1 + C_2}\right]$$

$$\Delta q_2 = \frac{C_2^2 \varepsilon}{C_1 + C_2}$$

At junction F,

$$\Delta q_1 = \Delta q - \Delta q_2 = \frac{C_2 \varepsilon}{C_1 + C_2} [C_2 + C_1] = C_2 \varepsilon$$

Hence, charge flowing in the direction of 1 is $\Delta q = C_2 \varepsilon$.

Alternative method: (See figure)

In loop ABDEG, $-\frac{(q_0 + \Delta q_1)}{C_1} - \varepsilon + \varepsilon = 0$

$$\text{or } \Delta q_1 = -q_0 = -\frac{C_1 C_2 \varepsilon}{C_1 + C_2}$$

In loop BFEDB,

$$-\frac{(q_0 + \Delta q_1)}{C_2} + \varepsilon = 0$$

$$\text{or } \Delta q = C_2 \varepsilon - q_0$$

At junction F,

$$\Delta q_2 = \Delta q_1 - \Delta q_1 = -q_0 - (C_2 \varepsilon - q_0) = -C_2 \varepsilon$$

Hence charge flow, in the direction of:

$$(1) \text{ is } -\frac{C_1 C_2 \varepsilon}{C_1 + C_2}$$

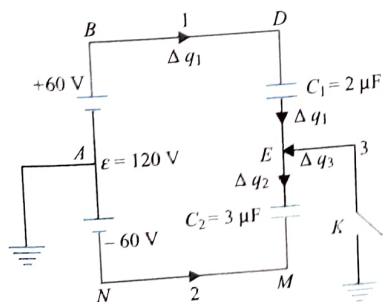
$$(2) \text{ is } C_2 \varepsilon$$

21. (1), 22. (2), 23. (4)

Before earthing charge on each capacitor (See figure).

$$q_0 = 120 \times \frac{6}{5}$$

$$q_0 = 24 \times 6 = 144 \mu\text{C}$$



After earthing, we get

$$\text{Path ABDE, } 0 + 60 - \frac{(q_0 + \Delta q_1)}{2} = 0$$

$$\Delta q_1 = 120 - q_0 = 24 \mu\text{C}$$

$$\text{Path ANME, } 0 - 60 + \frac{(q_0 + \Delta q_2)}{3} = 0$$

$$\text{or } \Delta q_2 = 180 - q_0 = 36 \mu\text{C}$$

$$\text{At junction E, } \Delta q_1 + \Delta q_3 = \Delta q_2$$

$$\text{or } \Delta q_3 = \Delta q_2 - \Delta q_1 = 36 + 24 = 60 \mu\text{C}$$

Hence, the charge flown in the direction

$$(1) \text{ is } -24 \mu\text{C}$$

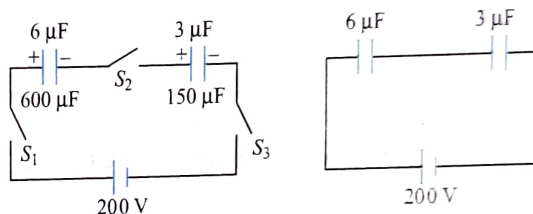
$$(2) \text{ is } -36 \mu\text{C}$$

$$(3) \text{ is } +60 \mu\text{C}$$

24. (4), 25. (2), 26. (4)

Plates 2 and 3 are joined together and they are neither connected to any of the terminals of the battery nor to any other source of charge. So, they jointly form an isolated system.

Before closing the switches, charges are shown in figure. After closing the switch, let q charge goes from battery. Then charge on each will increase by q .



Applying Kirchhoff's voltage law, we get

$$200 - \left(\frac{600 + q}{6}\right) - \left(\frac{150 + q}{3}\right) = 0$$

$$\text{or } q = 100 \mu\text{C}$$

So charge on $6 \mu\text{F}$ capacitor will be $700 \mu\text{C}$ and $3 \mu\text{F}$ capacitor will be $250 \mu\text{C}$.

From all the switches $100 \mu\text{C}$ of charge will flow.

27. (2), 28. (3), 29. (4)

When $C_3 \rightarrow \infty$, it means distance between the plates of C_3 is zero or both plates will be at the same potential. Then C_2 will be shorted. Entire potential V will be across C_1 . Hence, $V = V_1 = 10 \text{ V}$.

From graph, when $C_3 = 0$, $V_1 = 2 \text{ V}$. So, C_3 will act like open switch. C_1 and C_2 will be in series. Potential different across C_2 is $V_2 = 10 - V_1 = 8 \text{ V}$.

$$q = C_1 V_1 = C_2 V_2 \text{ or } C_{12} = C_{28} \text{ or } C_1 = 4C_2$$

$$V_1 = 4 \text{ V}, V_2 = 10 - 4 = 6 \text{ V}$$

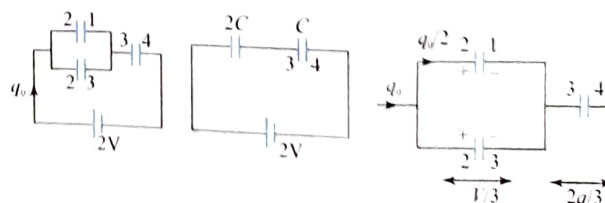
$$q = C_1 V_1 = (C_2 + C_3) V_2$$

$$\text{or } C_{14} = (C_2 + C_3) 6 \text{ or } 16C_2 = 6C_2 + 6C_3$$

$$\text{or } C_3 = 5C_2$$

30. (4), 31. (4), 32. (1)

When both the switches are closed, circuit is



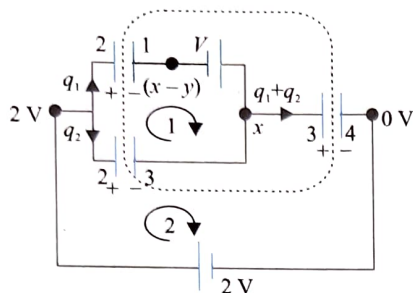
Charge on plate 2

$$C_{eq} = \frac{2C}{3} = \frac{2\varepsilon_0 A}{3d}$$

$$q_0 = \left(\frac{2C}{3}\right)(2V) = \frac{4CV}{3}$$

Charge on plate 1:

Where switch Sw_2 is opened, circuit is



$$\left[\frac{(x-V)}{-2V}\right]C + (x-2V)C + xC = 0$$

$$\text{or } x = \frac{5}{3}V$$

$$\text{Charge on plate 4 is } -\frac{5}{3}CV = -\frac{5\varepsilon_0 AV}{3d}$$

Alternative method:

$$\frac{-q_1}{C} + V + \frac{q_2}{C} = 0 \quad \text{or} \quad -q_1 + q_2 = -CV \quad \dots(i)$$

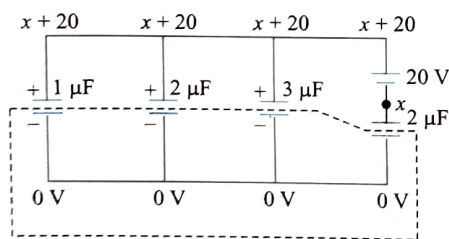
$$\frac{-q_1}{C} - \frac{(q_1 + q_2)}{C} + 2V = 0$$

$$\text{or } q_1 + 2q_2 = 2CV \quad \dots(ii)$$

$$q_1 = \frac{4CV}{3}, q_2 = \frac{CV}{3}$$

33. (3), 34. (1)

The circuit can be redrawn as



Conserving charges on lower plates, we get

$$-[(x+20)1 + (x+20)2 + (x+20)3 + 2x] = -60$$

$$\text{or } x = -\frac{15}{2}V$$

$$|Q_{1\mu F}| = 12.5 \mu C$$

$$|Q_{2\mu F}| = 25 \mu C$$

$$|Q_{3\mu F}| = 37.5 \mu C$$

$$|Q_{2\mu F}| = 15 \mu C$$

Matrix Match Type

1. i. $\rightarrow$ a., c.; ii. $\rightarrow$ b.; iii. $\rightarrow$ a., c.; iv. $\rightarrow$ d.

- By inserting dielectric slab, capacitance of 1 increases thereby increasing charge on capacitor. Thus, more charge flows through the battery. Energy stored in capacitor also increases.
- By increasing separation between the plates, capacitance of C_1 decreases. Charge on C_2 also decreases.

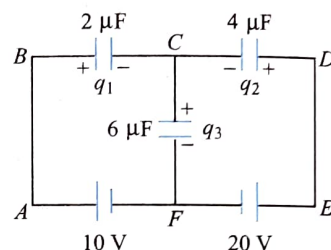
- By shorting capacitor 1, only capacitor 2 remains in the circuit. Potential difference across C_2 increases, thereby increasing charge on 2 as well as energy stored.
- By earthing plate of capacitor 1, potentials will change but there will be no change in potential difference, making no overall change in the circuit.

2. i. $\rightarrow$ b.; ii. $\rightarrow$ c.; iii. $\rightarrow$ a.; iv. $\rightarrow$ d.

The charges on three plates that are in contact add to zero, because these plates taken together form an isolated system, which cannot receive charges from the batteries. Thus, $q_3 - q_1 - q_2 = 0$. Applying Kirchhoff's law in loop $ABCFA$ and $CDEFC$, we get

$$-\frac{q_1}{2} - \frac{q_3}{6} + 10 = 0$$

$$\text{or } q_3 + 3q_1 = 60$$



$$\text{and } 20 - \frac{q_2}{4} - \frac{q_3}{6} = 0$$

$$\text{or } 3q_2 + 2q_3 = 240$$

Solving the above three equations, we have

$$q_1 = \frac{10}{3} \mu C, q_2 = \frac{140}{3} \mu C, q_3 = 50 \mu C$$

Potential difference across $6 \mu F$ capacitor is

$$\frac{q_3}{6} = \frac{50 \mu C}{6 \mu F} = \frac{25}{3} V$$

3. i. $\rightarrow$ b.; ii. $\rightarrow$ a.; iii. $\rightarrow$ d.; iv. $\rightarrow$ c.

These five plates constitute four identical capacitors in parallel, each of capacity $\varepsilon_0 A/d$. Now, as plate 1 is connected to the positive terminal of the battery and is a part of one capacitor only, so charge on it is

$$q_1 = +\left(\frac{\varepsilon_0 AV}{d}\right)$$

So, i. $\rightarrow$ b.

However, plate 4 is connected to the negative terminal of the battery and is common to the two identical capacitors in parallel. So,

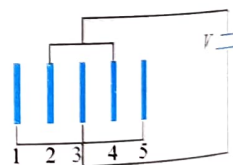
$$q_4 = -\frac{2\varepsilon_0 AV}{d}$$

Hence, ii. $\rightarrow$ a.

From the circuit, it is obvious that between the plates 2 and 3, the battery is connected, so the potential difference will be V . Plates 1 and 5 get connected through connecting wire, so the potential difference between 1 and 5 is zero.

Hence, iii. $\rightarrow$ d. iv. $\rightarrow$ c.

- i. $\rightarrow$ b., d.; ii. $\rightarrow$ a., c., d.; iii. $\rightarrow$ a., c., d.; iv. $\rightarrow$ a., c., d.
- Charge on the outer surfaces of the plates of capacitor will be always zero. If $V = E$, then no charge will flow in the circuit and hence no thermal energy will be dissipated. But if $V \neq E$, then charge will flow in the circuit and thermal energy will be dissipated.



6. i. → a.; ii. → b.; iii. → b., d.; iv. → b.
 Initial potential difference across C_1

$$V_1 = \frac{4V}{2+4} = \frac{2V}{3}$$

On doubling the distance between plates, C_1 becomes half. So, final potential difference across C_1

$$V_1' = \frac{4V}{1+4} = \frac{4V}{5}$$

This increases by a factor $\frac{V_1'}{V_1} = \frac{4V/5}{2V/3} = \frac{6}{5}$

(ii) Across C_2

$$\text{Initially } V_2 = \frac{2V}{2+4} = \frac{V}{3}$$

$$\text{Finally } V_2' = \frac{1V}{1+4} = \frac{V}{5}$$

This decreases by a factor of

$$\frac{V_2'}{V_2} = \frac{V/5}{V/3} = \frac{3}{5}$$

(iii) Energy in C_1

$$\text{Initially } U_1 = \frac{1}{2} \times 2 \left(\frac{2V}{3} \right)^2 = \frac{4V^2}{9}$$

$$\text{Finally } U_1' = \frac{1}{2} \times 1 \times \left(\frac{4V}{5} \right)^2 = \frac{8V^2}{25}$$

This decreases by a factor

$$\frac{U_1'}{U_1} = \frac{8V^2/25}{4V^2/9} = \frac{18}{25}$$

(iv) Energy in C_2

$$\text{Initially } U_2 = \frac{1}{2} \times 4 \left(\frac{V}{3} \right)^2 = \frac{2V^2}{9}$$

$$\text{Finally } U_2' = \frac{1}{2} \times 4 \left(\frac{V}{5} \right)^2 = \frac{2V^2}{25}$$

This decreases by a factor $\frac{U_2'}{U_2} = \frac{9}{25}$

6. i. → d.; ii. → d.; iii. → a.; iv. → a.;

(i) Voltage gets distributed among all capacitors.

(ii) The one with least capacitance

$$\Rightarrow S \Rightarrow \frac{C}{3}$$

(iii) The one with maximum capacitance

$$\Rightarrow 3C$$

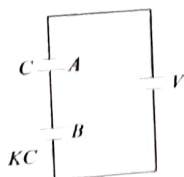
7. i. → a., d.; ii. → b., d.; iii. → b., d.; iv. → b., d.

As slab is inserted, capacitance of B increases and hence total capacitance of system decreases as a result change on both ↓.

$$Q = \frac{KC}{1+K} \times V$$

$$\text{or } V_A = \frac{Q}{C} = \frac{KV}{1+K} > \frac{V}{2}$$

$$V_B = \frac{Q}{KC} = \frac{V}{1+K} < \frac{V}{2}$$



8. i. → a.; ii. → b.; iii. → b.; iv. → c.

$$O = \frac{KQ}{2R} + \frac{K_C 2Q}{3R} + \frac{Kq_B}{2R}$$

$$q_B = - \left(Q + \frac{4Q}{3} \right) = - \frac{7Q}{3}$$

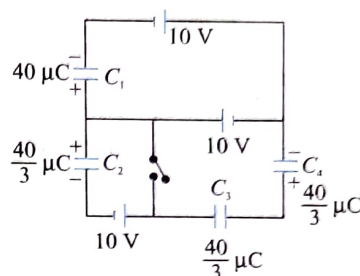
$$q_1 = -Q_A = -q$$

$$q_2 = - \frac{7Q}{3} + q = - \frac{4q}{3}$$

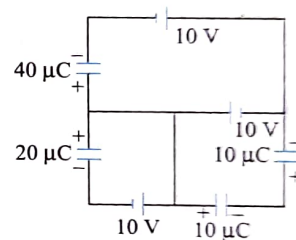
$$q_3 = \frac{4q}{3} \text{ and } q_3 + q_4 = 2q \text{ or } q_4 = - \frac{2q}{3}$$

9. i. → c.; ii. → a.; iii. → b.; iv. → b.

Before closing the switch:



After closing the switch



So charge on C_1 remains same, on C_2 increases, and on C_3 and C_4 decreases.

10. i. → a.; ii. → c.; iii. → b.; iv. → a.

The initial charge on capacitor is

$$q_i = CV_i = 1 \times 2 = 2 \mu C$$

The final charge on capacitor is

$$q_f = CV_f = 1 \times 4 = 4 \mu C$$

Net charge crossing the cell of emf 4 V is

$$q_f - q_i = 4 - 2 = 2 \mu C$$

The magnitude of work done by cell of emf 4 V is

$$W = (q_f - q_i) \times 4 = 8 \mu J$$

The gain in potential energy of capacitor is

$$\Delta U = \frac{1}{2} C (V_f^2 - V_i^2) = \frac{1}{2} \times [4^2 - 2^2] \mu J = 6 \mu J$$

Net heat produced in circuit is

$$\Delta H = W - U = 8 - 6 = 2 \mu J$$

Numerical Value Type

1. (4) To determine charge on 6 μF, we can remove 3 μF from the circuit. After this,

$$C_{eq} = 2 \mu F$$

$$Q = C_{eq} V = 2 \times 2 = 4 \mu C$$

2. (4) $\Delta V_{AB} = \frac{Q}{3}$, $\Delta V_{BC} = \frac{Q}{6}$ $\therefore (4+2)\mu F = 6\mu F$

$$\text{Also, } \frac{Q}{3} + \frac{Q}{6} = 12 \text{ or } Q = 24 \mu C$$

$$\Delta V_{AP} = \frac{24}{3} = 8 \text{ or } V_A - V_P = 8$$

$$\text{or } 12 - V_P = 8 \text{ or } V_P = 4 \text{ V}$$

$$3. (2) \frac{\epsilon_0 A}{d - \frac{d}{2} + \frac{d}{2k}} = \frac{4}{3} \left(\frac{\epsilon_0 A}{d} \right). \text{ Solve to get } k = 2.$$

4. (6) Volume of eight drops = Volume of big drop

$$8 \times \frac{4}{3} \pi r^3 = \frac{4}{3} \pi R^3 \text{ or } r = \frac{R}{2}$$

$$C = 4\pi\epsilon_0 R = 12 \mu\text{F}$$

Thus, capacitance of smaller drop is

$$C = 4\pi\epsilon_0 r = \frac{4\pi\epsilon_0 R}{2} = \frac{12}{2} = 6 \mu\text{F}$$

$$5. (4) \text{ Initial energy: } E = \frac{1}{2} C_0 V_0^2$$

(i) $E_1 = 2E$, because charge remains the same and the capacitance becomes half.

(ii) $E_2 = E/2$, because potential remains the same and the capacitance becomes half. So

$$\frac{E_1}{E_2} = 4$$

6. (9) Maximum charge C_1 can hold

$$Q_1 = C_1 V_1 = 1 \times 10^{-6} \times 6 \times 10^3 = 6 \times 10^{-3} \text{ C}$$

and maximum charge C_2 can hold

$$Q_2 = C_2 V_2 = 2 \times 10^{-6} \times 4 \times 10^3 = 8 \times 10^{-3} \text{ C}$$

When connected in series, both will have equal charges and so each can have charge Q_1 , which is smaller of the two. In this case, voltage across C_1 is 6 kV. And voltage across C_2 is

$$\frac{Q_1}{C_2} = \frac{6 \times 10^{-3}}{2 \times 10^{-6}} = 3 \text{ kV}$$

Therefore, the maximum voltage across the system is $6 + 3 = 9 \text{ kV}$.

7. (22) Let $C_1 = C$ and $C_2 = 2C$ and charges on different capacitors have been shown. Net charge on isolated system should be zero. Hence,

$$q_1 - q_2 - q_3 = 0 \quad \dots(i)$$

For left loop,

$$E - \frac{q_3}{2C} - \frac{q_1}{C} = 0 \quad \dots(ii)$$

$$\text{or } E - \frac{q_3}{2C} - \left(\frac{q_2 + q_3}{C} \right) = 0 \text{ (using } q_1 = q_2 + q_3)$$

$$\text{or } E - \frac{q_3}{2C} - \frac{q_2}{C} - \frac{q_3}{C} = 0$$

$$\text{or } E - \frac{q_2}{C} - \frac{3q_3}{2C} = 0 \quad \dots(iii)$$

For right loop,

$$E - \frac{q_2}{2C} - \frac{q_3}{2C} = 0$$

$$\text{or } 2E - \frac{q_2}{C} + \frac{q_3}{C} = 0 \quad \dots(iv)$$

Solving Eqs. (iii) and (iv), we get

$$q_3 = \frac{-2CE}{5}$$

$$\text{So } V_{MN} = \frac{-q_3}{2C} = \frac{2CE}{5 \times 2C} = \frac{E}{5} = \frac{110}{5} = 22 \text{ V}$$

8. (5) The distribution of charge is shown in figure in compliance with the point rule. Apply loop rules.

$$\text{Left loop: } -\frac{q_2}{5} + \frac{q_3}{0.75} + \frac{q_1}{15} = 0$$

$$\text{or } q_1 - 3q_2 + 20q_3 = 0 \quad \dots(i)$$

$$\text{Middle loop: } -\frac{q_2 + q_3}{15} - \frac{q_3}{0.75} + \frac{q_1 - q_3}{5} - \frac{q_3}{0.75} = 0$$

$$\text{or } 3q_1 - q_2 - 44q_3 = 0 \quad \dots(ii)$$

$$\text{Outermost loop: } 23 - \frac{q_2}{5} - \frac{q_2 + q_3}{15} - \frac{q_2}{5} = 0$$

$$\text{or } 345 = 7q_2 + q_3$$

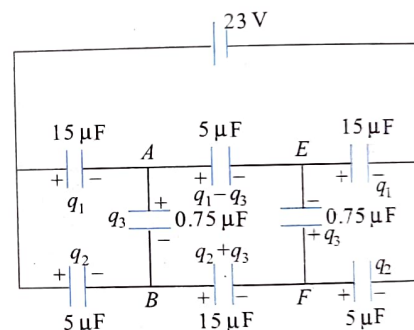
Solving for q_1 , q_2 , and q_3 , we get

$$q_1 = \frac{19 \times 345}{92}, q_2 = \frac{13 \times 345}{92}, q_3 = \frac{345}{92}$$

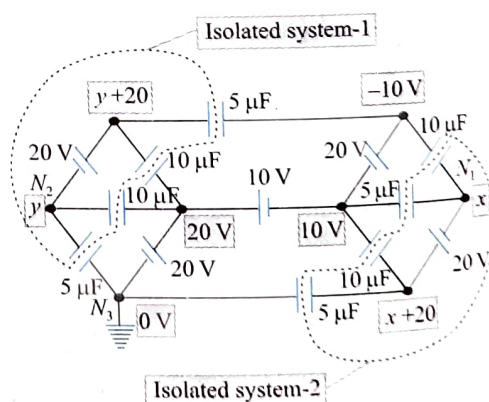
Therefore, potential difference between A and B is

$$\frac{q_3}{0.75} = \frac{345}{92} \times \frac{4}{3} = 5 \text{ V}$$

Potential difference between E and F is also 5 V but in the opposite direction.



9. (10) We assign node N_3 as 0 V and N_1, N_2 as x, y volt, respectively. Conservation of charge in isolated system 1.



$$5(y - 0) + 10(y - 20) + 10[(y + 20) - 20] + 5[(y + 20) - (-10)] = 0$$

$$\text{or } 30y = 50 \text{ or } y = \frac{5}{3} \text{ V}$$

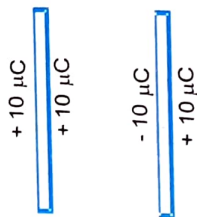
Conservation of charge in isolated system 2.

$$5[(x + 20) - 0] + 10[(x + 20) - 10] + 5[x - 10] + 10[x - (-10)] = 0$$

$$\text{or } 30x = -250 \text{ or } x = \frac{-25}{3} \text{ V}$$

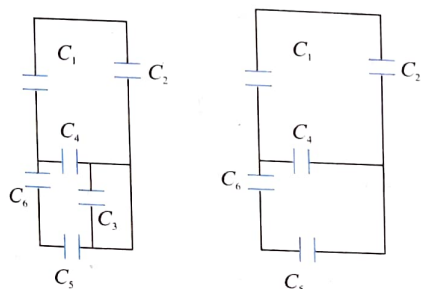
$$\text{Thus, } V_A - V_B = \frac{5}{3} - \left(\frac{-25}{3} \right) = 10 \text{ V}$$

10. (1) Given that capacitance $C = 10 \mu\text{F}$ and charge $= 20 \mu\text{C}$. But the effective charge $q = 10 \mu\text{C}$.



Potential difference is $q/C = 1 \text{ V}$.

11. (2) On joining two capacitors, charge on each will be half of the previous charge on the first capacitor. So energy stored on each capacitor will be one-fourth of energy stored in the previously in the first capacitor, that is, 1 J. Hence, the net energy stored in both will be $1 + 1 = 2 \text{ J}$.
12. (1.60) We begin with the circuit given, assigning names to each identical capacitor. C_1 begins with an unknown initial charge Q_0 , and achieves a charge Q after the circuit is connected and has re-established equilibrium.



As C_3 is clearly shorted, the potential difference across its plates is 0, and as such the capacitor will contain no charge and will not affect the circuit. We can therefore remove it from the schematic. This simplified circuit can be broken down into even simpler equivalent circuits. Leaving C_1 where it is, and recognizing C_5 and C_6 in series with each other (yielding an equivalent capacitance of $1/2 C$), and together in parallel with C_4 .

Acknowledging the combined capacitors are equal, C_{456} must have an equivalent capacitance of $3/2 C$. As C_{456} is simply in series with C_2 , the resultant equivalent circuit has simply one capacitor (C_{2456}) in series with C_1 , where C_{2456} must have an equivalent capacitance of $3/5 C$.

Since $Q = CV$, $V_1 = Q/C$

$$V_{2456} = 5Q'/3C,$$

where V_1 is the potential difference across C_1 , V_{2456} is the potential difference across C_{2456} , and Q' is the charge on C_{2456} .

For equilibrium conditions, $V_1 = V_2$. Therefore,

$$Q/C = 5Q'/3C, \quad 3CQ = 5CQ'$$

$$Q' = (3/5)Q.$$

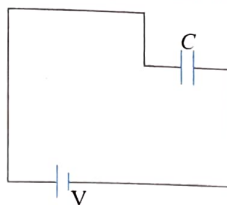
By the conservation of charge,

$$Q_0 = Q + Q' = Q + (3/5)Q = (8/5)Q.$$

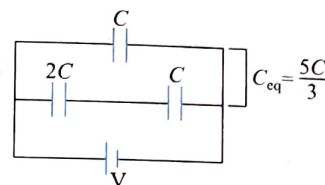
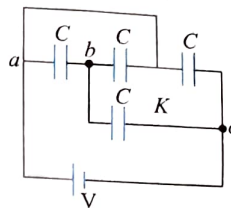
The initial charge on the capacitor was $8/5$ of the final charge on the capacitor after the switch was closed.

13. (100) We can redraw the circuit with K is open and when K is closed as shown in figures below

With key open circuit is



With key closed



Initial and final energies stored in capacitors are

$$U_i = \frac{1}{2} CV^2 \Rightarrow U_f = \frac{1}{2} \left(\frac{5C}{3} \right) V^2$$

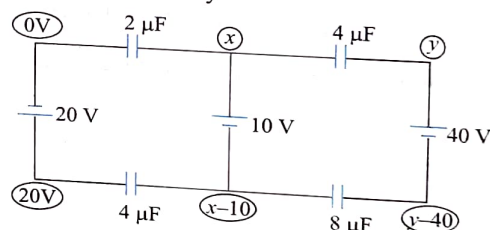
Change in stored energy is given as

$$\Delta U = U_f - U_i = \frac{1}{2} \left(\frac{5C}{3} - C \right) V^2$$

$$\Rightarrow \Delta U = \frac{1}{2} \left(\frac{2C}{3} \right) V^2 = \frac{1}{3} CV^2$$

$$\Rightarrow \Delta U = \frac{1}{3} \times 3 \times 10^{-6} \times 10^2 = 100 \mu\text{J}$$

14. (40) In the circuit we distribute the potentials as shown in figure below with reference to a zero potential considered at negative terminal of 20 V battery.



Writing nodal equations for x and y gives

$$2x + 4(x - y) + 4(x - 10 - 20) + 8(x - 10 - y + 40) = 0$$

$$9x - 6y = -60 \Rightarrow 3x - 2y = -20$$

$$\text{and } 4(y - x) + 8(y - 40 - x + 10) = 0 \quad \dots(i)$$

$$3y - 3x = 60 \Rightarrow y - x = 20$$

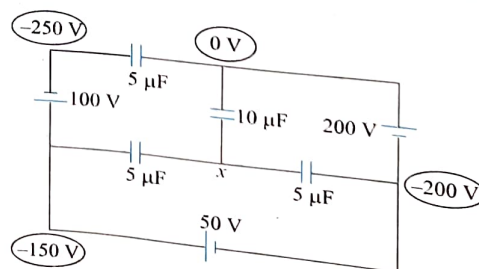
$$\text{Solving Eqs. (i) and (ii) gives} \quad \dots(ii)$$

$$x = 20V$$

This gives charge on $2 \mu\text{F}$ capacitor as

$$q_{2\mu\text{F}} = 2x = 2 \times 20 = 40 \mu\text{C}$$

15. (700) We distribute the potentials in the circuit shown in figure below.



Writing nodal equation for x gives

$$10x + 5(x + 200) + 5(x + 150) = 0$$

$$\Rightarrow 4x = -350 \Rightarrow x = -70 \text{ V}$$

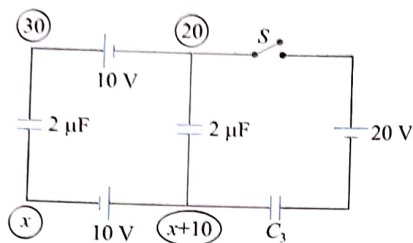
Thus charge on $10 \mu\text{F}$ capacitor is given as

$$q_{10\mu\text{F}} = 10 \times 70 = 700 \mu\text{C}$$

16. (0.30) When switch is open charge on capacitors are

$$q_{C_1} = q_{C_2} = 1 \times 20 = 20 \mu\text{C} \Rightarrow q_{C_3} = 0$$

After closing the switch circuit is shown figure,



Writing nodal equation for x gives

$$2(x + 10) + 2(x + 10 - 20) + 2(x - 30) = 0$$

$$3x = 30 \Rightarrow x = 10 \text{ V}$$

Thus find charge on capacitors is

$$q_{C_1} = 2 \times 20 = 40 \mu\text{C} \Rightarrow q_{C_2} = 0$$

$$q_{C_3} = 2 \times 20 = 20 \mu\text{C}$$

Thus heat produced on closing the switch is

$$H = \frac{\Delta q_1^2}{2C_1} + \frac{\Delta q_2^2}{2C_2} + \frac{\Delta q_3^2}{2C_3} = 300 \mu\text{J} = 0.3 \text{ mJ}$$

17. (24) For area of the plate to be minimum, the separation between the plates must be minimum. The separation between the plates is limited by the breakdown strength of the dielectric.

For air capacitor $\frac{V}{d_{\min}} = E_{\text{air}}$ [E_{air} = Breakdown field for air]

$$\therefore d_{\min} = \frac{V}{E_{\text{air}}}$$

$$\text{Now } \frac{\epsilon_0 A_{\min}}{d_{\min}} = C$$

$$\Rightarrow A_{\min} = \frac{C}{\epsilon_0} \frac{V}{E_{\text{air}}}$$

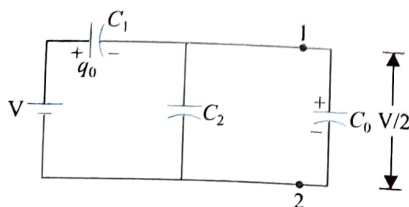
$$\Rightarrow A_1 = \frac{CV}{\epsilon_0 E_{\text{air}}}$$

With dielectric, similar calculation gives

$$A_2 = \frac{CV}{K\epsilon_0 E_{\text{dielect}}}$$

$$\therefore \frac{A_1}{A_2} = \frac{KE_{\text{dielect}}}{E_{\text{air}}} = 3 \times 8 = 24$$

18. (2) Let the equivalent capacitance be C_0 . Even if we remove one ladder, the equivalent will remain C_0 .



The potential drop across C_1 in figure shown is $\frac{V}{2}$.

$$\therefore \frac{q_0}{C_1} = \frac{V}{2} \Rightarrow q_0 = \frac{C_1 V}{2}$$

$$\text{Also } q_0 = C_0 V$$

$$\therefore \frac{C_1}{2} = C_0 \Rightarrow C_1 = 2C_0$$

From the given circuit

$$C_0 = \frac{(C_0 + C_2)C_1}{C_0 + C_2 + C_1}$$

$$\Rightarrow C_0^2 + C_2 C_0 - C_1 C_2 = 0$$

$$C_0 = \frac{-C_2 \pm \sqrt{C_2^2 + 4C_1 C_2}}{2}$$

Only positive sign is acceptable.

$$\therefore C_0 = \frac{-C_2 + \sqrt{C_2^2 + 4C_1 C_2}}{2}$$

$$\therefore \frac{C_1}{2} + \frac{C_2}{2} = \frac{\sqrt{C_2^2 + 4C_1 C_2}}{2}$$

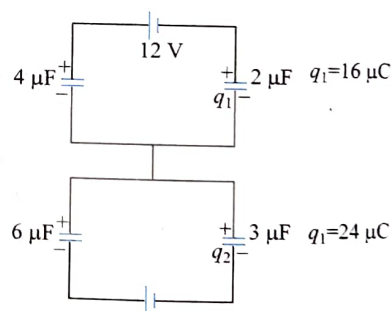
$$(C_1 + C_2)^2 = C_2^2 + 4C_1 C_2$$

$$C_1^2 = 2C_1 C_2 \Rightarrow C_1 = 2C_2$$

19. (240) With S open

Net emf = 0

$\therefore$ Charge on all capacitors = 0



With S closed

Energy stored in capacitors

$$U = 240 \mu\text{J}$$

Work done by cells = 480 μJ

Heat produced = 240 μJ

Archives

JEE Main

Single Correct Answer Type

- (1) $120C_1 = 200C_2$
 $6C_1 = 10C_2$
 $3C_1 = 5C_2$

- (3) By formula of electric field between the plates of a capacitor

$$E = \frac{\sigma}{k\epsilon_0}$$

$$\begin{aligned}\Rightarrow \sigma &= Ek\epsilon_0 = 3 \times 10^4 \times 2.2 \times 8.85 \times 10^{-12} \\ &= 6.6 \times 8.85 \times 10^{-6} = 5.841 \times 10^{-7} \\ &\approx 6 \times 10^{-7} \text{ C/m}^2\end{aligned}$$

$$(2) C_{eq} = \frac{3C}{3+C}$$

$$\text{Total charges } q = \left(\frac{3C}{3+C} \right) E$$

Charge upon capacitor $2 \mu\text{F}$,

$$Q_2 = \frac{2}{3} \times \frac{3CE}{(3+C)} = \frac{2CE}{3+C} = \frac{2E}{1+\frac{3}{C}}$$

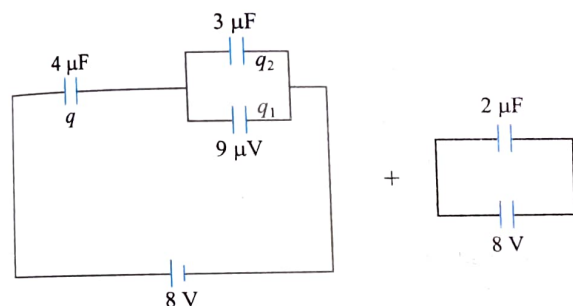
$$\text{If } C = 1 \mu\text{F}; q' = \frac{E}{4}$$

and if $C = 3 \mu\text{F}; q' = E$

Hence Q_2 is maximum at $C = 3 \mu\text{F}$

Hence graph (2) is correct.

4. (3) We can break this circuit into two parts



$$q = C_{eq}V = \left(\frac{4 \times 12}{4+12} \right) \times 8 = 24 \mu\text{C}$$

$$q_1 = \frac{9}{9+3} \times 24 \mu\text{C} = 18 \mu\text{C}$$

$$Q = q + q_1 = 24 + 18 = 42 \mu\text{C}$$

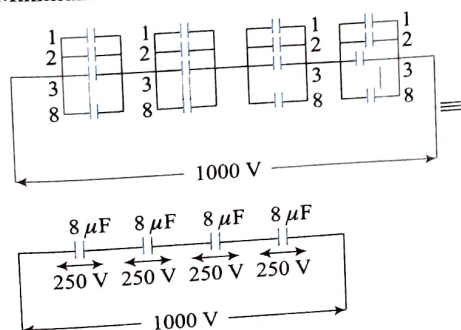
$$E = \frac{kQ}{r^2} = \frac{9 \times 10^9 \times 42 \times 10^{-6}}{(30)^2} = 420 \text{ N/C}$$

5. (2) To hold 1 kV potential difference minimum four capacitors are required in series

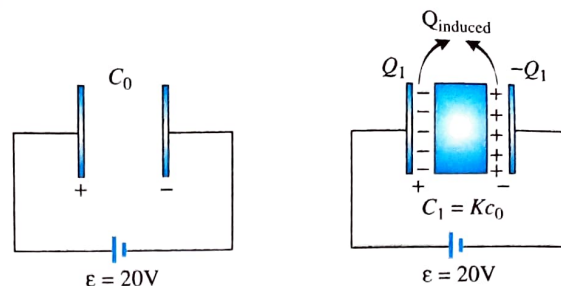
$$\Rightarrow C_1 = \frac{1}{4} \text{ for one series.}$$

So for C_{eq} to be $2 \mu\text{F}$, 8 capacitors of $1 \mu\text{F}$ in parallel with four such branches in series. Following arrangement will do the needful

$$\Rightarrow \text{Minimum number of capacitors} = 8 \times 4 = 32$$



6. (2)



The charge in capacitor after placing the dielectric

$$Q_1 = C_1 V$$

$$\text{or } Q_1 = (KC_0)\epsilon = \frac{5}{3} \times 90 \times 20 = 3000 \text{ pC}$$

$$\text{or } Q_1 = 3 \text{ nC}$$

The induced charge on dielectric surface

$$Q_{\text{induced}} = Q_1 \left(1 - \frac{1}{K} \right)$$

$$Q_{\text{induced}} = 3 \left(1 - \frac{1}{5/3} \right) = 3 \times \frac{2}{5} = \frac{6}{5} = 1.2 \text{ nC}$$

JEE Advanced

Single Correct Answer Type

$$1. (4) U_i = \frac{1}{2} (2)V^2 = V^2.$$

After connecting S to 2,

$$V_{\text{common}} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} = \frac{2V + 8 \times 0}{2 + 8} = \frac{V}{5}$$

$$U_f = \frac{1}{2} (2+8) \left(\frac{V}{5} \right)^2 = \frac{V^2}{5}$$

$$\text{Percent energy dissipated} = \frac{U_i - U_f}{U_i} \times 100$$

$$= \frac{V^2 - \frac{V^2}{5}}{V^2} \times 100 = 80\%$$

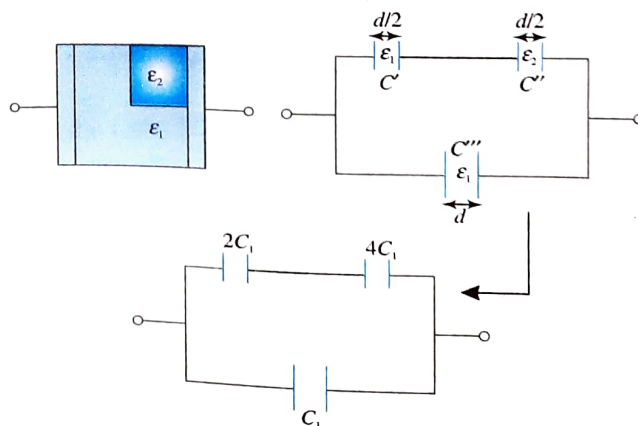
$$2. (3) q_3 = \frac{C_3}{C_2 + C_3} Q$$

$$q_3 = \frac{3}{3+2} \times 80 = \frac{3}{5} \times 80 = 48 \mu\text{C}$$

3. (4) A capacitance of the capacitor without dielectric.

$$C_1 = \frac{\epsilon_0 A}{d}$$

Now consider the case with filled dielectric. The given capacitor can be divided into three capacitors as shown.



$$C' = \frac{2\epsilon_0 \frac{S}{2}}{d/2} = \frac{2\epsilon_0 S}{d} = 2C_1$$

$$C'' = \frac{4\epsilon_0 \frac{S}{2}}{d/2} = \frac{4\epsilon_0 S}{d} = 4C_1$$

$$C''' = \frac{2\epsilon_0 \frac{S}{2}}{d} = \frac{\epsilon_0 S}{d} = C_1$$

$$C_2 = \frac{C'C''}{C' + C''} + C''' = \frac{4\epsilon_0 S}{3d} + \frac{\epsilon_0 S}{d} = \frac{7\epsilon_0 S}{3d}$$

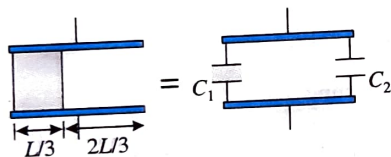
This gives : $\frac{C_2}{C_1} = \frac{7}{3}$

Multiple Correct Answers Type

1. (2), (4)

After switch S_1 is closed, C_1 is charged by $2CV_0$; when switch S_2 is closed, C_1 and C_2 both have upper plate charge CV_0 . When S_3 is closed, then upper plate of C_2 becomes charged by $-CV_0$ and lower plate by $+CV_0$.

2. (1), (4)



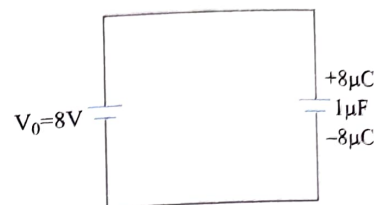
If $C = \frac{\epsilon_0 A}{3d}$ then $C_1 = kc$ $C_2 = 2c$

$$C_1 + C_2 = C \Rightarrow (k+2)c = C \Rightarrow c = \frac{C}{k+2}$$

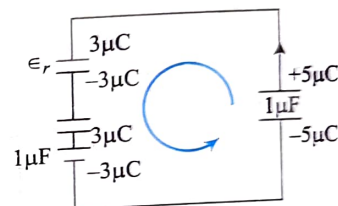
$$\therefore C_1 = \frac{kc}{k+2}, C_2 = \frac{2c}{k+2} \Rightarrow \frac{C}{C_1} = \frac{2+k}{k}$$

Numerical Value Type

1. (1.50) When S_1 is closed and S_2 is open charge in capacitor C_3
 $q_3 = C_3 V_0 = 1 \times 8 = 8 \mu C$



Now S_1 is open and S_2 is closed. Final charge in C_3 is found to be $5 \mu C$, hence the change in capacitors C_1 and C_2 should be $3 \mu C$.



Applying loop rule

$$\frac{5}{1} - \frac{3}{\epsilon_r \cdot 1} - \frac{3}{1} = 0 \Rightarrow \frac{3}{\epsilon_r} = 2$$

or $\epsilon_r = 1.50$

Chapter 5

Concept Application Exercises

Exercise 5.1

We know that $I = \frac{q}{t} = \frac{ne}{t}$; therefore, $n = \frac{It}{e} = \frac{0.7 \times 1}{1.6 \times 10^{-19}}$

So number of electrons is $n = 0.44 \times 10^{19}$

(a) $q = it = (7.5 \text{ A})(45 \text{ s}) = 337.5 \text{ C}$

(b) The number of electrons N is given by

$$N = \frac{q}{e} = \frac{337.5 \text{ C}}{1.6 \times 10^{-19} \text{ C}} = 2.1 \times 10^{21}$$

where $e = 1.6 \times 10^{-19} \text{ C}$ is the charge of an electron.

(a) The number N of electrons in 0.6 mol is

$$N = (0.6 \text{ mol})(6.02 \times 10^{23} \text{ electrons/mol}^{-1}) = 3.6 \times 10^{23} \text{ electrons}$$

$$q = Ne = (3.6 \times 10^{23})(1.6 \times 10^{-19}) = 5.78 \times 10^4 \text{ C}$$

(b) $t = (45 \text{ min})(60 \text{ s/min}^{-1}) = 2.7 \times 10^3 \text{ s}$

$$I = \frac{q}{t} = \frac{5.78 \times 10^4}{2.7 \times 10^3 \text{ Cs}} = 21.4 \text{ A}$$

4. Let dq be the charge that passes in a small interval of time dt , then $dq = i dt = (4 + 2t) dt$. Hence, the total charge passed between $t = 2$ and $t = 6$ is

$$q = \int_2^6 (4 + 2t) dt = 48 \text{ C}$$

5. In case of a resistance wire,

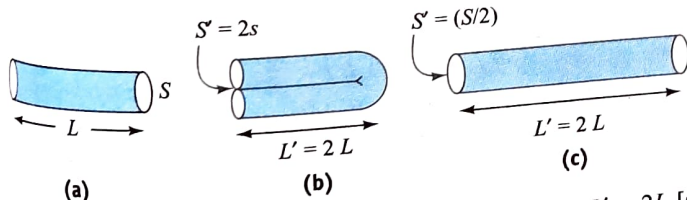
$$R = \rho \frac{L}{S} = \rho \frac{L}{\pi r^2} \quad [\text{as } S = \pi r^2]$$

And in changing dimensions without any change in mass, volume remains constant, i.e., $V = V'$ or $SL = S'L'$.

(a) When the wire is doubled on itself [as shown in Fig. (b)], its length will be halved, i.e., $L' = (L/2)$.

$$S \times L = S' \times (L/2), \quad \text{i.e., } S' = 2S$$

$$\therefore R' = \rho \left[\frac{L'}{S'} \right] = \rho \left[\frac{(L/2)}{(2S)} \right] = \frac{1}{4} \rho \frac{L}{S} = \frac{1}{4} R$$



(b) (i) When the length of the wire is doubled, i.e., $L' = 2L$ [as shown in Fig. (c)]

$$SL = 2LS', \quad \text{i.e., } S' = S/2$$

$$R' = \rho \left[\frac{L'}{S'} \right] = \rho \left[\frac{(2L)}{(S/2)} \right] = 4\rho \left[\frac{L}{S} \right] = 4R$$

(ii) When the radius of the wire is halved, i.e., $r' = r/2$,

$$S' = \pi r'^2 = \pi (r/2)^2 = \pi r^2/4 = S/4$$

And hence from $SL = S'L'$, i.e., $SL = (S/4)L'$,

$$\text{we have } L' = 4L, R' = \rho \frac{L'}{S'} = \rho \frac{(4L)}{(S/4)} = 16\rho \frac{L}{S} = 16R$$

$$6. \frac{R_f - R_i}{R_i} = \frac{\rho \frac{L_f}{A_f} - \rho \frac{L_i}{A_i}}{\rho \frac{L_i}{A_i}} = \frac{\frac{L_f}{A_f} - \frac{L_i}{A_i}}{\frac{L_i}{A_i}} \quad \dots(i)$$

Let the initial length of the wire be 100 cm, then the new length is

$$L_f = 100 + \frac{0.1}{100} \times 100 = 100.1 \text{ cm} \quad \dots(ii)$$

Let A_i and A_f be the initial and final areas of cross section. Then

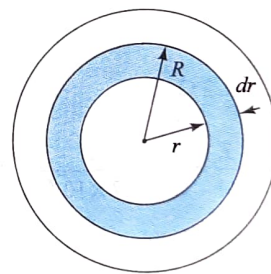
$$100 \times A_i = 100.1 A_f$$

$$\text{or } A_f = \frac{100}{100.1} A_i \quad \dots(iii)$$

From Eqs. (i), (ii), and (iii), we get

$$\begin{aligned} \frac{R_f - R_i}{R_i} \times 100 &= \frac{\frac{(100.1)^2}{100 A_i} - \frac{100}{A_i}}{\frac{100}{A_i}} \times 100 \\ &= \frac{(100.1)^2 - (100)^2}{(100)^2} \times 100 \\ &= \frac{200.1 \times 0.1}{100 \times 100} \times 100 = 0.2\% \end{aligned}$$

$$7. dL = J 2\pi r dr = J_0 \left(1 - \frac{r}{R} \right) 2\pi r dr = 2\pi J_0 \left(1 - \frac{r}{R} \right) r dr$$

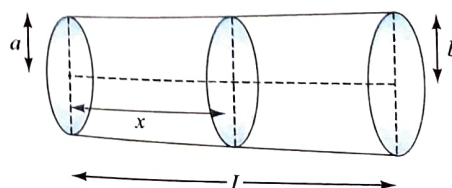


$$\begin{aligned} I &= \int dL = 2\pi J_0 \int_0^R \left(1 - \frac{r}{R} \right) r dr = 2\pi J_0 \left[\int_0^R r dr - \frac{1}{R} \int_0^R r^2 dr \right] \\ &= 2\pi J_0 \left[\frac{R^2}{2} - \frac{R^2}{3} \right] = 2\pi J_0 \frac{R^2}{6} = \frac{J_0 R^2}{3} \end{aligned}$$

8. Let at distance x , the radius is r . Then,

$$r = a + \Delta r = a + \left(\frac{b-a}{l} \right) x$$

$$R = \int dR = \int \frac{\rho dx}{\pi r^2} = \int_0^l \frac{\rho dx}{\pi \left[a + \left(\frac{b-a}{l} \right) x \right]^2}$$



$$\text{Let } a + \left(\frac{b-a}{l}\right)x = z \text{ or } \left(\frac{b-a}{l}\right)dx = dz$$

When z goes from a to b , we get

$$R = \int dR = \int_a^b \frac{\rho l}{\pi(b-a)z^2} dz = \frac{\rho l}{\pi(b-a)} \int_a^b \frac{dz}{z^2}$$

$$= \frac{\rho l}{\pi(b-a)} \left[\frac{1}{a} - \frac{1}{b} \right] = \frac{\rho l}{\pi ab}$$

9. Charge on the capacitor,

$$Q = CV = 20 \times 10^{-19} \times 2500 = 5 \times 10^{-5} \text{ C}$$

$$\text{Surface charge density is } \sigma_s = \frac{Q}{A} \quad [A \text{ is the plate area.}]$$

$$\text{Electric field strength between the plates is } E = \frac{\sigma_s}{K\epsilon_0} = \frac{Q}{KA\epsilon_0}$$

$$\text{Using } J = \sigma E = \frac{E}{\rho}$$

[Here σ is the conductivity and ρ is the resistivity.]

Current density is $J = Q/KA\epsilon_0\rho$ and current is

$$JA = \frac{Q}{K\epsilon_0\rho} = \frac{5 \times 10^{-5}}{6 \times (8.85 \times 10^{-22}) \times (2 \times 10^{11})}$$

$$= 4.7 \times 10^{-6} \text{ A} = 4.7 \mu\text{A}$$

10. Let L be the length and A the cross-sectional area of the wire and d be the density of the material of wire.

$$\text{Mass } M = \text{Volume} \times \text{density} = ALd$$

$$\text{or } AL = \frac{M}{d} = \frac{2.23 \times 10^{-3}}{8.92 \times 10^3} = \frac{1}{4} \times 10^{-6} \text{ m}^3 \quad \dots(i)$$

Resistance of wire,

$$R = \frac{V}{I} = \frac{1.7}{1} = 1.7 \Omega$$

$$R = \frac{\rho L}{A} \text{ or } \frac{L}{A} = \frac{R}{\rho} = \frac{1.7}{1.7 \times 10^{-8}} = 10^8 \text{ m}^{-1} \quad \dots(ii)$$

Multiplying (i) and (ii), we get

$$L^2 = \frac{1}{4} \times 10^{-6} \times 10^8 = \frac{10^2}{4} \text{ or } L = 5 \text{ m}$$

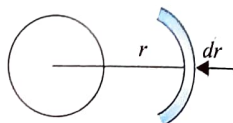
$$\text{From (i), } A = \frac{\frac{1}{4} \times 10^{-6}}{L} = \frac{\frac{1}{4} \times 10^{-6}}{5} = \frac{10^{-6}}{20} = 5 \times 10^{-8} \text{ m}^2$$

When wire is stretched to length L , its cross-sectional area decreases. So $R \propto L^2$. If R' is new resistance of wire, then

$$\frac{R'}{R} = \left(\frac{L'}{L}\right)^2$$

$$\text{or } R' = \left(\frac{L'}{L}\right)^2 R = (2)^2 \times 1.7 = 6.8 \Omega$$

$$11. (i) R = \int \frac{\rho dl}{A} = \int_a^b \frac{\rho \left(\frac{dr}{4\pi r^2}\right)}{\frac{\rho}{4\pi} \left(\frac{1}{a} - \frac{1}{b}\right)} = \frac{\rho}{4\pi} \left(\frac{b-a}{ab}\right)$$



$$(ii) \text{ The current through the sphere, } i = \frac{V}{R}$$

$$\Rightarrow i = \frac{V}{\frac{\rho}{4\pi} \left(\frac{b-a}{ab}\right)} = \frac{4\pi Vab}{\rho(b-a)}$$

(iii) Current density in function of radial distance

$$J = \frac{i}{A} = \frac{V}{RA} = \frac{V}{\frac{\rho}{4\pi} \left(\frac{1}{a} - \frac{1}{b}\right) \cdot 4\pi r^2} = \frac{Vab}{\rho(b-a)r^2}$$

(iv) Electric field in function of radial distance

$$= E = \rho J = \frac{Vab}{(b-a)r^2}$$

$$12. R = \int \rho \frac{dr}{A} = \int (\rho_0 r^3) \left(\frac{dr}{4\pi r^2}\right)$$

$$= \frac{\rho_0}{4\pi} \int_a^b r dr = \frac{\rho_0(b^2 - a^2)}{8\pi}$$

Exercise 5.2

- (a) As by definition, current and current density are related to each other through the relation $J = I/S$, so for steady current (i.e., $I = \text{constant}$) but nonuniform cross section J will not be constant but will vary with cross-sectional area S .

$$\text{As } J \propto \frac{I}{S} \quad [I \text{ being constant}]$$

Now, as J is related to electric field E and drift velocity v_d through the relations $J = \sigma E$ and $J = nev_d$

$$E = \frac{I}{\sigma S} \text{ and } v_d = \frac{I}{neS} \quad \left[\text{as } J = \frac{I}{S} \right]$$

But as for a given metal σ and n are constants, so

$$E \propto \frac{1}{S} \text{ and } v_d \propto \frac{1}{S} \quad [\text{as } I = \text{constant}]$$

i.e., in case of steady current flow through a metallic conductor of nonuniform cross section, current is constant while current density, electric field, and drift velocity are not constant and all vary inversely with area of cross section.

(b) Yes, current flows in a conductor only when electric field established within the conductor exerts force on the electrons.

- According to electron theory of metals, the drift speed of an electron inside a metal in the presence of an electric field E is given by

$$v_d = \left(\frac{e\tau}{m}\right)E = \left(\frac{e\tau}{m}\right)\left(\frac{V}{L}\right) \quad \left[\text{as } E = \frac{V}{L} \right]$$

- As $v_d \propto V$, on doubling V , drift velocity will be doubled.
- As $v_d \propto (1/L)$, on doubling L , drift velocity will be halved.
- As drift velocity is independent of diameter d , it will not change on doubling the diameter.

$$3. (a) E_1 = \frac{V}{L}, E_2 = \frac{2}{2L} = \frac{V}{L}, E_3 = \frac{2V}{3L}$$

Hence, $E_1 = E_2 > E_3$

$$(b) V_d \propto E. \text{ Hence, } V_{d1} = V_{d2} > V_{d3}$$

$$(c) I = neAv_d = \frac{n\pi d^2}{4} v_d = \frac{n\pi}{4} v_d d^2$$

$$\text{or } I \propto v_d d^2 \text{ or } I \propto E d^2 \text{ or } I = K E d^2$$

$$I_1 = \frac{KV}{L} (3d)^2 = \frac{9KVd^2}{L}, I_2 = K \frac{V}{L} d^2 A$$

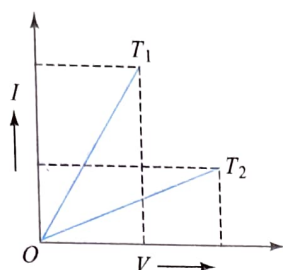
$$I_3 = K \frac{2V}{3L} (2d)^2 = \frac{8KVd^2}{3L}$$

Hence, $I_1 > I_3 > I_2$.

4. We know $R = V/I$ and slope of IV curve is $1/R$.

$$\text{or } R = \frac{1}{\text{slope}}$$

Since the slope of OT_2 is smaller than OT_1 , the resistance of wire at T_2 is greater than that at T_1 as resistance increases with temperature, so temperature T_2 is greater than T_1 .



$$5. (a) J = nqv_d = (2 \times 10^8 \times 10^6) \times (2 \times 1.6 \times 10^{-19}) \times 10^5 = 6.4 \text{ Am}^{-2}$$

As the velocity of positive ions is toward north, the direction of $\vec{J}$ will also be toward north.

(b) No, we need area of cross section of the wire.

$$6. \text{Number per second} = (\text{number/length}) (\text{velocity})$$

$$= (2 \times 10^{21})(0.05)$$

$$= 1 \times 10^{20} \text{ electrons/s}$$

$$I = Q/t = (1 \times 10^{20})(1.6 \times 10^{-19}) = 16 \text{ A}$$

$$7. R = R_0 [1 + \alpha(T - T_0)] \text{ or } \alpha = \Delta R / (R_0 \Delta T),$$

with $\Delta R = R - R_0 = 0.17 \Omega$ and $\Delta T = T - T_0 = 15^\circ\text{C}$. Then

$$\alpha = (0.17)/(25.00 \times 15) = 4.5 \times 10^{-4} \text{ } ^\circ\text{C}^{-1}$$

$$8. R_{150^\circ\text{C}} = 2.415 = R_0(1 + 150\alpha), R_{20^\circ\text{C}} = 1.6424 = R_0(1 + 20\alpha)$$

Solving these relations simultaneously, $\alpha = 3.9 \times 10^{-3} \text{ } ^\circ\text{C}^{-1}$ and

$R_0 = 1.5236 \Omega$. From $R_0 = \rho_0(L/A)$, we get

$$1.5236 = \frac{\rho_0(300)}{\pi(2 \times 10^{-3})^2/4} \text{ or } \rho_0 = 1.596 \times 10^{-8} \text{ Wm}$$

$$\text{Then, } \rho_{20^\circ\text{C}} = \rho_0(1 + 20\alpha) = (1.596 \times 10^{-8})[1 + (3.9 \times 10^{-3})(20)] = 1.720 \times 10^{-8} \Omega\text{m}$$

This indicates that the material is copper.

$$9. \text{We need } R_1(1 + \alpha_1\Delta t) + R_2(1 + \alpha_2\Delta t) = 20. \text{ Because } R_1 + R_2 = 20 \text{ and when } \Delta t = 0, \text{ we must have } R_1\alpha_1 = -R_2\alpha_2 \text{ with } \alpha_1 = -0.5 \times 10^{-3} \text{ and } \alpha_2 = 5 \times 10^{-3}. \text{ Solving the two equations } R_1 + R_2 = 20 \text{ and } R_1 = 10R_2 \text{ simultaneously leads to } R_1 = 18.18 \Omega \text{ and } R_2 = 1.82 \Omega.$$

$$10. \text{We assume that } \alpha \text{ is constant over the needed temperature range. Then } \Delta R = \alpha R \Delta t \text{ leads to } (35 - 10) = 0.0036(10) \Delta t. \text{ Solving, we get } \Delta t = 25/0.036 = 694^\circ\text{C}. \text{ Finally, } 694 + 20 = 714^\circ\text{C} \text{ (furnace temperature).}$$

11. In such a problem, term $\alpha\Delta T$ will have a larger value, so it could not be used directly in $R = R_0(1 + \alpha\Delta T)$. We need to go for basics. As we know that

$$\alpha = \frac{dR}{RdT}$$

$$\text{or } \int \frac{dR}{R} = \int \alpha dT \text{ or } \ln \frac{R_2}{R_1} = \alpha(T_2 - T_1)$$

$$\text{or } R_2 = R_1 e^{\alpha(T_2 - T_1)} = 10e^1 = 10e$$

...(i)

12. We know $I = n e A v_d$

Here, n is the number of electrons per unit volume of copper wire.

$$n = \frac{N}{V} = \frac{N_A}{M} \frac{m}{V} = \frac{N_A \rho}{M} \text{ } \dots\text{(ii)}$$

$$\text{From (i) and (ii), } I = \left(\frac{N_A \rho}{M} \right) e A v_d$$

$$\text{or } v_d = \frac{IM}{N_A \rho e A} = 4.9 \times 10^{-7} \text{ ms}^{-1}$$

13. (a) The resistance of a wire varies with temperature according to the relation $R = R_0(1 + \alpha\Delta\theta)$. So

$$\frac{R_2}{R_1} = \frac{R_0[1 + \alpha(t_2 - 0)]}{R_0[1 + \alpha(t_1 - 0)]} = \frac{(1 + \alpha t_2)}{(1 + \alpha t_1)}$$

According to given problem

$$R_2 = 2R_1 \text{ and } t_1 = 0$$

So, $2 = 1 + \alpha t_2$, i.e., $t_2 = (1/\alpha) = (1/4 \times 10^{-3}) = 250^\circ\text{C}$

(b) As here $t_2 (= 1/\alpha)$ does not include dimensions of the conductor (L and S), it is valid for all copper conductors whatever be their shape and size.

14. At a constant potential difference, since the final current is one-sixth of the initial current, it implies that the final resistance is six times the initial resistance, i.e., $R_f = 6R_i = 6R_{20}$ [Initial temperature of filament is 20°C].

Let the final temperature be $t^\circ\text{C}$ so that

$$R_f = R_i = 6R_{20} \dots\text{(i)}$$

We can express $R_t = R_{20} [1 + \alpha_{20}(t - 20)]$,

where α_{20} is the temperature coefficient of resistance at 20°C .

Using Eq. (i),

$$6R_{20} = R_{20} [1 + \alpha_{20}(t - 20)]$$

$$\text{or } t - 20 = \frac{5}{\alpha_{20}} = \frac{5}{0.0043} = 1162.3 \text{ or } t = 1182.8^\circ\text{C}$$

Exercise 5.3

1. The current will flow from the positive terminal to the negative terminal in the battery. During charging, the potential difference is $V + Ir = 2 + 5 \times 0.1 = 2.5 \text{ V}$.

2. (a) When no current is drawn from a cell, potential difference across the terminals of the cell is equal to its emf. It is clear that emf. = 1.4 V.

$$(b) \text{ Further, } V = \varepsilon - Ir \text{ or } r = \frac{\varepsilon - V}{I}$$

Here, $\varepsilon = 1.4 \text{ V}$. Consider any given value of potential difference from graph, say $V = 1.2 \text{ V}$. Current corresponding to this potential difference is 0.04 A. So

$$r = \frac{1.4 - 1.2}{0.04} = 5 \Omega$$

3. R_3 and R_4 are in series. The combined resistance is 400Ω . Now $R_2 (= 100 \Omega)$ and 400Ω resistances are in parallel. The combined resistance is $(100 \times 400)/500 = 80 \Omega$.

Total resistance is $R = 80 + 50 = 130 \Omega$

$$I = 65/130 = 1/2 \text{ A}$$

$$\text{So, } V = IR_1 = 1/2 \times 50 = 25 \text{ V}$$

4. Equivalent resistance R' of the parallel combination of 12Ω , 6Ω , and 4Ω resistances is given by

$$\frac{1}{R'} = \frac{1}{12} + \frac{1}{6} + \frac{1}{4} = \frac{1}{2} \text{ or } R' = 2 \Omega$$

Total resistance of the circuit is $R = 1 + 5 + 2 + 2 = 10 \Omega$

$$E = 20 - 8 = 12 \text{ V}$$

$$(a) \text{ Current in the circuit is } I = \frac{E}{R} = \frac{12}{10} = 1.2 \text{ A}$$

(b) Combined resistance of 6Ω and 4Ω in parallel is

$$\frac{6 \times 4}{6 + 4} = 2.4 \Omega$$

$$\text{Current in resistor of } 12 \Omega \text{ coil is } \frac{2.4}{12 + 2.4} \times 1.2 = 0.2 \text{ A}$$

(c) Potential difference across the 20 V battery is

$$(20 - 1.2 \times 1) = 18.8 \text{ V}$$

Potential difference across the 8 V battery is $(8 + 1.2 \times 2) = 10.4 \text{ V}$

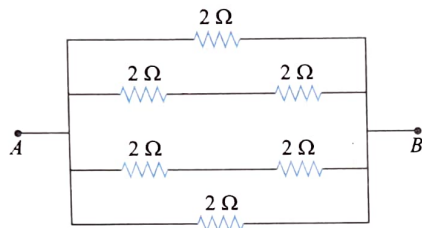
5. If V is the applied potential difference and R is the original resistance, then $V/R = 5$ and $V/(R + 2) = 4$. Dividing, we get

$$\frac{R+2}{R} = \frac{5}{4} \text{ or } R = 8 \Omega$$

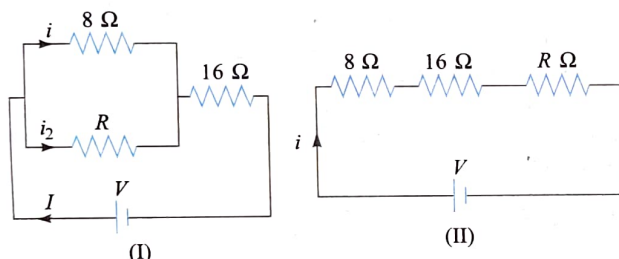
6. An equivalent of the given network is shown in figure. It is clear from the network of the figure that the resistances of 2Ω , 4Ω , 4Ω , and 2Ω are connected in parallel with each other. If R_p is the total resistance, then

$$\frac{1}{R_p} = \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{2} = \frac{2+1+1+2}{4} = \frac{3}{2}$$

$$\text{or } R_p = \frac{2}{3} \Omega$$



7. Let current through 8Ω be i in both cases.



In case (I), $V_1 = V_2$ or $i \times 8 = R \times i_2$ or $i_2 = \frac{8i}{R}$

$$I = i + i_2 = i + \frac{8i}{R}$$

$$V = I(R_{\text{Total}})$$

$$= \left(i + \frac{8i}{R}\right) \left(\frac{8R}{R+8} + 16\right) = i \left(\frac{8+R}{R}\right) \left(\frac{8R}{R+8} + 16\right) \quad \dots(i)$$

In case (II), $V = i(24 + R)$

Now, from Eqs. (i) and (ii), we get

$$\frac{24R + 128}{R} = 24 + R \text{ or } R = \sqrt{128} \Omega$$

$$8. V_{ab} = I_1 \left(6 + \frac{4 \times 6}{4+6}\right) = I_1(8.4) \quad \dots(i)$$

$$V_{ab} = I_2 \left(4 + \frac{8 \times 8}{8+8}\right) = I_2(8) \quad \dots(ii)$$

(a) From Eq. (i), we get

$$I_1 = \frac{12}{8.4} = \frac{10}{7} \text{ A}$$

I_1 is the current in the 6Ω resistor.

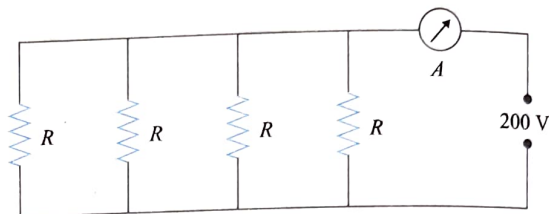
$$(b) I_{2,4} = \frac{4}{6+4} \times \frac{10}{7} = \frac{4}{7} \text{ A}$$

(c) From Eq. (ii), we get $I_2 = 3/2 \text{ A}$, so current in the 4Ω resistor is $3/2 \text{ A}$. It will be equally divided into 8Ω and 8Ω resistors, which are in parallel. Therefore,

$$I_8 = \frac{3}{2 \times 2} = \frac{3}{4} \text{ A}$$

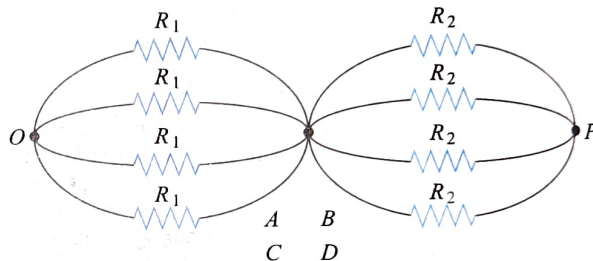
9. Here internal resistance is given by the slope of graph, i.e., x/y .

10. In parallel combination, each resistor has same potential difference, i.e., 200 V. The circuit, therefore, reduces to



$$\therefore R_{\text{eq}} = \frac{2000}{4} = 500 \Omega; I = \frac{200}{500} = 0.4 \text{ A}$$

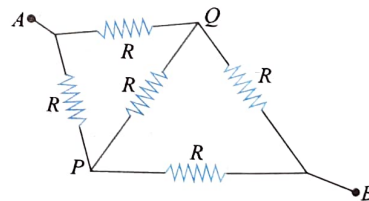
11. In the given circuit, A, B, C, and D are at the same potential by symmetry.



$$R_{\text{eq}} = \frac{R_1 + R_2}{4}, R_1 = ar, R_2 = \frac{\pi a}{2} r$$

$$\therefore R_{\text{eq}} = \frac{ar}{4} \left(1 + \frac{\pi}{2}\right) = \frac{ar}{8} (2 + \pi)$$

12. The given circuit can be redrawn as shown below. It is a balanced Wheatstone bridge. The resistance between P and Q can be removed. So $R_{AB} = R$.



13. From the circuit given in question, current through R_2 is

$$i - \frac{i}{10} = \frac{9i}{10}$$

Potential difference across R_2 is equal to potential difference across R. Therefore,

$$R_2 \times \frac{9}{10} i = R \times \frac{i}{10}, \text{ i.e., } R_2 = \frac{R}{9} = \frac{11}{9} \Omega$$

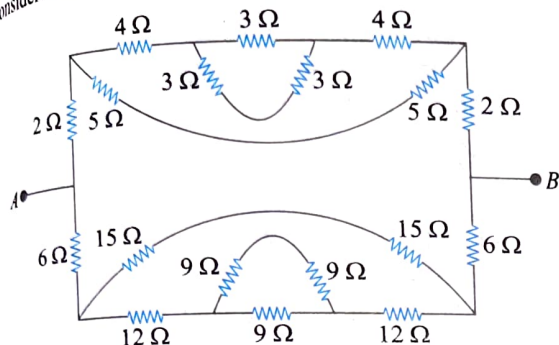
$$\therefore R_{\text{eq}} = \frac{R_2 \times R}{R_2 + R} = \frac{\frac{11}{9} \times \frac{11}{9}}{\frac{11}{9} + \frac{11}{9}} = \frac{11}{10} \Omega$$

Total circuit resistance is

$$\frac{11}{10} + R_1 = R = 11$$

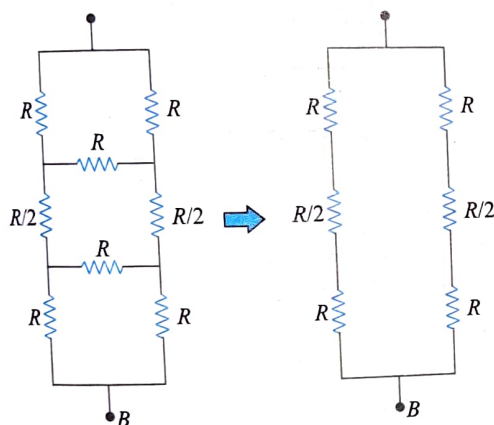
$$R_1 = 11 - \frac{11}{10} = 9.9 \Omega$$

After knowing the current distribution, the arrangement may be considered as shown in figure.



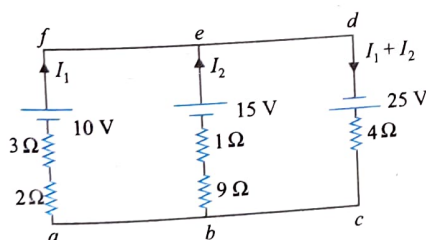
On solving, we get $R_{eq} = 6.75 \Omega$

We can redraw the circuit as shown in the figure. From figure it is clear that the equivalent resistance is $\frac{5R}{4}$.



Exercise 5.4

1. The circuit can be redrawn as follows:



$$\text{Loop } afdca: (2 + 3)I_1 + 4(I_1 + I_2) = 10 + 25 \quad \dots(i)$$

$$\text{or } 9I_1 + 4I_2 = 35$$

$$\text{Loop } edcbe: (1 + 9)I_2 + 4(I_1 + I_2) = 15 + 25 \quad \dots(ii)$$

$$\text{or } 4I_1 + 14I_2 = 40$$

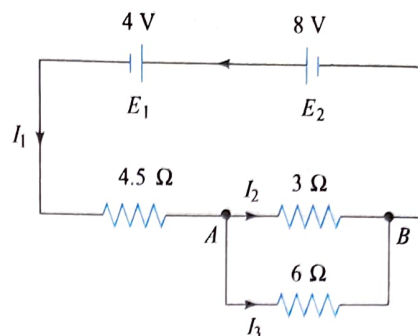
$$\text{From Eqs. (i) and (ii), we get } I_1 = 3A, I_2 = 2A.$$

2. Since the galvanometer shows no deflection, the Wheatstone bridge is balanced. Applying condition for balanced Wheatstone bridge,

$$\frac{100(100 + R)}{100R} = \frac{200}{40} = 5$$

$$\text{or } 5R = 100 + R \quad \text{or } R = 25 \Omega$$

$$3. \text{ Net emf} = 8 - 4 = 4 \text{ V}, R_{AB} = 6 \times 3/9 = 2 \Omega$$



Total resistance of the circuit is 8Ω . So

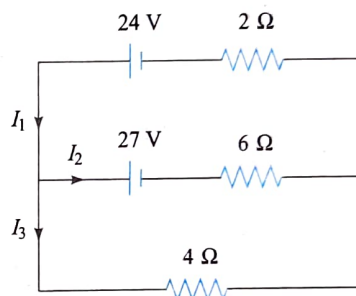
$$I_1 = 4/8 = 0.5 \text{ A}, V_{AB} = 0.5 \times 2 = 1 \text{ V}, I_2 = 1/3 \text{ A}, \text{ and } I_3 = 1/6 \text{ A}$$

4. Traversing the upper and lower loops anticlockwise, we get

$$2I_1 + 6I_2 = 24 - 27 \quad \text{or} \quad 2I_1 + 6I_2 = -3 \quad \dots(i)$$

$$\text{and } 4I_3 - 6I_2 = 27 \quad \text{or} \quad 4(I_1 - I_2) - 6I_2 = 27$$

$$\text{or } 4I_1 - 10I_2 = 27 \quad \dots(ii)$$



Solving Eqs. (i) and (ii), we get

$$I_1 = 3 \text{ A}, I_2 = -1.5 \text{ A},$$

$$I_3 = I_1 - I_2 = 3 + 1.5 = 4.5 \text{ A}$$

5. By using KVL, we get

$$-5 \times 2 - V_{PQ} + 15 = 0$$

$$\text{or } V_{PQ} = 5 \text{ V}$$

6. From Ohm's law, the voltage drop across the 6Ω resistor is

$$V = IR = (4 \text{ A})(6 \Omega) = 24 \text{ V}$$

The voltage drop across the 8Ω resistor is the same, since these two resistors are wired in parallel. The current through the 8Ω resistor is then

$$I = V/R = 24 \text{ V}/8 \Omega = 3 \text{ A}$$

The current through the 25Ω resistor is the sum of these two currents i.e., 7 A . The voltage drop across the 25Ω resistor is

$$V = IR = (7 \text{ A})(25 \Omega) = 175 \text{ V}$$

Total voltage drop across the top branch of the circuit is

$$175 + 24 = 199 \text{ V}$$

which is also the voltage drop across the 20Ω resistor. The current through the 20Ω resistor is then

$$I = V/R = 199 \text{ V}/20 \Omega = 9.95 \text{ A}$$

7. Current through 2Ω resistor is 6 A . Current through 1Ω resistor also is 6 A .

$$\xi = V = IR = (6 \text{ A})(3 \Omega) = 18 \text{ V}$$

Current through 6Ω resistor is $I = 18/6 = 3 \text{ A}$.

8. Applying KCL for the individual branches, we get

$$20 - i_1(2) + 10 = V \quad \dots(i)$$

$$0 - i_2\left(\frac{1}{2}\right) - 8 = V \quad \dots(ii)$$

$$-15 - i_3(1) - 6 = V \quad \dots(iii)$$

$$i_1 + i_2 + i_3 = 0 \quad \dots(iv)$$

Putting i_1 , i_2 , and i_3 from Eqs. (i), (ii), and (iii) in Eq. (iv), we have

$$\frac{30 - V}{2} + \frac{V + 8}{-1/2} + \frac{V + 21}{-1} = 0$$

or $V = -\frac{44}{7} \text{ V}$

9. Suppose current through different paths of the circuit is as follows.

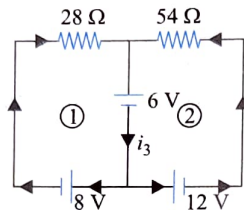
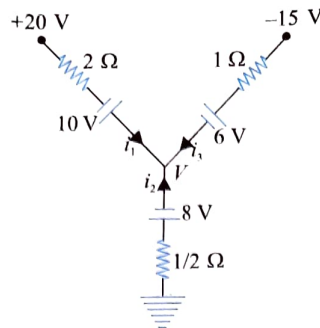
After applying KVL for loop (1) and loop (2), we get

$$28i_1 = -6 - 8$$

or $i_1 = -\frac{1}{2} \text{ A}$

And $54i_2 = -6 - 12$ or $i_2 = -\frac{1}{3} \text{ A}$

Hence $i_3 = i_1 + i_2 = -\frac{5}{6} \text{ A}$



10. (a) The sum of the currents that enter the junction below the resistor equals

$$3 \text{ A} + 5 \text{ A} = 8 \text{ A}$$

- (b) Using the lower left loop, we get

$$\varepsilon_1 - (4 \Omega)(3 \text{ A}) - (3 \Omega)(8 \text{ A}) = 0 \quad \text{or} \quad \varepsilon_1 = 36 \text{ V}$$

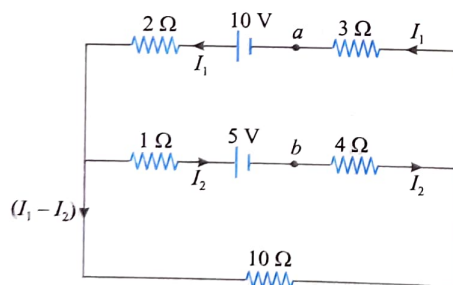
Using the lower right loop, we get

$$\varepsilon_2 - (6 \Omega)(5 \text{ A}) - (3 \Omega)(8 \text{ A}) = 0 \quad \text{or} \quad \varepsilon_2 = 54 \text{ V}$$

- (c) Using the top loop, we get

$$54 \text{ V} - R(2 \text{ A}) - 36 \text{ V} = 0 \quad \text{or} \quad R = \frac{18 \text{ V}}{2 \text{ A}} = 9 \Omega$$

11. (a)



Upper loop (clockwise)

$$2I_1 - 10 + 3I_1 + 4I_2 + 5 + 1I_2 = 0$$

$$5I_1 + 5I_2 = 5$$

or $I_1 + I_2 = 1$

Lower loop (clockwise)

$$-1 \cdot I_2 - 5 - 4I_2 + 10(I_1 - I_2) = 0$$

$$10I_1 - 15I_2 = 5$$

or $2I_1 - 3I_2 = 1$

Solving (i) and (ii) we get $I_1 = \frac{4}{5} \text{ A}$ and $I_2 = \frac{1}{5} \text{ A}$

Hence current through lower branch

$$i_{10 \Omega} = I_1 - I_2 = \frac{4}{5} - \frac{1}{5} = \frac{3}{5} \text{ A}$$

$$(b) V_a + 3I_1 + 4I_2 = V_b$$

$$\Rightarrow V_a - V_b = -3 \times \frac{4}{5} - 4 \times \frac{1}{5} = -\frac{16}{5} \text{ V}$$

12. (i) Here $I_1 = \frac{36 - 0}{9} = 4 \text{ A}$

Also $I_2 = \frac{36 - 0}{9} = 4 \text{ A}$

Also, $36 - V_a = I_1 \times 6 = 6I_1$
 $= 6 \times 4 = 24 \text{ V}$

$\therefore V_a = 36 - 24 = 12 \text{ V}$

$$36 - V_b = 3 \times 4 = 12 \text{ V}$$

and $V_b = 36 - 12 = 24 \text{ V}$

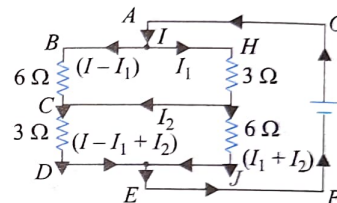
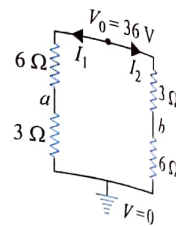
$\therefore V_a - V_b = 12 - 24 = -12 \text{ V}$

- (ii) In loop ABCIHA.

$$-6(I - I_1) + 3I_1 = 0 \quad \text{or} \quad -6I + 6I_1 + 3I_1 = 0$$

or $9I_1 = 6I$

$\therefore I = \frac{9}{6}I_1 = \frac{3}{2}I_1$



In loop CDEJIC,

$$-3(I - I_1 + I_2) + 6(I_1 - I_2) = 0$$

or $-3I + 3I_1 - 3I_2 + 6I_1 - 6I_2 = 0$

or $-3I + 9I_1 - 9I_2 = 0$

In loop EFGHJE,

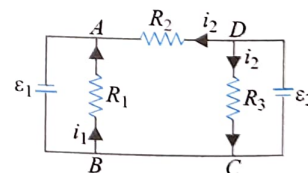
$$36 - 3I_1 - 6(I_1 - I_2) = 0$$

or $9I_1 - 6I_2 = 36$

After solving Eqs. (i), (ii), and (iii), we get

$$I_2 = 3 \text{ A from } b \text{ to } a.$$

13. We can redraw the circuit as shown in figure



From the figure it is clear

The current through R_1 , $i_1 = \frac{6}{1} = 6 \text{ A}$ (From B to A)

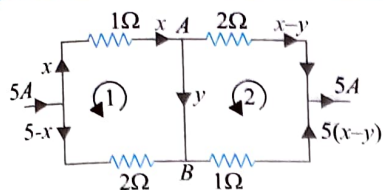
The current through R_3 , $i_3 = \frac{3}{3} = 1 \text{ A}$ (From D to B)

Applying KCL for loop in outer loop, $\varepsilon_2 - i_2 R_2 + \varepsilon_1 = 0$

$$\Rightarrow i_2 = \frac{\varepsilon_1 + \varepsilon_2}{R_2}$$

$$\Rightarrow i_2 = \frac{6 + 3}{3} = 3 \text{ A (From D to A)}$$

14. (a) The current, $i = \frac{\varepsilon}{R_{eq}} = \frac{10 \text{ V}}{\frac{2}{3} + \frac{2}{3} + \frac{2}{3}} = \frac{10}{2} = 5 \text{ A}$



(b) Applying KCL for loop 1:

$$-x + 2(5 - x) = 0 \text{ or } x = \frac{10}{3} \text{ A}$$

...(i)

For loop 2: $-2(x - y) + 1[5 - (x - y)] = 0$

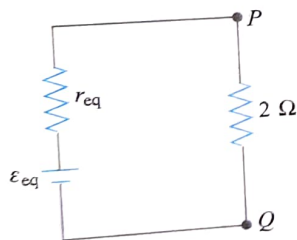
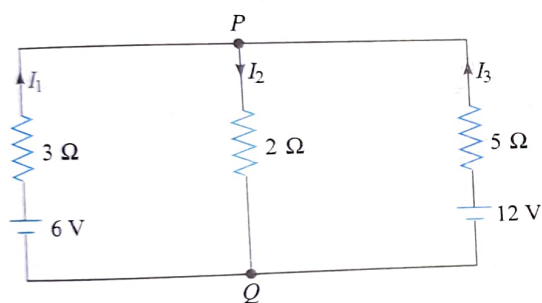
$$-3x + 3y + 5 = 0$$

...(ii)

From (i) and (ii), $y = \frac{5}{3} \text{ A}$

Exercise 5.5

$$1. \epsilon_{eq} = \frac{\frac{6}{\frac{1}{3} + \frac{1}{5}} + \frac{12}{\frac{1}{3} + \frac{1}{5}}}{\frac{1}{\frac{1}{3} + \frac{1}{5}} + \frac{1}{\frac{1}{3} + \frac{1}{5}}} = \frac{33}{4} \text{ V}, r_{eq} = \frac{1}{\frac{1}{3} + \frac{1}{5}} = \frac{15}{8} \Omega$$



$$\text{Hence, } I_2 = \frac{\frac{33}{4}}{2 + \frac{15}{8}} = \frac{66}{31} \text{ A}$$

Potential difference across PQ is

$$V_{PQ} = I_2 R = \frac{66}{31} \times 2 = \frac{132}{31} \text{ V}$$

Current through 6 V battery:

$$\frac{132}{31} = 6 - 3I_1 \text{ or } I_1 = \frac{18}{31} \text{ A}$$

$$\text{Hence, } I_3 = I_2 - I_1 = \frac{66}{31} - \frac{18}{31} = \frac{48}{31} \text{ A}$$

$$2. nm = ?, 30 = \frac{n \times 1}{m} \text{ or } n = 30m,$$

$$1.5 = \frac{n \times 1.5}{30 + \frac{n}{m}}$$

On solving, we get $n = 60$, $m = 2$, total number of batteries = $nm = 120$.

3. Let potential at point B be 0.

The potentials at other points are mentioned. So potential at E is not known numerically. Let potential at E be x .

Now applying Kirchhoff's current law at junction E (this can be applied at any other junction also), we get

$$\frac{x - 10}{1} + \frac{x - 30}{2} + \frac{x + 14}{2} = 0$$

$$\text{or } 4x = 36 \text{ or } x = 9$$

$$\text{Current in } EF = \frac{10 - 9}{1} = 1 \text{ A (from F to E)}$$

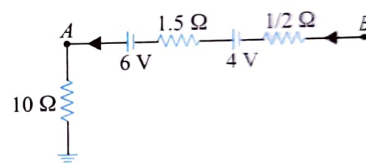
$$\text{Current in } BE = \frac{30 - 9}{2} = 10.5 \text{ A (from B to E)}$$

$$\text{Current in } DE = \frac{9 - (-14)}{2} = 11.5 \text{ A (from E to D)}$$

4. (i) $\epsilon_{eq} = \epsilon_2 - \epsilon_1 = 4 - 6 = -2 \text{ V}$ (directed to left)

$$(ii) 6 - i\left(\frac{1}{2} + 1.5 + 10\right) - 4 + 6 = 0$$

$$\text{or } i = \frac{8}{12} = \frac{2}{3} \text{ A}$$



$$5. \epsilon_{eq} = \frac{\frac{6}{\frac{1}{2} + \frac{1}{2}} + \frac{4}{\frac{1}{2} + \frac{1}{2}}}{\frac{1}{\frac{1}{2} + \frac{1}{2}} + \frac{1}{\frac{1}{2} + \frac{1}{2}}} = 2 \text{ V}$$

$$r_{eq} = \frac{1}{\frac{1}{2} + \frac{1}{2}} + 3 = 4 \Omega$$

$$6. (a) \epsilon = \frac{-\frac{12}{4} + \frac{8}{2}}{\frac{1}{4} + \frac{1}{2}} = \frac{4 - 3}{3/4} = \frac{4}{3} \text{ V}$$



$$\frac{1}{r} = \frac{1}{4} + \frac{1}{2} = \frac{3}{4} \text{ or } r = \frac{4}{3} \Omega$$

$$\text{Then, } i = \frac{\epsilon}{r + R} = \frac{4/3}{4/3 + 4} = \frac{1}{4} \text{ A}$$

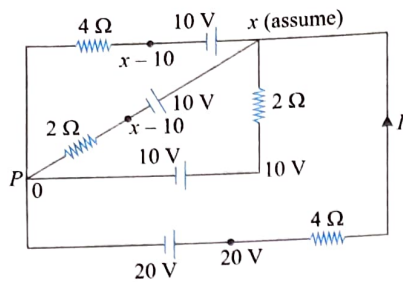
$$V_{ab} = 4i = 4 \times \frac{1}{4} = 1 \text{ Volt}$$

$$V_b - i_1(4) + 12 = V_a \text{ or } V_a - V_b = 12 - 4i_1$$

$$1 = 12 - 4i_1 \Rightarrow i_1 = \frac{11}{4} \text{ A}$$

$$\text{Since, } i + i_2 = i_1, i_2 = i_1 - i = \frac{11}{4} - \frac{1}{4} = 2.5 \text{ A}$$

7. The simplified circuit is shown in figure. We have to find I .

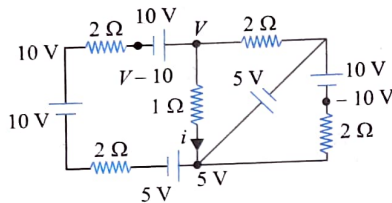


Let the potential of point P be zero. Potential at other points is shown in figure. Applying Kirchhoff's current law at X_x , we get

$$\frac{x-10}{4} + \frac{x-10}{2} + \frac{x-20}{4} + \frac{(x-10)}{2} = 0 \text{ or } x = \frac{35}{3} \text{ V}$$

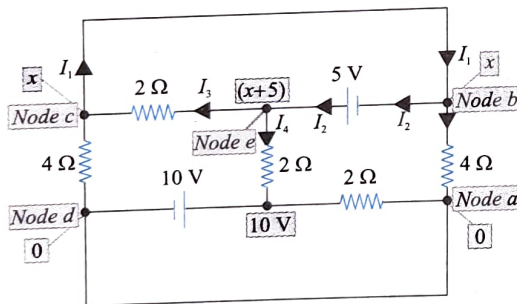
$$I = \frac{20 - \frac{35}{3}}{4} = \frac{25}{12} \text{ A}$$

8. $\frac{(V-10)-10}{2} + \frac{V-0}{2} + \frac{V-5}{1} = 0$ or $V = \frac{30}{4} = \frac{15}{2}$ V



Current through 1Ω resistor is $i = \frac{V-5}{1} = \frac{15}{2} - 10 = \frac{5}{2}$ A

9. Assign potential at each node as shown in figure.



At node 'e', $I_2 = I_3 + I_4$

where $I_3 = \frac{(x+5)-x}{2}$ and $I_4 = \frac{(x+5)-10}{2}$

Hence, $I_2 = \frac{(x+5)-x}{2} + \frac{(x+5)-10}{2}$... (i)

Note that outgoing current at node c is equal to incoming current at node b . Thus our equation is at node c and node b .

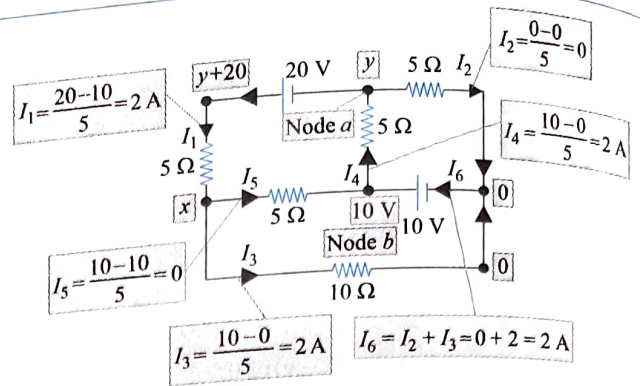
$\frac{(x+5)}{2} + \frac{(0-x)}{4} = I_2 + \frac{(x-0)}{4}$... (ii)

From Eqs. (i) and (ii), we get $x = -2.5$ V

Current through R_4 is 3.75 A.

10. At node a , $\frac{(y+20)-x}{5} + \frac{y-10}{5} + \frac{y-0}{5} = 0$

$\Rightarrow x - 3y = 10$... (i)



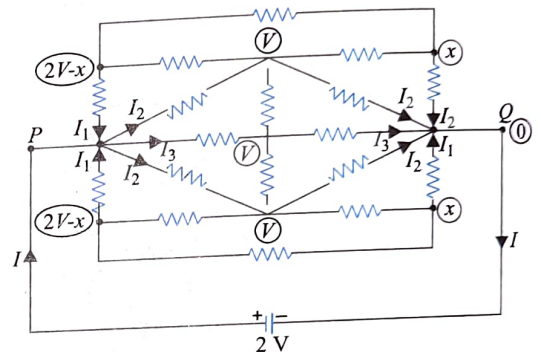
$$\frac{x-(y+20)}{5} + \frac{x-10}{5} + \frac{x-0}{5} = 0$$

$\Rightarrow 3x - y = 30$... (ii)

On solving, we get $x = 10$ V and $y = 0$ V

From figure it is clear that current through each battery will be 2 A.

11. To find the equivalent resistance of the circuit, let us connect a battery of emf '2V' across the terminals P and Q and distribute potentials at different junctions of the circuit as shown in figure.



To find junction potentials we write KCL equation for x as

$$\frac{V-x}{R} + \frac{(2V-x)-x}{R} = \frac{x}{R}$$

$\Rightarrow 4x = 3V \Rightarrow x = \frac{3V}{4}$... (i)

Equivalent resistance of the circuit across points A and B is given as

$$R_{eq} = \frac{V_{\text{battery}}}{I_{\text{battery}}} = \frac{2V}{2I_1 + 2I_2 + I_3}$$

$\Rightarrow R_{eq} = \frac{2V}{\frac{2x}{R} + \frac{3(V-0)}{R}} = \frac{2V \cdot R}{2x + 3V}$... (ii)

From (i) and (ii) we get

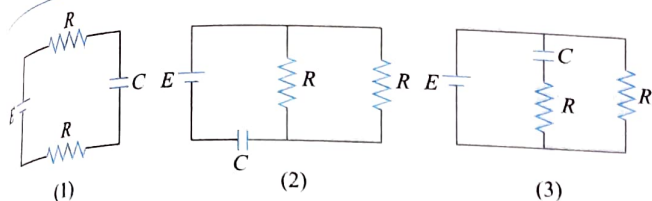
$\Rightarrow R_{AB} = \frac{4R}{9}$

Exercise 5.6

- (a) At $t = 0$, C will behave as a short circuit, so no current passes through R_2 . And $I = I_2 = \mathcal{E}/R_1$.
- (b) At $t = \infty$, C will block the current. So $I_2 = 0$. And

$$I = I_1 = \frac{\mathcal{E}}{R_1 + R_2}$$

- (a) In each case, in steady state (equilibrium), potential difference across capacitor will be E . So charge in each case will be finally $q = CE$.



$$(b) \tau_1 = 2RC, \tau_2 = \frac{R}{2}C, \tau_3 = 2RC$$

For (1), time constant is the largest so it will take maximum time.
For (2), it is the smallest. So time taken will be in the order (1) > (3) > (2).

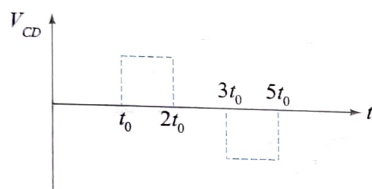
3. (a) Immediately after the switch is closed, C_1 will act as a simple wire due to which R_2 and R_3 will be short-circuited. So, $I = E/R_1$

(b) After a long time, C_1 and C_2 will block the current. Current will pass through only R_1 and R_3 . $I = \frac{E}{R_1 + R_3}$

4. For $t = t_0$ to $t = 2t_0$, $V = at + b$

Charge on capacitor $q = CV = C[at + b]$

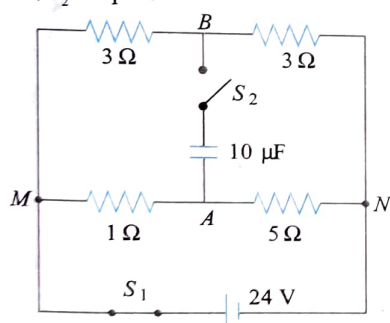
Current, $I = \frac{dq}{dt} = Ca \rightarrow \text{constant}$



Potential across $CD = IR = CaR \rightarrow \text{constant}$

From $2t_0$ to $3t_0$, V is constant, so no current flows through the capacitor or resistor. Hence, V_{CD} is zero.

5. (a) S_1 is closed, S_2 is open.



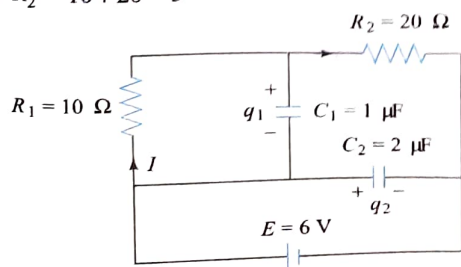
$$V_M - V_A = \frac{1 \times 24}{1 + 5} = 4 \text{ V}, \quad V_M - V_B = \frac{3 \times 24}{3 + 3} = 12 \text{ V}$$

$$V_A - V_B = 8 \text{ V}$$

(b) (i) Just after closing S_2 , A and B will come to the same potential, so $V_A - V_B = 0$.

(ii) After a long time, no current will flow through AB , so $V_A - V_B = 8 \text{ V}$.

$$6. I = \frac{E}{R_1 + R_2} = \frac{6}{10 + 20} = \frac{1}{5} \text{ A}$$



$$q_1 = C_1 V_1 = C_1 I R_1 = 1 \times \frac{1}{5} \times 10 = 2 \mu\text{C}$$

$$q_2 = C_2 V_2 = 2 \times 6 = 12 \mu\text{C}$$

7. (a) At $t = 0$, capacitor acts as short-circuit. There will not be any current in the 40Ω resistance. Therefore,

$$I_0 = \frac{10}{20} = 0.5 \text{ A}$$

(b) Capacitor acts as open circuit, so

$$I = \frac{10}{60} = \frac{1}{6} \text{ A}$$

(c) Voltage across capacitor is

$$V_{40} = IR = \frac{1}{6} \times 40, \quad Q = CV = 0.50 \times \frac{1}{6} \times 40 = \frac{10}{3} \mu\text{C}$$

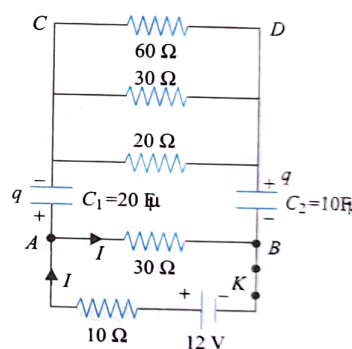
(d) $q = q_0(e^{-t/RC})$, capacitor will discharge through the 40Ω resistor when S is open, i.e.,

$$\frac{20}{100} q_0 = q_0 \left(e^{-\frac{t}{40 \times 0.5 \times 10^{-6}}} \right) \text{ or } \frac{1}{5} = \frac{1}{e^{\frac{t}{20 \times 10^{-6}}}}$$

$$\text{or } t = 20 \times 10^{-6} \ln(5)$$

8. In the steady state, no current passes through upper three resistors.

$$\text{So } I = \frac{12}{10 + 30} = 0.3 \text{ A}$$



Potential difference between A and B is $V = 30 \times 0.3 = 9 \text{ V}$.

Now in loop $ACDBA$:

$$\frac{q}{C_1} + \frac{q}{C_2} = 9$$

$$\text{or } \frac{q}{20} + \frac{q}{10} = 9 \text{ or } q = 60 \mu\text{C}$$

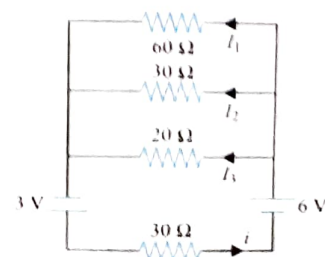
(a) Potential difference across C_1 :

$$V_1 = \frac{q}{C_1} = \frac{60}{20} = 3 \text{ V}$$

Potential difference across C_2 :

$$V_2 = \frac{q}{C_2} = \frac{60}{10} = 6 \text{ V}$$

(b) After K is opened, 12 V will be out of circuit, and capacitors will act as batteries as shown in figure.



Now one can find $I_1 = 0.0375 \text{ A}$.

9. The current distribution is indicated in figure. When the condenser has been fully charged, there will be no current in this branch. Applying Kirchhoff's law to meshes $NLKMN$ and $PNMOP$, we get

$$30i_2 - 20i_3 = 1 \quad \dots(i)$$

$$10i_1 + 20i_3 = 4 \quad \dots(ii)$$

Applying Kirchhoff's first law at junction M , we get

$$i_2 + i_3 = i_1 \quad \dots(iii)$$

Solving Eqs. (i), (ii), and (iii), we get

$$i_2 = i_3 = 0.1 \text{ A}; i_1 = 0.2 \text{ A}$$

Considering mesh $PBALP$, we have

$$V_A - 1 + 10i_1 = V_B \text{ or } V_A - V_B = 1 - 10i_1 = -1 \text{ V}$$

10. (a) In steady state, no current will flow through the circuit if S is opened. So the potential at a will be 18 V, and that at b will be zero. Hence, $V_a - V_b = 18 - 0 = 18 \text{ V}$.

(b) Obviously, a is at the higher potential.

(c) When S is closed, finally in steady state, current I will flow as shown in figure.

$$I = \frac{18}{6+3} = 2 \text{ A}, V_1 = 6I = 6 \times 2 = 12 \text{ V}$$

$$V_b = 18 - V_1 = 18 - 12 = 6 \text{ V}$$

$$(d) q_1 = C_1 V_1 = 6 \times 12 = 72 \mu\text{C}$$

$$V_1 + V_2 = 18 \text{ or } V_2 = 6 \text{ V}, q_2 = C_2 V_2 = 3 \times 6 = 18 \mu\text{C}$$

Before closing S , the potential on each capacitor is 18 V

and charge is $q_{10} = 6 \times 18 = 108 \mu\text{C}$, $q_{20} = 3 \times 18 = 54 \mu\text{C}$

Change in charge is $q_1 - q_{10} = -36 \mu\text{C}$ and $q_2 - q_{20} = -36 \mu\text{C}$

11. (a) For $t = 0$ to $t = (2R_2 + R_1)C$, capacitor gets charged from all the three resistors. So

$$q = CE \left[1 - e^{-\frac{t}{(2R_2 + R_1)C}} \right]$$

Putting $t = (2R_2 + R_1)C$, we get

$$q_1 = CE [1 - e^{-1}] = \frac{CE(e-1)}{e}$$

Now battery is disconnected and the capacitor gets discharged through R_1 after S_1 is opened and S_2 is closed. So

$$\int_{q_1}^{q_2} \frac{dq}{q} = \int_0^{t_2} \frac{dt}{R_1 C} \text{ or } q_2 = \frac{q_1}{e} = \frac{CE(e-1)}{e^2}$$

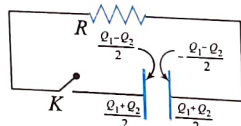
(b) Since battery is disconnected at this time, there is no current in R_2 .

12. $q_0 = CE$, $q_0/2 = q_0 e^{-t_1/RC}$ or $t_1 = RC \ln 2$

$$\text{Now } \int_{q_0/2}^{3q_0/4} \frac{dq}{CE - q} = \int_0^{t_2} \frac{dt}{3RC} \text{ or } t_2 = 3RC \ln 2$$

From here, we get $t_1/t_2 = 1/3$.

13. Initially charges on the various surfaces of the capacitor plates are as shown in the figure. When the switch is closed, the outer surface charges will remain intact. Change will occur only in inner surface charges. Charge at any time on the inner surface of left plate is given by



$q = \left(\frac{Q_1 - Q_2}{2} \right) e^{-t/RC}$ and the charge on the inner surface of the right plate will be $-q$. So the net charge at any time on left plate is

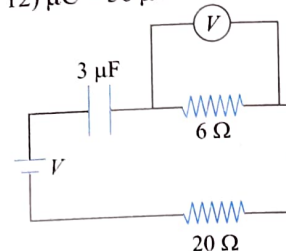
$$q_1 = \left(\frac{Q_1 + Q_2}{2} \right) + \left(\frac{Q_1 - Q_2}{2} \right) e^{-t/RC}$$

and the net charge at any time on right plate is

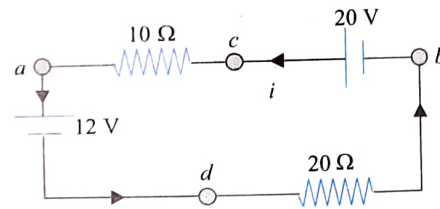
$$q_2 = \left(\frac{Q_1 + Q_2}{2} \right) - \left(\frac{Q_1 - Q_2}{2} \right) e^{-t/RC}$$

14. (a) As explained earlier that a capacitor is an open circuit to DC in steady state. The effective circuit through which current will pass initially after closing switch S_1 is shown in figure. When steady state is reached, the capacitor will not allow current. Thus, there is no current from battery. Reading of voltmeter is zero.

(b) Without current, the resistors act as conducting wires. Hence, potential difference across capacitor is 12 V. Charge on capacitor, $3 \mu\text{F}$, is $Q = (3 \times 12) \mu\text{C} = 36 \mu\text{C}$



(c) When all the switches are closed, the branch containing capacitor will not allow current after steady state is reached. The equivalent circuit from point of view of current is redrawn as in figure.



Let the current in the single loop circuit be I . We begin at left lower corner and traverse the circuit anticlockwise while applying KVL. We get $-20I + 20 - 10I - 12 = 0$

$$I = \frac{4}{15} \text{ A}$$

Power received by 12 V battery is $P = VI = 12 \times \frac{4}{15} = \frac{16}{5} \text{ W}$

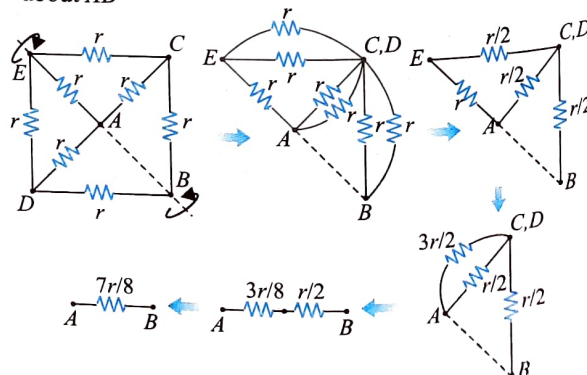
$$(d) V_c - 10I - 12 = V_d$$

$$\text{or } V_c - V_d = \frac{44}{3} \text{ V or } Q = CV = \frac{88}{3} \mu\text{C}$$

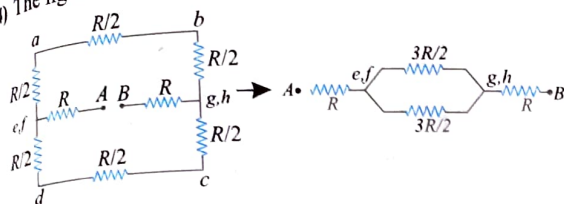
Exercises

Single Correct Answer Type

1. (2) Parallel axis of symmetry about line AB , we can fold the circuit about AB

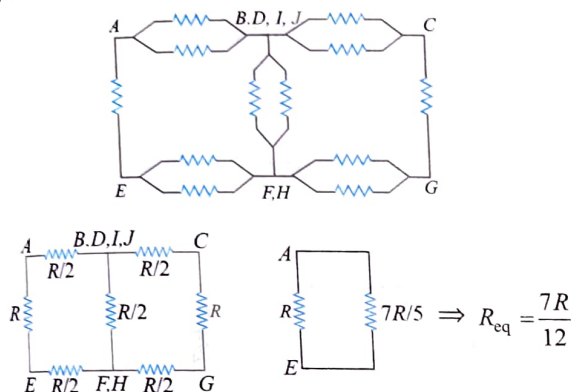


4. (4) The figure can be redrawn as follows:



$$R_{AB} = R + \frac{3R}{4} + R = \frac{11R}{4}$$

5. (1) The figure can be redrawn as follows:



4. (3) A careful observation will reveal that one end of each resistor is connected to A and the other end of each resistor is connected to B. Hence, the resistors are in parallel. So $R_{eq} = R/5$.

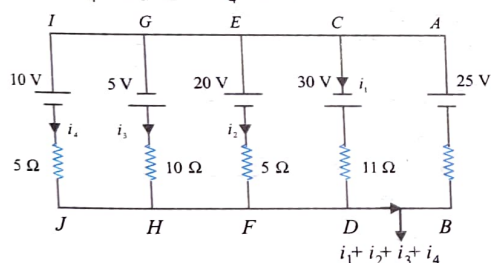
5. (3) Applying KVL in loops CDBAC, EFBAE, GHBAE, and IJBAI, we get

$$30 - i_1 \times 11 = -25 \text{ or } i_1 = 5 \text{ A}$$

$$20 + i_2 \times 5 = 25 \text{ or } i_2 = 1 \text{ A}$$

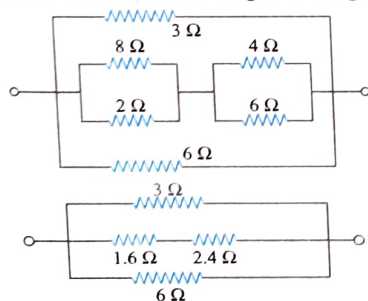
$$5 - i_3 \times 10 = -25 \text{ or } i_3 = 3 \text{ A}$$

$$10 + i_4 \times 5 = 25 \text{ or } i_4 = 3 \text{ A}$$



Current through 25 V cell is $i_1 + i_2 + i_3 + i_4 = 12 \text{ A}$.

6. (1) The equivalent of the network is given in figure.



The equivalent of the above network is a parallel combination of 3 Ω, 4 Ω, and 6 Ω, i.e.,

$$\frac{1}{R} = \frac{1}{3} + \frac{1}{4} + \frac{1}{6} \text{ or } R = \frac{4}{3} \Omega$$

7. (3) Current through 1 Ω resistance will be 2 A in the upward direction.

$$V_G - 2 \times 4 + 3 - 2 \times 2 + 2 \times 1 = V_H \text{ or } V_G - V_H = 7 \text{ V}$$

8. (2) For series connection, $I_{\min} = \frac{N\varepsilon}{Nr} = \frac{\varepsilon}{r}$

$$\text{For parallel connection, } I_{\max} = \frac{\varepsilon}{(r/N)} = \frac{N\varepsilon}{r}$$

$$9. (1) i = \frac{(n-2)\varepsilon}{nr}$$

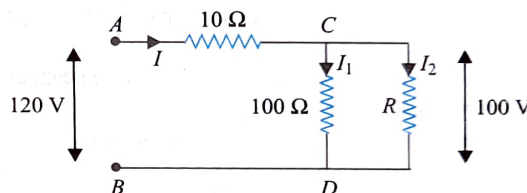
$$V_B - V_A = -ir + \varepsilon = \varepsilon - \frac{(n-2)\varepsilon}{nr} r = \varepsilon \left[1 - \frac{n-2}{n} \right] = \frac{2\varepsilon}{n}$$

10. (3) For cell A, the current i flows opposite to the direction of its emf.

$$\begin{aligned} \text{Potential difference is } \varepsilon + ir &= \varepsilon + \frac{(n-2)\varepsilon}{nr} r \\ &= \varepsilon \left[1 + \frac{n-2}{n} \right] = \varepsilon \left(\frac{2n-2}{n} \right) \end{aligned}$$

$$11. (1) \frac{(20R)/(20+R)}{5} = \frac{20}{10} \text{ or } R = 20 \Omega$$

12. (1) Potential difference across C and D is 100 V.



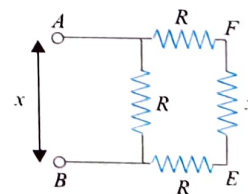
$$\text{Hence, } I_1 = \frac{100}{100} = 1 \text{ A, } V_{AC} = 120 - 100 = 20 \text{ V}$$

$$\text{And } I = \frac{20}{10} = 2 \text{ A. Hence, } I_2 = 2 - 1 = 1 \text{ A}$$

$$R = 100/I_2 = 100 \Omega$$

13. (2) Let the resistor to be connected across CD be x . Then the equivalent resistance across EF should be x and also across AB should be x . So we get $\frac{(2R+x)R}{3R+x} = x$.

$$\text{Solve to get } x = (\sqrt{3} - 1)R$$



14. (2) Let R be the resistance of the wire and let R' be the resistance of the wire. Energy released in t seconds is $(3V^2)/R \times t$.

$$R' = 2R \quad (\text{as length is twice})$$

$$\text{Therefore, energy released in } t \text{ seconds is } \frac{(N^2 V^2)}{2R} \times t$$

$$\text{But } Q = mc\Delta T$$

$$\therefore Q' = \frac{(N^2 V^2)}{2R} \times t$$

$$\therefore mc\Delta T = \frac{(9V^2)}{R} \times t$$

$$\text{Applying } Q' = m'c\Delta T,$$

...(i)

$$2mc\Delta T = \frac{(N^2 V^2)}{2R} \times t \quad \dots(ii)$$

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{mc\Delta T}{2mc\Delta T} = \frac{9V^2 \times t/R}{N^2 V^2 t/2R}$$

$$\therefore \frac{1}{2} = \frac{9 \times 2}{N^2} \text{ or } N = 6$$

15. (1) In the steady state, no current is flowing through the capacitor branch, so the capacitor branches may be removed from the circuit, the equivalent circuit is shown in figure.

$$\therefore R_{AB} = R_1 + R_2 = r + r = 2r$$

16. (1) The current in the circuit is $I = (12/12) = 1 \text{ A}$.

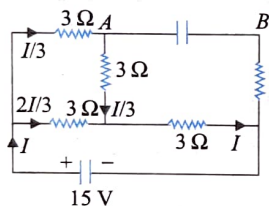
Potential across d and c is $12 \text{ V} - 3 \times 1 \text{ V} = 9 \text{ V}$

$$\text{Capacitance across } d \text{ and } e \text{ is } C = \frac{1 \times 2}{1 + 2} = \frac{2}{3} \mu\text{F}$$

Therefore, charge on either capacitor is $\frac{2}{3} \times 9 = 6 \mu\text{C}$.

17. (4) No current will flow in the branch containing capacitor. Hence, no energy is stored in the capacitor.

18. (3) In the steady state, no current flow through C .

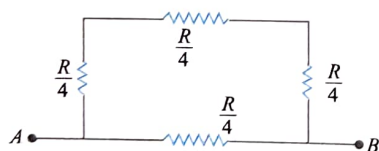


$$15 = 3 \times 2I/3 + 3I \text{ or } I = 3 \text{ A}$$

$$V_A - 3I/3 - 3I = V_B$$

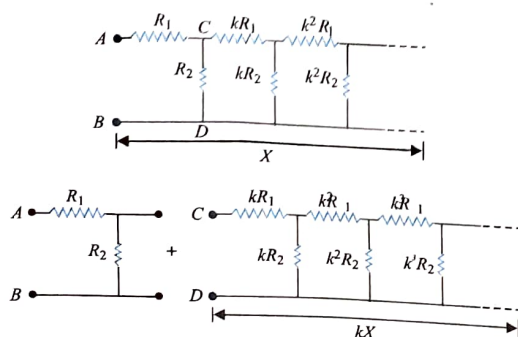
$$\text{or } V_A - V_B = 4I = 4 \times 3 = 12 \text{ V}$$

19. (3)

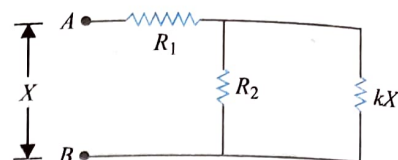


$$R_{AB} = \frac{3R}{16} = \frac{3(16)}{16} = 3\Omega$$

20. (3) When each element of the circuit is multiplied by a factor k , then the equivalent resistance also becomes k times. Let the equivalent resistance between A and B be x .



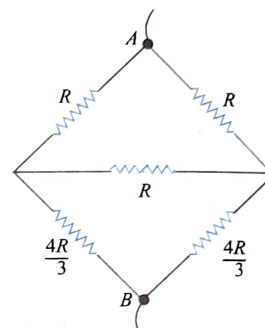
So the equivalent circuit becomes



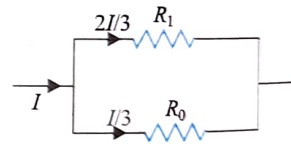
$$X = R_1 + \frac{kXR_2}{kX + R_2}$$

$$\text{For } k = \frac{1}{2}, X = \frac{(R_1 - R_2) + \sqrt{R_1^2 + R_2^2 + 6R_1R_2}}{2}$$

$$21. (2) R_{AB} = \frac{7}{6} R$$



22. (3) We will solve this problem by the superposition principle. Let current I enter at A , then current $I/6$ will flow through AB . Now, in the second case, let the current be extracted from B , again current $I/6$ will flow through AB . Now, if simultaneously current enters at A and leaves at B , then the current $I/6 + I/6 = I/3$ should flow through AB and current $I - I/3 = 2I/3$ will flow through the remaining part. Let the resistance of the remaining part be R_1 .



$$\text{Now } \frac{R_0 I}{3} = \frac{R_1 2I}{3} \text{ or } R_1 = \frac{R_0}{2}$$

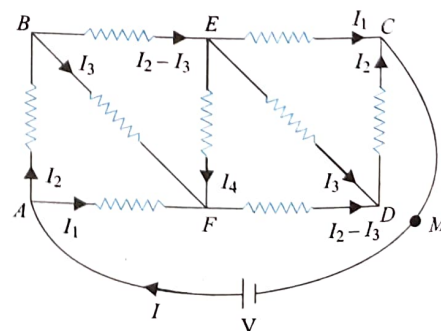
$$\text{or } R_{eq} = \frac{R_1 R_0}{R_1 + R_0} = \frac{(R_0/2) R_0}{(R_0/2) + R_0} = \frac{R_0}{3}$$

23. (4) At junction E or F :

$$I_1 + I_3 + I_4 = I_2 - I_3 \text{ or } I_1 - I_2 + 2I_3 + I_4 = 0$$

Loop $AFBA$ or loop $ECDE$:

$$rI_1 = rI_2 + rI_3 \text{ or } I_1 = I_2 + I_3$$



Loop $BEFB$ or loop $EDFE$:

$$r(I_2 - I_3) + rI_4 = rI_3 \text{ or } I_2 - 2I_3 + I_4 = 0$$

Loop $AFDCMA$ or $ABECMA$:

$$V = rI_1 + r(I_2 - I_3) + rI_2 = rI_1 + 2rI_2 - rI_3$$

Solve to get $I_1 = 2V/5r$, $I_2 = V/3r$

$$R_{eq} = \frac{V}{I} = \frac{V}{I_1 + I_2} = \frac{V}{2V/5r + V/3r} = \frac{15r}{11}$$

(2) From the given situation, we have

$$i_1 = \frac{V}{R}(e^{-t/RC}) \text{ and } i_2 = \frac{V}{R}(e^{-t/3RC})$$

$$\frac{i_1}{i_2} = (e)^{\frac{2t}{3RC}} \text{ which increases with time}$$

(4) Since total number of cell is 45,

$$mn = 45$$

Now n cells are in series, then resistance is $n \times 0.5 \Omega$

Now there are m rows with such series, then

$$\frac{1}{R_{eq}} = \frac{1}{n \times 0.5} + \frac{1}{n \times 0.5} + \dots (m \text{ times})$$

$$R_{eq} = \frac{n \times 0.5}{m} = 2.5 \Omega \quad \left[\because I = \frac{nE}{R + r_{eq}} \right]$$

$$n = 5m$$

Using Eqs. (i) and (ii), $m = 3$ and $n = 15$

(4) In the steady state condition, no current will flow through the capacitor C.

$$\text{Current in the outer circuit is } I = \frac{2V - V}{2R + R} = \frac{V}{3R}$$

Potential difference between A and B is

$$V_A - V + V + IR = V_B$$

$$\therefore V_B - V_A = IR = \left(\frac{V}{3R}\right)R = \frac{V}{3}$$

$$2. (2) V = IR_{eq} = I \frac{4R}{3}$$

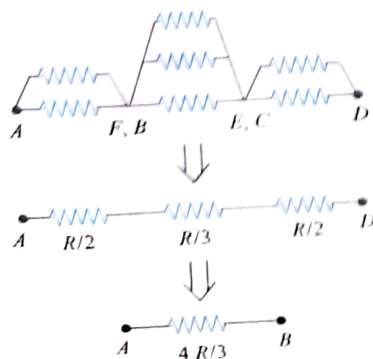
$$\text{or } I = \frac{3V}{4R}$$

$$V_1 = V_A - V_F = I \frac{R}{2}$$

$$V_2 = V_A - V_C = I \left(\frac{R}{2} + \frac{R}{3} \right) = \frac{5}{6} IR$$

$$V_2 - V_1 = V_F - V_C = IR \left(\frac{5}{6} - \frac{1}{2} \right) = \frac{IR}{3}$$

$$V_F - V_C = \frac{IR}{3} = \left(\frac{4IR}{3} \right) \frac{1}{4} = \frac{V}{4}$$



28. (3) Let $V_A - V_F = V$
if $V_A - V_C = V_A - V_B$

Equivalent resistance along ADCEF is

$$R \left(\frac{2}{7} + \frac{3}{7} + \frac{1}{7} + \frac{1}{7} \right) = R \quad (\text{Remove CB})$$

Along ADCEF, $V_A - V_F = V = I_1 R$

$$V_1 = \frac{V}{R}$$

$$V_A - V_C = I_1 \frac{5}{7} R = \frac{5V}{7} = V_A - V_B \quad \dots (ii)$$

[From Eq. (i)]

Along path ABF, equivalent resistance is

$$R_{AB} + \frac{2}{3} R$$

$$V_A - V_F = V = I_2 \left(R_{AB} + \frac{2}{3} R \right)$$

$$\text{or } I_2 = \frac{V}{R_{AB} + \frac{2}{3} R} \quad \text{or } V_A - V_B = I_2 R_{AB} = V_A - V_C$$

$$\left[\frac{V}{R_{AB} + \frac{2}{3} R} R_{AB} \right] = \frac{5}{7} R \quad [\text{from Eq. (ii)}]$$

$$\text{or } R_{AB} = \frac{5}{3} R$$

29. (4) Let current in AB be I_1 , in BF be I_2 , in BC be $2I_2$, in FC be I_3 , in FE be I_3 , and in CD be $(I_3 + 2I_2)$.

At point B, $I_1 = 3I_2$

At point F, $I_2 = 2I_3$

Along ABFCD,

$$V_A - V_D = V = I_1 \left(\frac{2R}{5} \right) + I_2 \left(\frac{R}{5} \right) + I_3 \left(\frac{R}{5} \right) + (I_3 + 2I_2) \frac{R}{5} = 2I_2 R$$

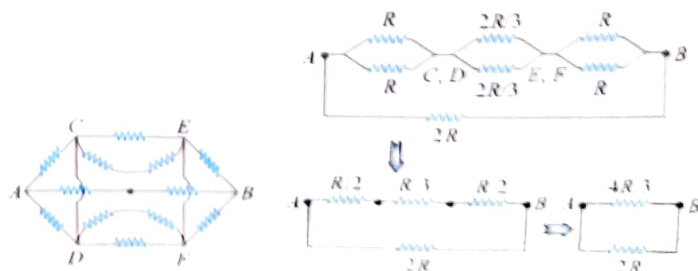
Along ABCD,

$$V_A - V_D = V = 2I_2 R$$

$$= I_1 \left(\frac{2R}{5} \right) + 2I_2 R' + (I_3 + 2I_2) \frac{R}{5}$$

$$\text{or } 2I_2 R' = \frac{3}{10} I_2 R \quad \text{or } R' = \frac{3}{20} R$$

30. (4) The current in CO, OE, DO, and OF will be same. So these branches will not touch each other at O.



31. (2) No current will pass through C_1 and C_2

$$R_{eq} = 10 \Omega, \quad I = 1 \text{ A}$$

$$V_A - V_C = 1 \times 6 = 6 \text{ V}, \quad V_C = V_D = 6 = 4 \text{ V}$$

$$V_C - V_B = \text{voltage difference across } C_2 = 4 \text{ V}$$

32. (2) Here we will apply superposition principle

(i) Let us feed a current I to point 1, but do not draw the current from 2 and allow the current to spread to infinity. Then current in branch 12 will be $I/3$.

(ii) Secondly, feed current I to the current at take it out from 2, now again current through branch 12 will be $I/3$.

(iii) Now carry out both the above steps (i) and (ii) simultaneously. Then current in branch 12 will be sum of above two currents, i.e., $\frac{I}{3} + \frac{I}{3} = \frac{2I}{3}$

Resistance between 1 and 2 is R . Let remaining resistance is R_1 . Then

$$R_1 \frac{I}{3} = R \frac{2I}{3} \quad \text{or} \quad R_1 = 2R$$

$$R_{eq} = \frac{R R_1}{R + R_1} = \frac{R \cdot 2R}{R + 2R} = \frac{2R}{3}$$

33. (2) The given circuit in steady state reduces to

$$I = \frac{12 - 4}{4} = 2 \text{ A}$$

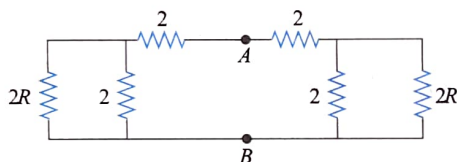
$$V_{CD} = 3 \times 2 = 6 \text{ V}$$

Now change on $6 \mu\text{F}$ capacitor is

$$\frac{6V}{\left(\frac{1}{3} + \frac{1}{6}\right) \frac{1}{\mu\text{F}}} = 12 \mu\text{C}$$

Potential difference across $6 \mu\text{F}$ capacitor is $12 \mu\text{C} / 6 \mu\text{F} = 2 \text{ V}$.

34. (1) The equivalent circuit can be redrawn as shown in figure.



$$\text{Let } R = \frac{2R \times 2}{2R + 2} + 2 = \frac{2R}{R+1} + 2$$

$$\text{or } R = \frac{2R + 2R + 2}{R+1} = \frac{2+4R}{R+1}$$

$$R^2 - 3R - 2 \text{ or } R = \frac{3 + \sqrt{17}}{2}$$

$$R_{AB} = \frac{R}{2} = \frac{3 + \sqrt{17}}{4}$$

$$35. (2) U = \frac{1}{2} CV^2 = \frac{1}{2} C (2IR)^2 = 2I^2 R^2 C$$

36. (1) Potential difference = 0. Therefore, charge = 0.

37. (4) Potential difference across the central limb = 16 V = potential difference across 16Ω = potential difference across the left limb.

Current through $16 \Omega = 1 \text{ A}$

Current through the left limb = 1 A and $R = 11 \Omega$

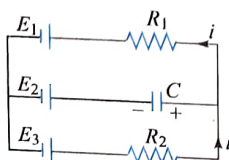
$$\therefore \tau_c = \left(\frac{16}{2} + 2\right) \Omega \times (4 \times 10^{-6}) \text{ F} = 40 \mu\text{s}$$

$$38. (3) i = \frac{2.5 - 1}{30} = 0.05 \text{ A}$$

$$-20i + 2.5 = V_{\text{capacitor}} + 1$$

$$V_{\text{capacitor}} = 0.5 \text{ V}$$

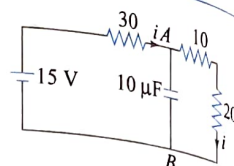
$$q^{(-)} = CV = 10 \times 0.5 = 5$$



$$39. (1) i = \frac{15}{60} = \frac{1}{4} \text{ A}$$

$$V_{AB} = \frac{1}{4} \times 30 = 7.5 \text{ V}$$

$$q = eV = 10 \times 7.5 = 75 \mu\text{C}$$



40. (1) Resistance of each part = $R/2n$. For n such parts connected in series, equivalent resistances, say

$$R_1 = n \left[\frac{R}{2n} \right] = \frac{R}{2}$$

Similarly, equivalent resistance, say R_2 , for another set of n identical, respectively, in parallel resistances would be

$$\frac{1}{n} \left(\frac{R}{2n} \right) = \frac{R}{2n^2}$$

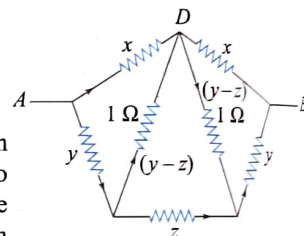
For getting maximum of R_1 and R_2 , the resistances should be connected in series and hence,

$$R_{eq} = R_1 + R_2 = \frac{R}{2} \left(1 + \frac{1}{n^2} \right)$$

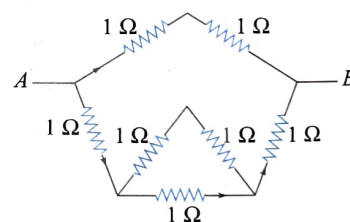
41. (1) In the steady state, no current flows through the capacitor.

42. (1) Recall the condition for maximum power flow through a circuit.

$$43. (3) R_{\text{equal}} = \frac{\frac{8}{3} \times 2}{\frac{8}{3} + 2} = \frac{16}{14} = \frac{8}{7} \Omega$$



The charge x passing between A and D will entirely pass to $DA2$. So, we can detach the circuit at point D and shown in figure.



44. (2) The equivalent capacitance C_{eq} is

$$\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

$$\text{or } C_{eq} = \frac{6}{5} \mu\text{F}$$

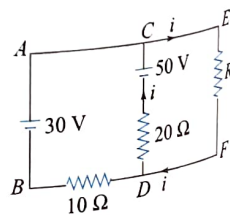
The charge that flows from P to Q is

$$Q = C_{eq} V = \frac{6}{5} \times 5 = 6 \mu\text{C}$$

$$45. (3) i = \frac{50}{20 + R}$$

Potential drop across R = potential drop across AB

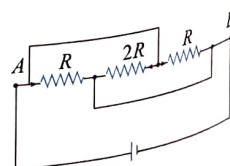
$$\frac{50}{20 + R} R = 30 \quad \text{or} \quad R = 30 \Omega$$



46. (2) In figure, all resistances are connected in parallel.

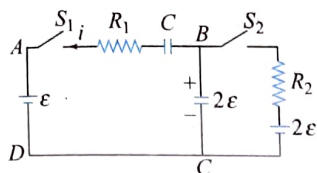
So, $R_{eq} = \frac{2R \times R/2}{2R + R/2}$ and current

in all resistances flows from



positive terminal of battery (means A end) to negative terminal of battery (means B end).

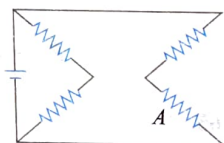
47. (2) Just before S_1 is closed, the potential difference across capacitor 2 is 2ϵ .



Just after S_1 is closed, the potential differences across capacitors 1 and 2 are 0 and 2ϵ , respectively. Applying KVL to loop ABCD immediately after S_1 is closed,

$$\epsilon = -iR_1 + 0 + 2\epsilon, i = \frac{\epsilon}{R_1} \text{ toward left}$$

48. (4) This is a dc circuit because the battery is the only source of voltage. Hence, the capacitors behave like open circuits. An equivalent circuit is then two parallel sets of two identical series resistors (see figure). The voltage drop across each parallel branch must be the battery voltage of 3 V. Since the resistors are identical, there is an equal voltage drop of 1.5 V across each resistor. In particular, there is a drop of 1.5 V across resistor A.



49. (3) $U_i = \frac{1}{2} C \epsilon_1^2$

$$U_f = \frac{1}{2} C \epsilon_2^2, \quad \Delta U = \frac{1}{2} C (\epsilon_2^2 - \epsilon_1^2)$$

$$Q_{in} = +C\epsilon_1, \quad Q_f = -C\epsilon_2, \quad \Delta Q = |Q_f - Q_i| = C(\epsilon_2 + \epsilon_1)$$

$$\text{Work done by battery } W_b = \epsilon_2 \Delta Q = C(\epsilon_2 + \epsilon_1)\epsilon_2$$

$$\text{Heat generated} = W_b - \Delta U$$

$$= C\epsilon_2^2 + C\epsilon_1\epsilon_2 - \frac{1}{2} C (\epsilon_2^2 - \epsilon_1^2)$$

$$= \frac{1}{2} C (\epsilon_2^2 + \epsilon_1^2 + 2\epsilon_1\epsilon_2) = \frac{1}{2} C (\epsilon_1 + \epsilon_2)^2$$

50. (3) Applying KVL, we get

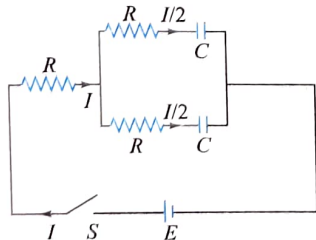
$$\frac{I}{2} = \frac{dq}{dt} \text{ or } I = 2 \frac{dq}{dt}$$

$$E - IR - \frac{I}{2} R - \frac{q}{C} = 0$$

$$\text{or } E - \frac{3}{2} IR - \frac{q}{C} = 0$$

$$\text{or } E - \frac{3}{2} \times 2 \frac{dq}{dt} R - \frac{q}{C} = 0$$

$$q_{(t)} = EC(1 - e^{-t/3RC})$$



51. (2) $I_1 = \frac{5}{2+3} = 1 \text{ A}$

$$I_2 = \frac{5}{3+3} = \frac{5}{6} \text{ A}$$

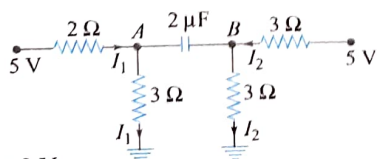
$$V_A - 0 = 3I_1 \text{ or } V_A = 3 \text{ V}$$

$$V_B - 0 = 3I_2 \text{ or } V_B = 2.5 \text{ V}$$

Potential difference across capacitor is

$$3 \text{ V} - 2.5 \text{ V} = 0.5 \text{ V} = 1/2 \text{ V}$$

$$q = CV = 2 \times 1/2 = 1 \mu\text{C}$$



52. (3) After long time, current through the capacitor = 0.

Therefore, current through the 6Ω resistor, $i = \frac{12-4}{8} = 1 \text{ A}$

Voltage across capacitor, $V = 4 + (6)(1) = 10 \text{ V}$

Charge on the capacitor, $Q = (10)(10) = 100 \mu\text{C}$

After the insertion of the dielectric, we get

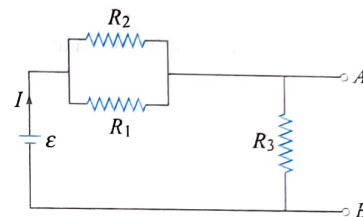
$$C' = \frac{A\epsilon_0}{\frac{d}{3} + \frac{d}{3} + \frac{d}{3(2)}} = \frac{A\epsilon_0}{d} \left[\frac{1}{\left(\frac{1}{3} + \frac{1}{3} + \frac{1}{6} \right)} \right]$$

$$= (10) \left(\frac{1}{(5/6)} \right) = \frac{5}{6} (10) = 12 \mu\text{F}$$

Hence, voltage across the capacitor remains the same.

Charge on the capacitor, $Q' = (12)(10) = 120 \mu\text{C}$

53. (4) At $t = \infty$, the equivalent circuit is



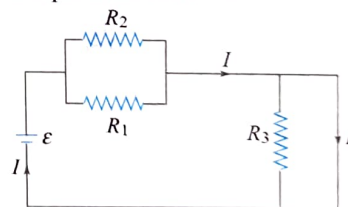
$$\therefore I = \frac{\epsilon}{R_3 + \frac{R_1 R_2}{R_1 + R_2}} = \frac{10}{1 + \frac{(2)(2)}{2+2}} = \frac{10}{1+1} = 5 \text{ A}$$

$$\text{Also, } I_{R_1} R_1 = I_{R_2} R_2 \text{ or } \frac{I_{R_1}}{I_{R_2}} = \frac{R_2}{R_1}$$

$$\therefore V_{AB} = IR_3 = (5)(1) = 5 \text{ V}$$

$$\therefore Q_{\max} = CV_{AB} = 5 \mu\text{C}$$

At $t = 0$, the equivalent circuit is



$$\therefore I_{R_3} = 0$$

$$\text{and } I_{R_1} R_1 = I_{R_2} R_2 \text{ or } \frac{I_{R_1}}{I_{R_2}} = \frac{R_2}{R_1}$$

54. (3) Initially, all three capacitors are in parallel

$$E_i = \frac{1}{2} \times 3CV^2 = \frac{3}{2} CV^2$$

When key is closed, two capacitors are in series.

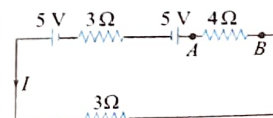
$$E_f = \frac{1}{2} C \left(\frac{V}{2} \right)^2 \times 2 = \frac{CV^2}{4}$$

$$\Delta E = E_i - E_f = \frac{5}{4} CV^2$$

55. (3) Equivalent circuit:

$$I = \frac{5-5}{8} = 0 \text{ or } V_A - V_B = 0$$

So, current will be only in the smaller loop containing the 6 V and 4 V battery. So, current through the 6 V battery is



$$I' = \frac{6-4}{4} = \frac{1}{2} \text{ A}$$

56. (2) Using KVL, we get

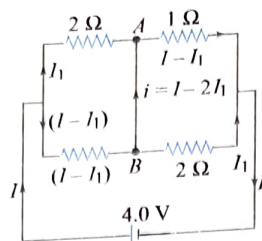
$$2I_1 - (I - I_1) = 0 \text{ or } I = 3I_1$$

$$\text{and } (I - I_1) + 2I_1 = 4$$

Solving, we get $I_1 = 1 \text{ A}$ and $I = 3 \text{ A}$

Therefore, current through

$$AB = I - 2I_1 = (3 - 2) \text{ A} = 1 \text{ A}$$



57. (4) For maximum current, net resistance of cells must be equal to
- 2.5Ω
- , i.e.,

$$\frac{n(0.5)}{m} = 25 \quad \dots(i)$$

$$\text{and } m \times n = 45$$

Solving, we get $n = 15$, $m = 3$

58. (3) From work-energy theorem, we get
- $\Delta = \varepsilon i - i^2 r$
- .

59. (3)
- $E = \lambda / 2\pi\epsilon_0 r'$
- , where
- λ
- is the linear charge density of the inner cylinder.

$$\text{And } V = \int_a^b E dl = \frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{b}{a}\right) \quad \dots(ii)$$

$$\text{Now, } I = \int \vec{J} \cdot d\vec{A} = \sigma \int \vec{E} \cdot d\vec{A} = \sigma \int \frac{\lambda}{2\pi\epsilon_0 r} 2\pi r dr$$

Current per unit length will be

$$I = \frac{\sigma\lambda}{\epsilon_0} \quad \dots(ii)$$

From Eq. (i), we get

$$I = \frac{2\sigma\pi\epsilon_0}{\epsilon_0 \ln(b/a)} v = \frac{2\pi\sigma}{\ln(b/a)} v$$

Alternatively,

$$I = \frac{V}{R} \text{ or } R = \int_{x=a}^b \frac{1}{\sigma} \frac{dx}{2\pi x l} = \frac{1}{2\pi r} \ln\left(\frac{b}{a}\right)$$

$$\therefore I = \frac{2\pi\sigma V}{\ln(b/a)}$$

60. (2)
- $50 = 10[R + r]$

$$\text{or } R + r = 5 \Omega$$

$$\eta = \frac{R}{R+r} \text{ or } 0.25 = \frac{R}{R+r}$$

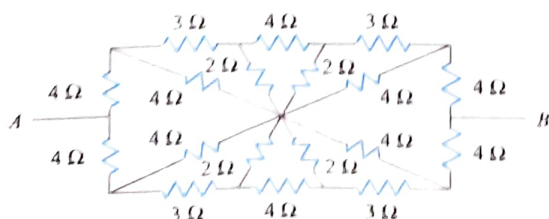
$$R + r = 4R$$

$$\text{or } r = 3R$$

$$\text{Then } R = 5/4 = 1.2 \Omega \text{ and } r = 3.75 \Omega$$

61. (1) Across the dotted line, the circuit is symmetric.

$$R_{eq} = 6 \Omega$$



62. (3) Potential drop across
- $1 \Omega = 2 \times 1/1.5 \text{ V}$
- . This potential drop exists across capacitor. So

$$Q = CV = \frac{4}{3} \mu\text{C}$$

63. (2) The current will pass through only resistance part of the circuit. Hence,

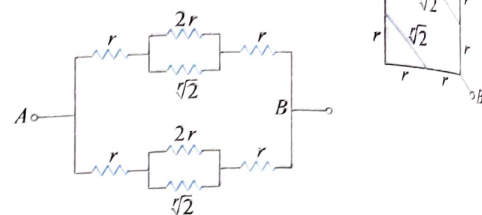
$$I = \frac{9}{1+6+2} \text{ A}$$

Potential drop across $3 \mu\text{F}$ capacitance is

$$V_{AB} = V_{AB} + V_{BC} = 1 \times 6 + \frac{1}{2} \times 1 = 6.5 \text{ V}$$

(Apply current division for current in branch BC which is $1/2 \text{ A}$.)

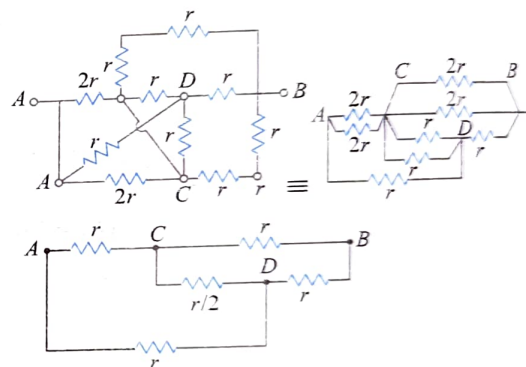
64. (1) The circuit is equivalent to Let each half side has resistance
- $\rho r d/2$
- .



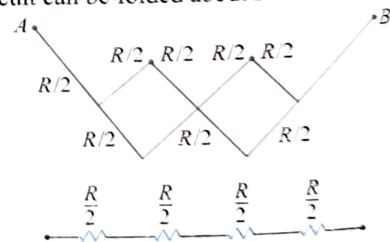
On solving, we get

$$R = \frac{1}{2} \left[2r + \frac{(2r)(r\sqrt{2})}{(2+\sqrt{2})r} \right] = r\sqrt{2} = \rho d / \sqrt{2}$$

65. (4)

 $\equiv r$ (By Wheatstone balanced bridge concept)

66. (2) The circuit can be folded about B and redrawn as

Hence, the equivalent resistance between A and B is $2R$.

67. (4)
- $R = \frac{1}{\sigma} \times \frac{l}{4\pi R^2}$

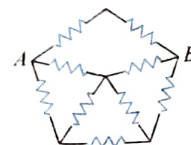
Using values, $R = 5 \times 10^{-11} \Omega$

68. (2) Since current
- $neAv_d$
- through both the rods is the same, i.e.

$$2(neAv_d) = ne(2A)v_R$$

$$\frac{v_d}{v_R} = 1$$

69. (1) Equivalent circuit,
- $R_{\text{net}} = \frac{8R}{11}$

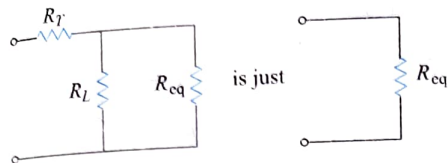


(2) If P is disconnected, R_{eq} of circuit increases, hence less current is drawn. Therefore, $0 + 2iR_1 - x$

Therefore, as i decreases, x increases.

(2) From the hint, the equivalent resistance of the circuit is

$$R_T + \frac{1}{1/R_L + 1/R_{eq}} = R_{eq}$$



$$R_T + \frac{R_L R_{eq}}{R_L + R_{eq}} = R_{eq}$$

$$\text{or } R_T R_L + R_T R_{eq} + R_L R_{eq} = R_L R_{eq} + R_{eq}^2$$

$$\text{or } R_{eq}^2 - R_T R_{eq} - R_T R_L = 0$$

$$\text{or } R_{eq} = \frac{R \pm \sqrt{R_T^2 + 4R_T R_L}}{2}$$

Only the positive sign is physical.

$$R_{eq} = \frac{1}{2} \left(\sqrt{4R_L R_T + R_T^2} + R_T \right)$$

For example, if $R_T = 1 \Omega$ and $R_L = 20 \Omega$, $R_{eq} = 5 \Omega$

72. (2) Final charge on capacitor $q_f = CE$

Charge flown = $CE - CE/2 = CE/2$

Work done by battery: $W_b = E(CE/2) = \frac{1}{2} CE^2$

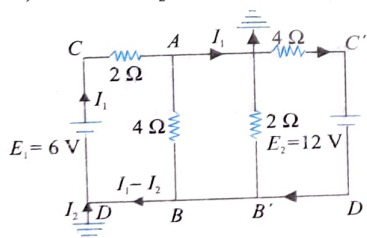
Change in energy of capacitor is

$$\Delta U = \frac{1}{2C} [q_f^2 - q_0^2]$$

$$= \frac{1}{2C} [(CE)^2 - (CE/2)^2] = \frac{3CE^2}{8}$$

$$\text{Heat} = W_b - \Delta U = \frac{1}{8} CE^2$$

73. (4) $E_1 = (2 + 4) \times 1 = 6 \text{ V}$, $E_2 = (2 + 4) \times 2 = 12 \text{ V}$



There will be no current in AB and $A'B'$, because all these points are at the same potential. From D to A' :

$$V_D + 6 - 2I_1 = V_{A'}, \text{ but } V_D = V_{A'} = 0$$

$$\Rightarrow I_1 = 3 \text{ A}$$

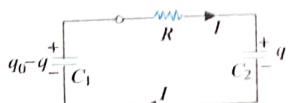
For outermost loop: $6 - 12 = 2I_1 + 4(I_1 - I_2)$ or $I_2 = 6 \text{ A}$

Current through $C'A' = I_1 - I_2 = 3 - 6 = -3 \text{ A}$

74. (2) $I = \frac{dq}{dt}$; apply Kirchhoff's law: $\frac{q_0 - q}{C_1} = IR + \frac{q}{C_2}$

$$\frac{q_0 - q}{C_1} \left[\frac{C_1 + C_2}{C_1 C_2} \right] = \frac{dq}{dt} R$$

$$\int_0^q \frac{dq}{\frac{q_0 - q}{C_1} \left(\frac{C_1 + C_2}{C_1 C_2} \right)} = \int_0^t \frac{dt}{R}$$



$$\text{Solve to get } q = \frac{q_0 C_2}{C_1 + C_2} \left[1 - e^{-\frac{(C_1 + C_2)t}{C_1 C_2 R}} \right]$$

75. (4) In steady state, there will be no current in any branch. So A and D will be at the same potential. Hence, charge on capacitor between A and D will be zero.

$$76. (3) \text{ Initially } I_1 = \frac{E_1 + E_2}{(1 + 2 + 7)} = \frac{E_1 + E_2}{10}$$

$$\text{When } E_2 \text{ is shorted } I_2 = \frac{E_1}{(1 + 7)} = \frac{E_1}{8}$$

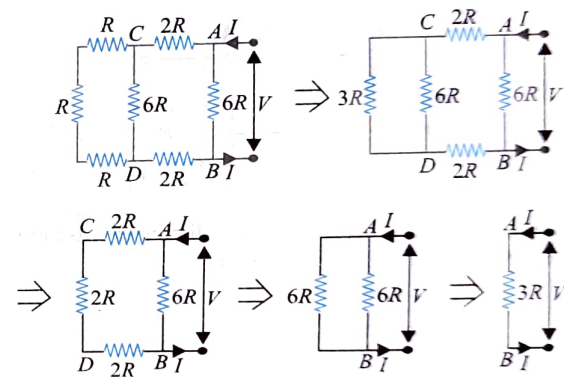
If $I_2 > I_1$

$$\frac{E_1}{8} > \frac{E_1 + E_2}{10} \quad \text{or} \quad \frac{5}{4} > \frac{E_1 + E_2}{E_1}$$

$$\text{or } \frac{5}{4} > 1 + \frac{E_2}{E_1} \quad \text{or} \quad \frac{E_2}{E_1} < \frac{1}{4}$$

$$\text{or } \frac{E_1}{E_2} > 4$$

$$77. (3) R_{eq} = \frac{\frac{11}{4} RR}{(15/4)R} = \frac{11}{15} R$$



$$V = I R_{eq}$$

Hence, current through resistance is $V/6R$.

78. (2) A careful observation will reveal that one end of each resistor is connected to A and the other end of each resistor is connected to B . Hence, resistors are in parallel.

$$R_{eq} = R/5$$

When the key is open, current will pass through resistance 1 and 2 only, which are in parallel. $R_{eq} = R/2$

$$79. (2) \text{ Current in } EC = \frac{40}{20} = 2 \text{ A}$$

$$\text{Current at } C = 2 + 2 = 4 \text{ A}$$

$$\text{Equivalent resistance of } CD = \frac{5 \times 5}{10} = 2.5 \Omega$$

$$\text{Potential drop in } CD = 4 \times 2.5 = 10 \text{ V}$$

$$100 - V_{EC} + V_{CD} + V_{DF}$$

$$V_{DF} = 100 - 40 - 10 = 50 \text{ V}$$

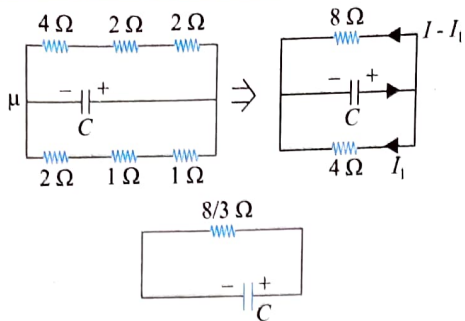
$$\text{Current in wire } DG = 2 \text{ A}$$

$$\text{Current in resistance } DF = 2 \text{ A}$$

$$R \times 2 = 50 \text{ or } R = 25 \Omega$$

$$80. (3) q_0 = 160 \mu\text{C}$$

$$C = 6 \mu\text{F}$$



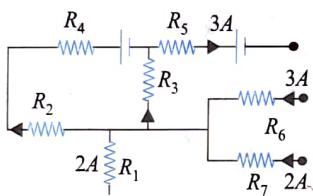
$$q = 160e^{-\frac{t}{\frac{8}{3} \times 6}} = 160 \times e^{-t/16} \text{ or } I = \frac{dq}{dt}$$

$$I = \frac{160}{16} e^{-t/16} = 10e^{-t/16} \text{ or } 10e^{-1} = \frac{10}{e}$$

$$I_1 \times 4 = \left(\frac{10}{e} - I_1 \right) \times 8$$

$$\text{or } 12I_1 = \frac{80}{e} \text{ or } I_1 = \frac{80}{12e} = \frac{20}{3e} \text{ A}$$

81. (3) Since 3A is in upper part of circuit, out of 5A coming in lower part, 3A has to go to the upper part. Out of which some part will flow through R_2 and rest through the resistance R_3 . 2A will go through R_1 .



82. (1) The two opposite cells A and B will cancel two more cells, so net emf will be $n - 4$. So current is

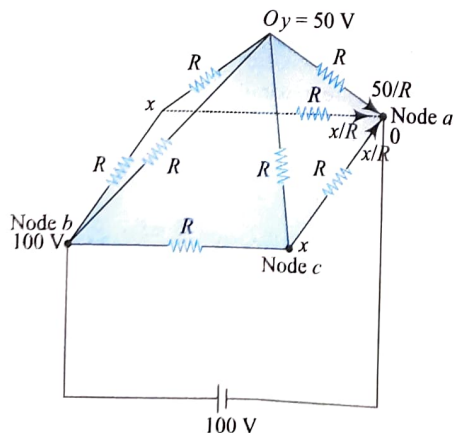
$$I = \frac{(n-4)\epsilon}{nr}$$

Now pd across A or B is

$$i\epsilon + Ir \text{ (as they will be in charging state)}$$

$$= \epsilon + \frac{(n-4)\epsilon}{n} = 2\epsilon \left(1 - \frac{2}{n} \right) = \frac{2\epsilon(n-2)}{n}$$

83. (2) Step 1: Attach a 100 V battery and assign potential at each node.



Step 2: By symmetry $100 - y = y - 0$ or $y = 50$ V

Step 3: Now apply KCL at node c

$$\frac{x-100}{R} + \frac{x+50}{R} + \frac{x-0}{R} = 0$$

$$\text{or } x = 50 \text{ V}$$

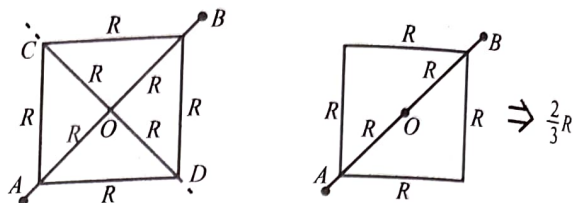
Step 4: If R_{eq} is equivalent resistance of this circuit, we have

$$\frac{100}{R_{eq}} = \frac{x}{R} + \frac{50}{R} + \frac{50}{12} \text{ or } R_{eq} = \frac{2}{3} R$$

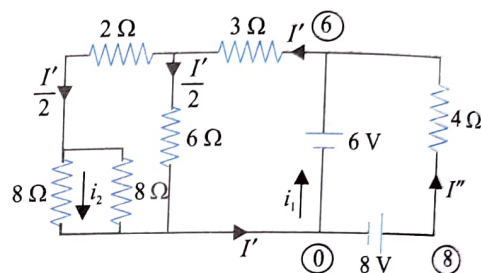
Alternative method

Making 2D projection

There is perpendicular axis of symmetry, i.e., the points CO will be equipotential then the circuit will be



84. (3) The circuit can be reduced as



$$\text{From Fig. (2), } I' = \frac{6}{6} = 1.0 \text{ A}$$

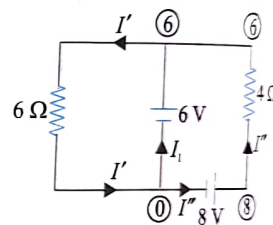
$$\text{Hence } I_2 = \frac{(I'/2)}{2} = \frac{I'}{4} = \frac{1}{4} \text{ A}$$

From Fig. (2) the current through 4 Ω branch

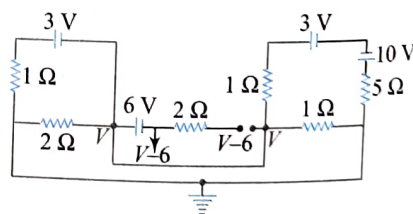
$$I'' = \frac{8-6}{4} = \frac{1}{2} \text{ A}$$

$$\text{Hence } I_1 = I' - I'' = 1 - \frac{1}{2} = \frac{1}{2} \text{ A}$$

$$\text{or } \frac{I_1}{I_2} = \frac{1/2}{1/4} = \frac{1}{2} \times \frac{4}{1} = 2$$



85. (2) In steady state, the capacitor is fully charged and is treated as open circuit, so no current flows through branch containing capacitor in steady state. So the circuit can be redrawn as:



Potential difference across the capacitor in steady state is

$$V - 6 - V = -6 \text{ V}$$

(Negative sign signifies that left hand plate is of negative polarity.)

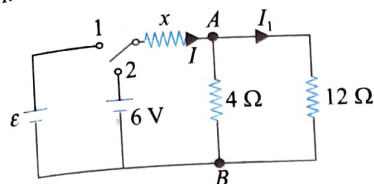
$$\text{Charge} = CV = 1 \times 6 = 6 \text{ C}$$

$$86. (2) \text{ Here } I_1 = \frac{V}{R} e^{-\frac{t}{RC_1}} \text{ and } I_2 = \frac{V}{R} e^{-\frac{t}{RC_2}}$$

$$\frac{I_1}{I_2} = \frac{e^{-\frac{t}{RC_1}}}{e^{-\frac{t}{RC_2}}} = e^{-\frac{t}{R} \left(\frac{1}{C_1} + \frac{1}{C_2} \right)} = e^{-\frac{t(C_1+C_2)}{RC_1C_2}}$$

Hence option (2) is correct.

(c) When switch (1) is closed



$$I_1 = I \left(\frac{4}{4+12} \right) \text{ or } \frac{3}{4} = I \left(\frac{1}{4} \right)$$

$$I = 3 \text{ A}$$

When switch (2) is closed the current through resistance x is 1 A. Hence

$$V_{AB} = 1 \times \left(\frac{4 \times 12}{4+12} \right) \text{ or } V_{AB} = 3 \text{ V}$$

Hence potential difference across x will be 3 V.

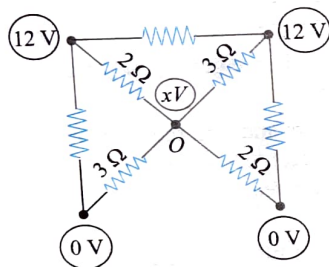
$$x = \frac{3}{7} = 3 \Omega$$

$$\text{Hence } \varepsilon = 3(3+3) = 18 \text{ V}$$

(4) The potential difference across C_3 is zero hence at steady state we have two capacitors (C_1 and C_2) in series.

$$C_{eq} = \frac{6 \times 3}{6+3} = 2 \mu\text{F}$$

(4) The circuit can be redrawn as



At junction O , let potential of this point is x .

$$\frac{(12-x)}{2} + \frac{(12-x)}{3} = \frac{(x-0)}{3} + \frac{(x-0)}{2} \text{ or } x = 6 \text{ V}$$

Multiple Correct Answers Type

1. (1), (3)

As the cross sections are in series, so the current in them will be the same. From $I = neAv_d$, we have $v_d \propto 1/A$. As one cross section is more, hence the drift velocity is less.

2. (1), (3), (4)

$$I = \frac{dQ}{dt} = a - 2bt$$

$$I = 0 \text{ for } t = (a/2b) \text{ and } (dI/dt) = -2b$$

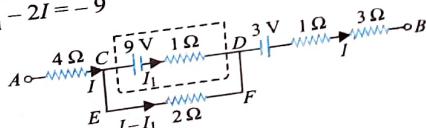
3. (2), (3), (4)

The 7Ω resistor is short-circuited, so no current will flow through it. Potential difference across each of 3Ω and 6Ω is 12 V, so we can find current in them.

4. (1), (3), (4)

For loop $CEFD$, $-2(I-I_1) + 1I_1 + 9 = 0$

$$\text{or } 3I_1 - 2I = -9$$



From A to B via CD , $V_A - 4I - 9 + 3 - 1I_1 - 1I - 3I = V_B$

But $V_A - V_B = 16 \text{ V}$, so $8I + I_1 = 10$

$$\text{or } I_1 = -2\text{ A}, I = 1.5 \text{ A}$$

$$\text{or } I - I_1 = 1.5 + 2 = 3.5 \text{ A}$$

$$V_C - 9 - 1I_1 = V_D \text{ or } V_C - V_D = 7 \text{ V}$$

5. (1), (4)

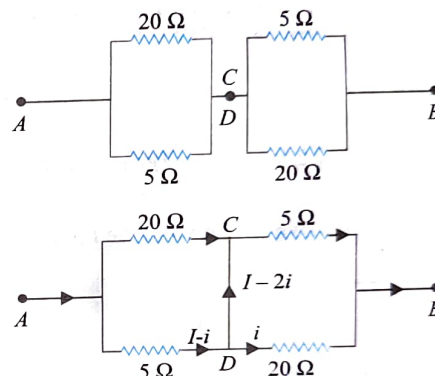
The last three resistors 2Ω , 4Ω , and 2Ω are in series having the equivalent resistance as $2 + 4 + 2 = 8 \Omega$. This will be in series with the 8Ω next to them. So their equivalent resistance becomes 4Ω . In this way, net equivalent resistance of the circuit becomes $R_{eq} = 9 \Omega$. This will be in series with $r = 1 \Omega$. So the current through 3Ω is

$$I = e/(R+r) = 10/(9+1) = 1 \text{ A}$$

Further, current will get divided at C and E into half at each point. So finally the current reaching in 4Ω will be 0.25 A .

6. (1), (2), (4)

As C and D are joined, they must be at the same potential and may be treated as the same point. This gives the equivalent resistance as 8Ω . We distribute current in the network using symmetry.



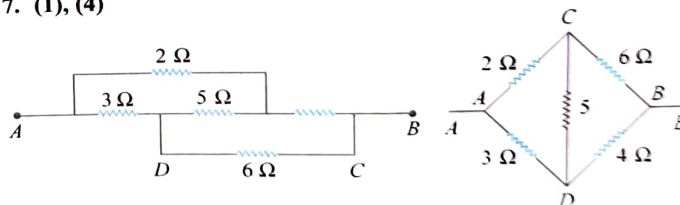
$$V_A - V_D = V_A - V_C$$

$$\text{or } 20i = 5(I-i)$$

$$\text{or } i = I/5$$

$$I - 2i = I - \frac{2I}{5} = \frac{3I}{5} = \text{current flowing from } D \text{ to } C$$

7. (1), (4)



Rearrangement of the circuit as shown in figure gives a balanced Wheatstone bridge, and no current flows through the 5Ω resistor. It can thus be removed from the circuit.

8. (1), (2), (3)

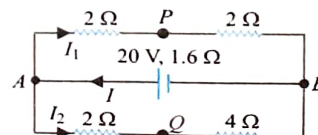
Potential difference across resistance is equal to the potential difference across the terminals of the battery. So $V = E - Ir$. This is an equation of a straight line. Comparing this with the given graph, we can see that $E = 10 \text{ V}$ and $r = 5 \Omega$. Also

$$I_{\max} = \frac{E}{r} = \frac{10}{5} = 2 \text{ A}$$

When external resistance $R = 0$.

9. (1), (4)

Resistors 2Ω and 2Ω are in series, and 2Ω and 4Ω are in series, then their resultant are in parallel. Produce net resistance $R = 2.4 \Omega$.



$$I = \frac{20}{r+R} = \frac{20}{1.6+2.4} = 5 \text{ A}$$

$$I_1 + I_2 = 5 \text{ A}$$

$$V_{AB} = 4I_1 = 6I_2 \text{ or } 2I_1 = 3I_2$$

From Eqs. (i) and (ii), $I_1 = 3 \text{ A}$, $I_2 = 2 \text{ A}$

$$V_A - V_P = 6 \text{ V}, V_A - V_Q = 2I_2 = 4 \text{ V}$$

$$\text{Thus, } V_Q - V_P = 2 \text{ V}$$

10. (3), (4)

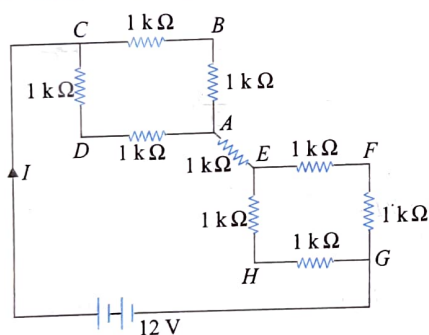
Note that points a , h , g , and f have the same potential. They are connected by conducting wires without any circuit elements between them. Similarly, points b , c , d , and e have the same potential. Hence, the potential drop across branch e and f , and a and b is the same. The two resistors (6Ω and 4Ω in series) are directly connected across the terminals of 12 V battery.

$$I = \frac{V}{R_1 + R_2} = \frac{12}{10} = 1.2 \text{ A}$$

$$\therefore V_1 = (1.2)(6) = 7.2 \text{ V}, V_2 = (1.2)(4) = 4.8 \text{ V}$$

11. (1), (2), (4)

The circuit can be redrawn as shown in the figure.



$$R_{eq} = 3 \text{ k}\Omega, I = \frac{12}{3 \times 10^3} = 4 \text{ mA}$$

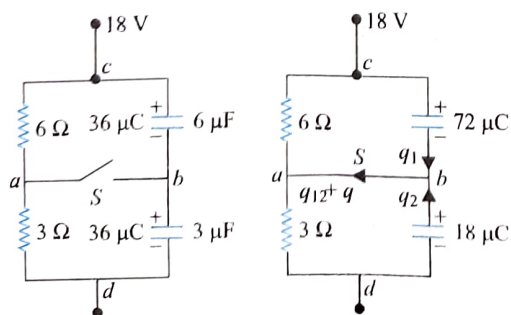
$$V_E - V_G = I(1 \text{ k}\Omega) = 4 \times 10^{-3} \times 1 \times 10^3 = 4 \text{ V}$$

12. (1), (3)

When S is opened

$$V_c - V_a = \frac{18 \times 6}{6+3} = 12 \text{ V}$$

$$\text{or } V_b - V_a = 12 - 6 = 6 \text{ V}$$



Charges flowing after S is closed:

$$q_1 = 72 - 36 = 36 \mu\text{C}, q_2 = 36 - 18 = 18 \mu\text{C}$$

Charges flowing through S after it is closed:

$$36 + 18 = 54 \mu\text{C}$$

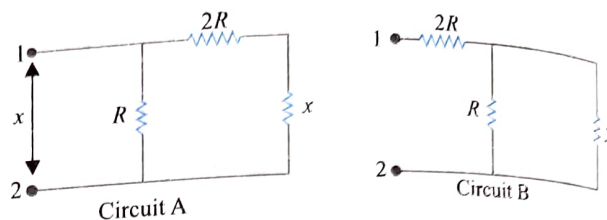
Final potential of b is 6 V .

13. (1), (2), (3), (4)

$$\text{Circuit A: } x = \frac{R(2R+x)}{3R+x} \text{ or } 3Rx + x^2 = 2R^2 + Rx$$

$$\text{or } x^2 + 2Rx - 2R^2 = 0 \text{ or } x = \frac{-2R + \sqrt{4R^2 + 8R^2}}{2}$$

$$\text{or } x = -\frac{2R + 2\sqrt{3}R}{2} = (\sqrt{3} - 1)R$$



$$\text{Circuit B: } y = \frac{yR}{y+R} + 2R \text{ or } y^2 + Ry = yR + 2Ry + 2R^2$$

$$\text{or } y^2 - 2Ry - 2R^2 = 0 \text{ or } y = (\sqrt{3} + 1)R$$

14. (1), (3)

Use symmetry.

15. (1), (4)

Equivalent resistance of the circuit $= (2.5 + 1.5) = 4 \Omega$

Current through battery is $I = 20/4 = 5 \text{ A}$

Current through P = Current through $Q = 5/2 = 2.5 \text{ A}$

$$V_A - V_Q = 3 \times 2.5 = 7.5 \text{ V}$$

$$V_A - V_Q = 2 \times 2.5 = 5 \text{ V}$$

From Eqs. (i) and (ii), we get $V_Q - V_P = 2.5 \text{ V}$.

16. (2), (3), (4)

Disregard capacitors and find the current through G . The potential difference across each capacitor is then found from the potential difference across the resistances in parallel with them.

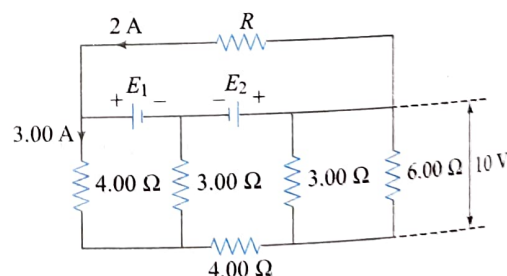
17. (1), (2), (3)

Initial charge (before filling the dielectric slab) is $10 \times 10 = 100 \mu\text{C}$.

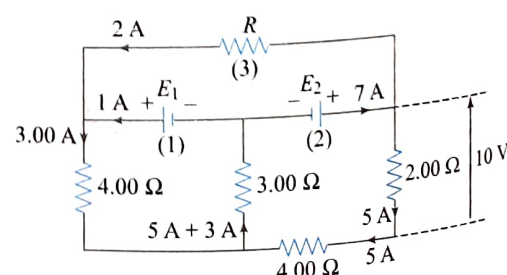
Final charge (after filling the dielectric slab) is $10 \times 30 = 300 \mu\text{C}$.

Increase in charge is $20 \mu\text{C}$

18. (1), (2), (3), (4)



After redrawing the circuit



$$(1) I_4 = 5 \text{ A}$$

(2) From loop (1),

$$-8(3) + E_1 - 4(3) = 0 \text{ or } E_1 = 36 \text{ V}$$

From loop (2),
 $+4(5) + 5(2) - E_2 + 8(3) = 0$

or $E_2 = 54 \text{ V}$

(3) From loop (3),
 $-2R - E_1 + E_2 = 0$

or $R = \frac{E_2 - E_1}{2} = -36 = 9 \Omega$

19. (2), (3)
 (Moderates) As the length is doubled, the cross-sectional area of the wire becomes half, thus the resistance of the wire $R = \rho L/A$ becomes four times the previous value. Hence after the wire is elongated, the current becomes one-fourth. Electric field is potential difference per unit length and hence becomes half the initial value. The power delivered to resistance is $P = V^2/R$ and hence becomes one-fourth.

20. (1), (2), (3), (4)
 2Ω resistance will be short-circuited as potential drop across it is 0 also 4Ω and 5Ω are in parallel.

$$\frac{1}{R} = \frac{4+5}{4 \times 5} = \frac{9}{20}$$

$$I = \frac{20}{20} \times 9 = 9 \text{ A} \Rightarrow I_1 \times 4 = (9 - I_1) \times 5$$

$$I_1 = 5 \text{ A}, I - I_1 = 4 \text{ A}$$

21. (2), (3)

Let a current I flows in circuit, then

$$V_G + E - IR_1 = V_X \text{ or } V_X - V_G = E - IR_1$$

It switch is opened, then equivalent resistance increases so I decreases. Hence $V_X - V_G$ increases.

22. (1), (3)

Immediately after closing the switch, the capacitor will behave like conducting wires, so all the resistances between B and G will be short-circuited and current will flow through $ABFE$. Hence current $= \mathcal{E}/R$. After long time when the capacitors are fully charged, current will go through $ABDG$. Hence current $= \mathcal{E}/2R$.

23. (2), (3), (4)

When S_2 is closed and S_1 is open

$$I = \frac{20 - 10}{3} = \frac{10}{3} \text{ A}$$

When S_1 is open and S_2 is closed

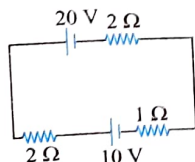
Taking outer loop

$$-2I - 2I + 20 = 0$$

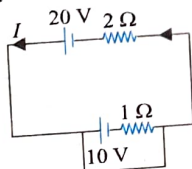
or $I = \frac{20}{4} \text{ A} = 5 \text{ A}$

When S_1 and S_2 both are open

$$I = \frac{20 - 10}{5} = 2 \text{ A}$$



When both are closed

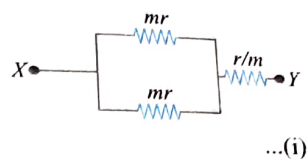


Again taking outer loop $I = \frac{20}{2} = 10 \text{ A}$

24. (2), (3)

$$R_q = \frac{m}{2} + \frac{r}{m}$$

$$I = \frac{V}{\frac{mr}{2} + \frac{r}{m}}$$



For maximum current $\frac{mr}{2} + \frac{r}{m}$ should be minimum

$$\text{But } \left(\frac{mr}{2} + \frac{r}{m} \right) = \left(\sqrt{\frac{mr}{2}} - \sqrt{\frac{r}{m}} \right)^2 + 2 \frac{mr}{2} \frac{r}{m}$$

$$= \left(\sqrt{\frac{mr}{2}} - \sqrt{\frac{r}{m}} \right)^2 + r^2$$

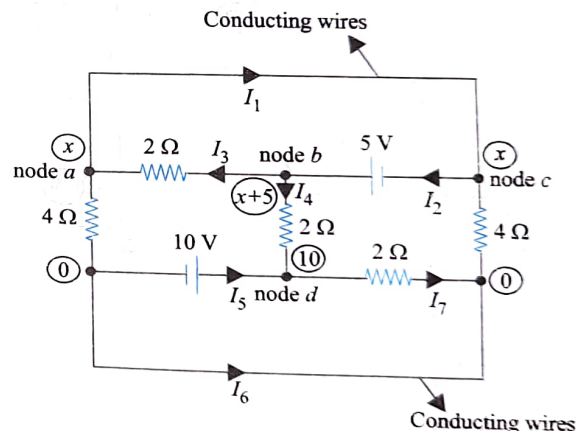
For minimum value $\sqrt{\frac{mr}{2}} = \sqrt{\frac{r}{m}}$

$$m = \sqrt{2}$$

And maximum current from (i),

$$I = \frac{V}{\frac{\sqrt{2}r}{2} + \frac{r}{\sqrt{2}}} = \frac{V}{\sqrt{2}r}$$

25. (2), (3)



From diagram it is clear $I_3 = \frac{(x+5) - x}{2} = \frac{5}{2} \text{ A}$

and $I_7 = \frac{10 - 0}{2} = 5 \text{ A}$

At node a

$$I_3 = I_1 + \frac{x}{4} \text{ or } \frac{5}{2} = I_1 + \frac{x}{4}$$

At node b

$$I_3 + \frac{(x+5-10)}{2} = I_2 \text{ or } \frac{5}{2} + \left(\frac{x-5}{2} \right) = I_2$$

$$\frac{x}{2} = I_2$$

At node c

$$I_1 = I_2 + \frac{x-0}{4}$$

$$\frac{5}{2} - \frac{x}{4} = \frac{x}{2} + \frac{x}{4} \text{ or } x = \frac{5}{2}$$

$$\text{or } I_1 = \frac{5}{2} - \frac{5}{8} = \frac{15}{8} \text{ A (current through upper conducting wire)}$$

$$\text{And } I_2 = \frac{5}{4} \text{ A (current through 5 V battery)}$$

The current (I_4) in 2Ω wire

$$I_4 = \frac{\left(\frac{5}{2} + 5\right) - 10}{2} = \frac{\frac{5}{2} - 5}{2} = -\frac{5}{4} \text{ A}$$

$$\text{At node } d: I_5 - \frac{5}{4} = 5 \text{ or } I_5 = 5 + \frac{5}{4} = \frac{25}{4} \text{ A}$$

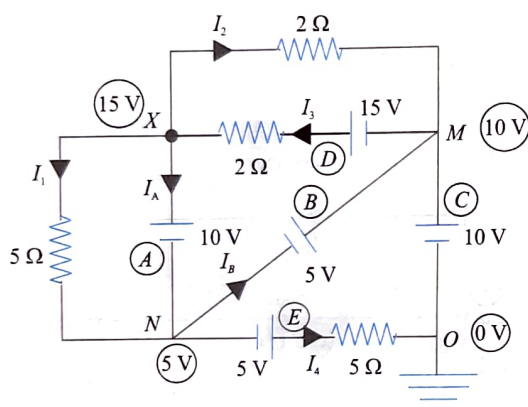
(current through 10 V battery)

$$\text{Hence } I_6 - I_5 = \frac{x - 0}{4} = \frac{\frac{5}{2} - 0}{4}$$

$$\text{or } I_6 - \frac{25}{4} = \frac{5}{8}$$

$$\text{or } I_6 = \frac{5}{8} + \frac{25}{4} = \frac{55}{8} \text{ A (current through lower conducting wire)}$$

26. (1), (3), (4)



From circuit diagram it is clear

$$I_1 = \frac{15 - 5}{6} = 2 \text{ A}, I_2 = \frac{15 - 10}{2} = 2.5 \text{ A}$$

Taking branch $x M$

$$15 + 2I_3 - 15 = 10 \text{ or } I_3 = 5 \text{ A}$$

Taking branch $N O$

$$5 + 5 - 5I_4 = 0 \text{ or } I_4 = 2 \text{ A}$$

In junction x : $I_3 = I_1 + I_2 + I_A$

$$5 = 2 + 2.5 + I_A \text{ or } I_A = \frac{1}{2} \text{ A}$$

In junction B : $I_1 + I_A = I_4 + I_B$

$$\text{or } 2.0 + 0.5 = 2 + I_B \text{ or } I_B = \frac{1}{2} \text{ A}$$

27. (1), (3)

Since 1, 2, and 3 all at same potential so equivalent resistance between 1 and 2, 2 and 3, 3 and 1 are all zero.

28. (1), (2)

Since 1, 2, and 3 are all at same potential, so equivalent resistance between 0 and 1, 0 and 2, and 0 and 3 are all equal.

Further, all three side resistances will have no current in them, so they can be removed. The middle three resistances will be in parallel between 0 and 1. So equivalent resistance between 0 and 1 is $R/3$.

29. (1), (2)

R_{01} , R_{02} , and R_{03} are in parallel having same potential difference, so they will have same current. Also current at O is equally divided among three resistances.

30. (2), (3), (4)

The equivalent circuit can be redrawn as shown in figure. From the figure it is obvious that power dissipated by R_1 is maximum. Potential difference

$$\text{across } R_2 \text{ is } \frac{25}{25 + 50} \times 3 \text{ V} = 1 \text{ V}$$

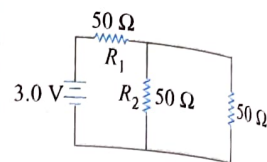
Therefore, potential difference across R_3 or R_4 is

$$\frac{20}{20 + 30} \times 1 \text{ V} = 0.4 \text{ V}$$

The equivalent resistance of circuit across the cell is $50 + 25 = 75 \Omega$

Therefore, current drawn through cell is

$$\frac{3}{75} \times 1000 \text{ mA} = 40 \text{ mA}$$



Linked Comprehension Type

1. (1), 2. (3), 3. (1)

Left loop:

$$V_F + 6 + 4I_1 - 30 + 2I_1 = V_F$$

or $I_4 = 4 \text{ A}$

Right loop:

$$V_F + 6 + 1I_2 - 15 + 2I_2 = V_F$$

or $I_2 = 3 \text{ A}$

Net current through 6 V battery is

$$I = I_1 + I_2 = 4 + 3 = 7 \text{ A}$$

The 6 V battery is getting charged. Both 15 V and 30 V batteries produce current in 6 V battery.

4. (4), 5. (2), 6. (4)

$$15 - IR_3 - IR_4 = 0$$

Given $R_3 = 4 \Omega$ and $R_4 = 2 \Omega$

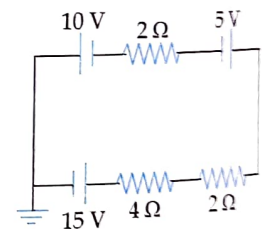
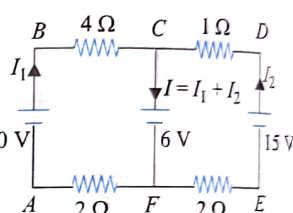
$$I = 2.5 \text{ A through } R_3 \text{ and } R_4$$

$$-10 - IR_1 + 5 = 0$$

$$\text{or } I = \frac{5}{R_1} = 2.5 \text{ A}$$

$$15 - 6I - 5 - 2I + 10 = 0$$

$$\text{or } I = \frac{20}{8} = 2.5 \text{ A}$$

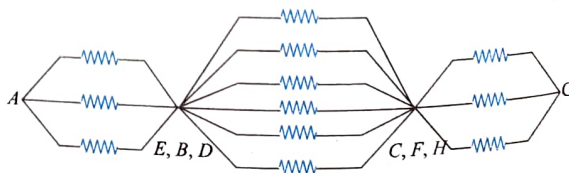
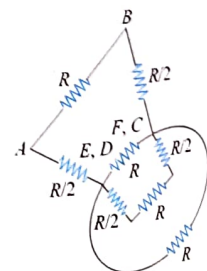


7. (3), 8. (2), 9. (1)

If we connect a battery across A and B , then E and D will be at the same potential and F and C will be at the same potential.

$$R_{eq(AB)} = \frac{7R}{12}$$

If we connect the battery across A and G , then B , E , and D will be at the same potential. C , F , and H will be at the same potential.



$$R_{eq} = \frac{5R}{6}$$

If we connect the battery across A and C , then no current will flow in the resistances between B and F and between H and D . So they can be removed. Thus,

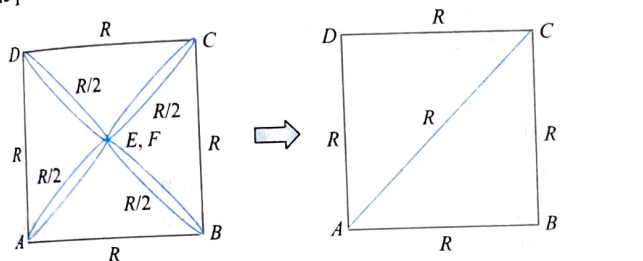
$$R_{eq} = \frac{3R}{4}$$

11. (1), 12. (3)

If we connect battery between E and F , then A , B , C , and D will be all at the same potential.

$$R_{eq} = \frac{R}{4} + \frac{R}{4} = \frac{R}{2}$$

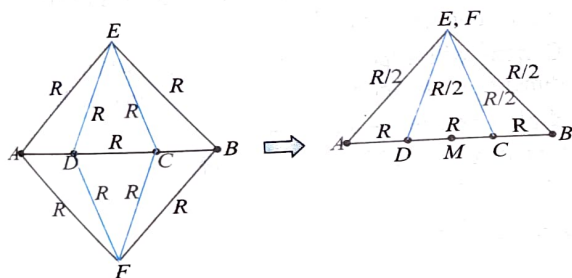
If we connect a battery between A and C , then E and F will be at the same potential.



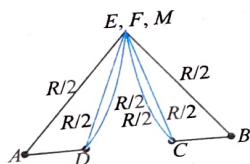
Now resistances between DE and EB will be useless. Thus,

$$R_{eq} = R/2$$

Let us remove the resistance between A and B , then E and F will be at the same potential; we can draw the remaining as follows:



M is the midpoint of DC , so M will be at the same spot as E and F .



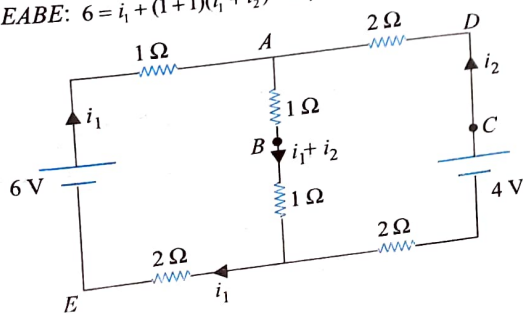
Now find $R_{AB} = 5/7R$. The resistance that we had removed will be in parallel to it, so

$$R_{eq} = \frac{R_{AB}R}{R_{AB} + R} = \frac{(5/7)RR}{(5/7)R + R} = \frac{5}{12}R$$

13. (2), 14. (2), 15. (4)

In the steady state, there will be no current in capacitor branch.

Loop $EABE$: $6 = i_1 + (1+1)(i_1 + i_2) + 2i_2$ or $5i_1 + 2i_2 = 6$... (i)



Loop $ADCBA$: $4 = 2i_2 + (1+1)(i_1 + i_2) + 2i_2$

$$\text{or } 6i_2 + 2i_1 = 4$$

Solving Eqs. (i) and (ii), $i_1 = 14/13$ A, $i_2 = 4/13$ A

Given $V_E = 0$, $V_E + 6 - 1i_1 - 1(i_1 + i_2) = V_B$

$$\text{or } V_B = 6 - 2i_1 - i_2 = 6 - 2 \times \frac{14}{13} - \frac{4}{13} = \frac{46}{13} \text{ V}$$

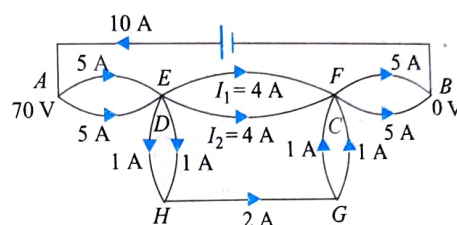
Now along $BADC$: $V_B + 1(i_1 + i_2) + 2i_2 = V_C$

$$\text{or } V_C - V_B = i_1 + 3i_2 = \frac{14}{13} + 3 \times \frac{4}{13} = 2 \text{ V}$$

Charge on capacitor is $q = C(V_C - V_B) = 4 \times 2 = 8 \mu\text{C}$

16. (2), 17. (3), 18. (1)

The equivalent circuit diagram is as shown.



$$R_{AB} = \frac{7r}{5} \quad \text{or} \quad I = \frac{5V}{7r} = \frac{5 \times 70}{7 \times 5} = 10 \text{ A}$$

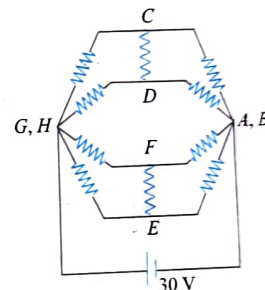
19. (4), 20. (4), 21. (1)

D , B , and E are symmetrical to A , hence potential at these points is same. Similarly, C , F , and H are symmetrical to G . So potential at these points is the same.

Resulting diagram will be as shown: C , D , F , and E will be at the same potential.

Potential difference between C and G will be half of the applied potential, i.e.,

$$V_{CG} = 30/2 = 15 \text{ V}$$



22. (4), 23. (3)

Initially capacitors act as conducting wire. So initially all the three resistors will be in parallel.

After long time, resistors will be in series,

$$R_{eq} = 2 + 3 + 4 = 9 \Omega$$

$$I = \frac{24}{9} = \frac{8}{3} \text{ A}$$

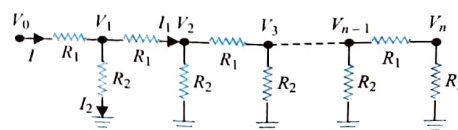
Potential difference $3 \mu\text{F}$ = potential difference across $(3 \Omega + 4 \Omega)$

$$\Rightarrow V_{3\mu\text{F}} = (3 + 4)I = \frac{56}{3} \text{ V}$$

$$q = CV_{3\mu\text{F}} = \frac{3 \times 56}{3} = 56 \mu\text{C}$$

24. (4), 25. (3), 26. (4)

Given $V_1 = \frac{V_0}{k}$, $V_2 = \frac{V_1}{k}$, $V_3 = \frac{V_2}{k}$, $I = I_1 + I_2$



$$\frac{V_0 - V_1}{R_1} = \frac{V_1 - V_2}{R_1} + \frac{V_1 - 0}{R_2}$$

$$\frac{I_0 - V_0/k}{R_1} = \frac{V_0/k - V_0/k^2}{R_1} + \frac{V_0/k}{R_2}$$

$$\frac{R_1}{R_2} = \frac{(k-1)^2}{k}$$

Current in R_1 and R_2 will be the same:

$$\frac{I_{n-1} - I_n}{R_1} = \frac{I_n}{R_2}$$

or
$$\frac{I_{n-1} - \frac{I_{n-1}}{k}}{R_1} = \frac{I_{n-1}}{k R_2}$$

or
$$R_1 = R_2 (k-1)$$

Put the value of R_1 in the above expression, we get

$$\frac{R_2}{R_2} = \frac{k}{k-1}$$

Current in R_2 nearest to V_0 is

$$I_2 = \frac{V_1}{R_2} = \frac{V_0/k}{R_2 \left(\frac{k}{k-1} \right)} = \left(\frac{k-1}{k^2} \right) \frac{V_0}{R_2}$$

27. (2), 28. (2), 29. (4)

$$i_0 = 0.1 \text{ A}, E_2 = 4 \text{ V}, i_2 = 0$$

$$\text{As } 0.1 R_1 + 0.1 R_2 - E_1 = 0$$

$$0.1 R_2 - 4 \text{ V} = 0$$

$$R_2 = 40 \Omega$$

$$\text{Now } i_2 = 0.3 \text{ A}, i_1 = 0.1 \text{ A}, E_2 = 8 \text{ V}$$

$$\text{Now } 0.1 R_1 + E_1 - 8 = 0$$

$$0.1 + 4 - E_1 = 0$$

$$0.2 R_1 - 4 = 0$$

or
$$R_1 = \frac{4}{0.2} = 20 \Omega$$

$$0.2 E_1 = 2 + 4 = 6 \text{ V}$$

30. (4), 31. (3), 32. (2)

At steady state, the circuit will be simplified to the following circuit:

Using Kirchhoff's law, we can calculate $I = 1 \text{ A}$.

Further using Kirchhoff's law in original circuit, we can calculate charges on different capacitors.

33. (3), 34. (4), 35. (4)

$$\text{Given, } R_2 = 2R_1$$

$$\text{For } AO, 100 - V = i_1 R_1 \quad \dots(i)$$

$$\text{For } BO, 100 - V = i_2 (2R_1) \quad \dots(ii)$$

$$\text{For } OD, V - 0 = i_3 (20) \quad \dots(iii)$$

$$\text{For } OC, V - 50 = \left(i_1 + \frac{i_2}{2} - i_3 \right) (50) \quad \dots(iv)$$

$$VR_1 - 50R_1 = \left(\frac{3i_1}{2} \times 50 - i_3 \times 50 \right) R_1$$

From Eqs. (i) and (iii),

$$VR_1 = 50R_1 = 75(100 - V) - 2.5 VR_1$$

$$3.5 VR_1 - 50R_1 = 7500 - 75 V$$

If $V < 75 \text{ V}$, then RHS > 0 .

So, LHS > 0

$$3.5 V > 50 \quad (\text{since } R_1 \neq 0)$$

$$V > \frac{50}{3.5}$$

From Eq. (v), $V = 20 \text{ V}$

$$70 R_1 - 50 R_1 = 7500 - 1500 = 6000$$

$$R_1 = 300 \Omega$$

$$R_2 = 2R_1 = 600 \Omega$$

From Eq. (v),

$$3.5 VR_1 + 75 V = 7500 + 50 R_1$$

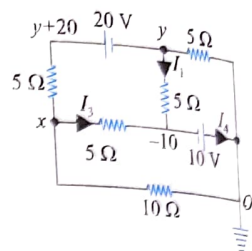
For any value of R_1 , $V > 0$.

36. (2), 37. (1)

$$\frac{x}{10} + \frac{x+10}{5} + \frac{x-y-20}{5} = 0$$

$$\frac{y}{5} + \frac{y+10}{5} + \frac{y+20-x}{5} = 0$$

On solving for x and y we get $x = 0$, $y = -10$. Hence $I_1 = 0$ and $I_3 = 2 \text{ A}$, i.e., I_4 will also be 2 A .



Matrix Match Type

1. i. $\rightarrow$ b., c.; ii. $\rightarrow$ a., b., c.; iii. $\rightarrow$ d.; iv. $\rightarrow$ a., b., c.

- Electrical conductivity of a conductor depends on temperature of the material. It decreases with the increase in temperature. It also depends upon the nature of material.
- Conductance (the reciprocal of resistance) also depends upon dimensions of material apart from temperature and nature of material.
- $J = \sigma E$, so current density depends upon electric field. For a given conductor, σ is fixed.
- Current will depend upon resistance, which depends upon dimensions, temperature, and nature of material.

2. i. $\rightarrow$ d.; ii. $\rightarrow$ d.; iii. $\rightarrow$ a.; iv. $\rightarrow$ b.

Let current through various branches is as shown in the figure.

$$i_1 + i_2 + i_3 = 0 \quad \dots(i)$$

$$V_A - V_B = V_D - V_C = V_E - V_F$$

$$1 - i_1 = 2 - 2i_2 = 3 - Ri_3 \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$i_1 + \frac{2-1+i_1}{2} + \frac{3-1+i_1}{R} = 0 \quad \text{or} \quad i_1 = \frac{-(R+4)}{3R+2}$$

Again from Eqs. (i) and (ii), we get

$$\frac{2i_2-1}{1} + i_2 + \frac{1+2i_2}{R} = 0 \quad \text{or} \quad i_2 = \frac{R-1}{3R+2}$$

$$\text{Now } i_3 = -i_1 - i_2 = \frac{5}{3R+2}$$

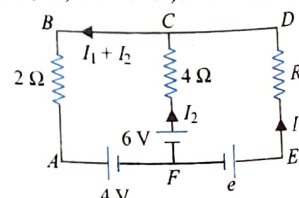
(i) Current i_3 cannot be zero.

(ii) Current i_1 cannot be zero.

(iii) $i_2 = 0 \Rightarrow R = 1 \Omega$

(iv) $i_3 = \frac{5}{8}$ or $\frac{5}{3R+2} = \frac{5}{8}$ or $R = 2 \Omega$

3. i. $\rightarrow$ b.; ii. $\rightarrow$ a., b., c.; iii. $\rightarrow$ b.; iv. $\rightarrow$ a., b., c., d.



Loop FEDCF: $e - 6 = RI_1 - 4I_2$... (i)
 Loop AFCBA: $6 - 4 = 4I_2 + 2(I_1 + I_2)$... (ii)
 $2 = 2I_1 + 6I_2$

or Solving them, we get

$$I_1 = \frac{3e - 14}{4 + 3R}, I_2 = \frac{R + 6 - e}{4 + 3R}$$

(i) $I_2 = 0$ or $e = R + 6$
 $e > 6 \text{ V}$ ($\because R \neq 0$)

(ii) For current from F to C direction,
 $I_2 > 0$ or $R + 6 > e$ or $e < R + 6$
 Possible for any finite value of e , because R is finite.

(iii) For current from C to F direction,
 $I_2 < 0$ or $e > R + 6$

(iv) For current in 2Ω from B to A direction,

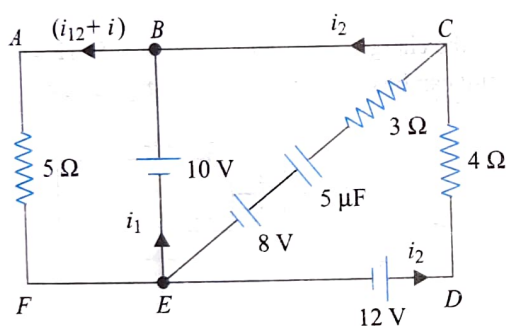
$$I_1 + I_2 = \frac{R - 8 + 2e}{4 + 3R} > 0$$

$$R - 8 + 2e > 0 \text{ or } e > 4 - \frac{R}{2}$$

Depending on the value of R , e can take any value from zero to infinity.

4. i. $\rightarrow$ c.; ii. $\rightarrow$ b.; iii. $\rightarrow$ b.; iv. $\rightarrow$ a.

When a steady state is reached, no current passes through the capacitor or the branch CE.



Considering the loop ABEFA

$$5 \times (i_1 + i_2) = 10 \text{ or } i_1 + i_2 = 2 \text{ A} \quad \dots (i)$$

Considering the loop BCDEB

$$4i_2 = 12 - 10 = 2 \text{ or } i_2 = 0.5 \text{ A}$$

$$\text{So, } i_1 = 2 - 0.5 = 1.5 \text{ A}$$

To find the charge on capacitors, we must know potential difference across the plates. Consider the loop CEDC:

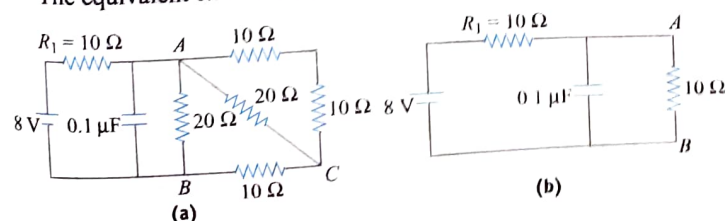
$$-12 + 4i_2 + 3 \times 0 - V_C + 8 = 0$$

$$\text{or } V_C = -2 \text{ V}$$

So charge on capacitor is $Q = CV = 10 \mu\text{C}$

5. i. $\rightarrow$ a.; ii. $\rightarrow$ c.; iii. $\rightarrow$ b.; iv. $\rightarrow$ d.

The equivalent circuit is shown in Fig. (b).



The current through R_1 is $8/20 = 0.4 \text{ A}$.

In the steady state, the potential difference across AB is 4 V.

Charge on capacitor in steady state is $q = CV = 0.4 \mu\text{C}$.

Current through resistor R is $I = \frac{V}{R} = \frac{4}{20} = 0.2 \text{ A}$

6. i. $\rightarrow$ c.; ii. $\rightarrow$ b.; iii. $\rightarrow$ b.; iv. $\rightarrow$ b.

Current through any cross section should remain the same.

$I = nev_d$, as area increases, drift velocity decreases.

E is directly proportional to v_d , so E also decreases.

If E decreases, then pd across a segment of fixed length also decreases.

7. i. $\rightarrow$ c.; ii. $\rightarrow$ c.; iii. $\rightarrow$ d.; iv. $\rightarrow$ b.

$$J = \sigma E$$

... (i)

Multiplying both sides of Eq. (i) by cross-sectional area of rod A , we get $JA = \sigma EA$ or $i = \sigma \phi$

$$\frac{\phi}{i} = \frac{1}{\sigma} = \text{resistivity of the rod}$$

$$\frac{E}{J} = \frac{1}{\sigma} = \text{resistivity of the rod}$$

$$\sigma V = iV = \text{power delivered to the rod}$$

$$\frac{V}{\sigma \phi} = \frac{V}{i} = \text{resistance of the rod.}$$

8. i. $\rightarrow$ a.; ii. $\rightarrow$ c.; iii. $\rightarrow$ d.; iv. $\rightarrow$ c.

Since current in both rods is the same,

$$n_1 e v_1 A_1 = n_2 e v_2 A_2$$

$$\text{or } \frac{v_1}{v_2} = \frac{n_2 A_2}{n_1 A_1} = \frac{1}{2} \times \frac{2}{1} = 1$$

$$E = \rho J = \rho \frac{1}{A}$$

$$\text{or } \frac{E_1}{E_2} = \frac{\rho_2}{\rho_1} = \frac{A_2}{A_1} = \frac{2}{1} \times \frac{2}{1} = 4$$

$$\frac{\text{pd across rod I}}{\text{pd across rod II}} = \frac{E_1 \times AB}{E_2 \times BC} = 4$$

$$\frac{\text{Average time taken by free electron to move from A to B}}{\text{Average time taken by free electron to move from B to C}}$$

$$= \frac{AB}{V_1} \times \frac{V_2}{BC} = 1$$

9. i. $\rightarrow$ a., b., c., d.; ii. $\rightarrow$ a., b., c., d.; iii. $\rightarrow$ d.; iv. $\rightarrow$ d.

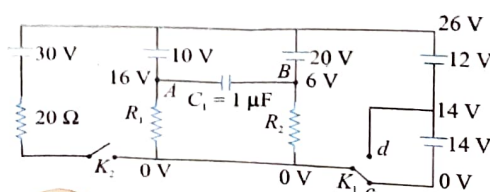
$$i = \frac{E}{\left(R_3 + \frac{R_1 R_2}{R_1 + R_2} \right)}$$

$$i_1 = i \left(\frac{R_2}{R_1 + R_2} \right) = \frac{E \left(\frac{R_2}{R_1 + R_2} \right)}{\left(R_3 + \frac{R_1 R_2}{R_1 + R_2} \right)}$$

$$i_2 = i \left(\frac{R_1}{R_1 + R_2} \right)$$

$$\frac{i_1}{i_2} = \frac{R_2}{R_1} \text{ and } \frac{i}{i_1} = \frac{i}{i \left(\frac{R_2}{R_1 + R_2} \right)} = \frac{R_1 + R_2}{R_2}$$

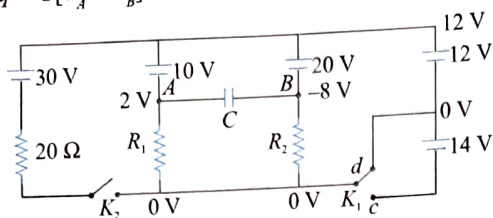
10. i. $\rightarrow$ a., b., d.; ii. $\rightarrow$ a., b., c., d.; iii. $\rightarrow$ a., b., d.; iv. $\rightarrow$ a., b., c., d.



Current through R_1 and R_2 should be downward

$$V_A > V_B$$

$$q = C[V_A - V_B] = 10^{-6}[16 - 6] = 10 \mu\text{C}$$

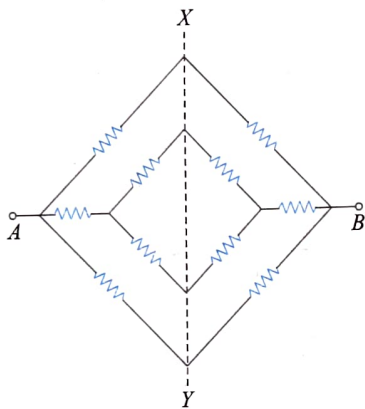


Now current in R_2 is upward.

(iii) and (iv) similar to (i) and (ii), respectively. Opening and closing of K_2 does not affect the above results.

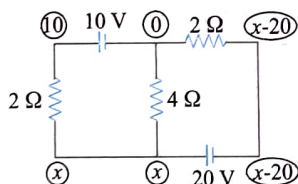
Numerical Value Type

1. (9) By applying perpendicular axis symmetry. Points lying on the line 'XY' will have same potential therefore, Resistance connected along line XY can be removed after which we can use series and parallel combination of resistances which gives $R_{AB} = 9 \Omega$.



2. (3) Distributing potentials at junctions of circuit as shown in figure, we write KCL equation for unknown potential x as

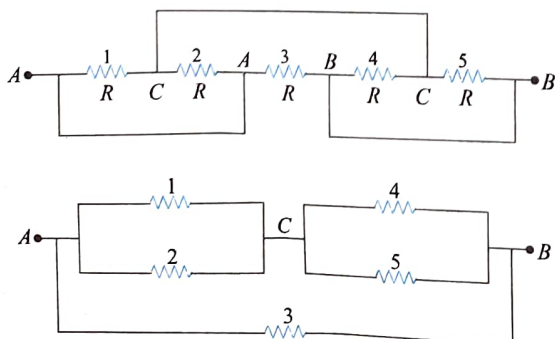
$$\frac{x}{4} + \frac{x-20}{2} + \frac{x-10}{2} = 0 \Rightarrow x = 12 \text{ V}$$



The current in 4Ω resistance is given as

$$I = \frac{12}{4} = 3 \text{ A}$$

3. (5) The circuit can be redrawn as shown in the figure.



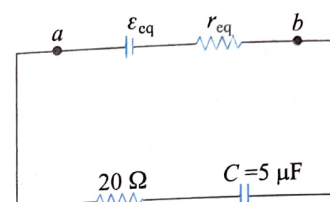
Now from simplify circuit we can observe that the resistances 1 and 2 are in parallel; similarly, the resistances 4 and 5 are in parallel. Equivalent of each pair is $\frac{R}{2}$. They add to become R which is in parallel to the resistance 3.

$$\text{Hence, } R_{eq} = \frac{R}{2} = \frac{10}{2} = 5 \Omega$$

4. (60) Three cells are in parallel combination, the equivalent emf of the three cells

$$\epsilon_{eq} = \frac{\frac{\epsilon_1}{r_1} + \frac{\epsilon_2}{r_2} + \frac{\epsilon_3}{r_3}}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}} = \frac{\frac{10}{1} + \frac{20}{2} + \frac{10}{1}}{\frac{1}{1} + \frac{1}{2} + \frac{1}{1}} = 12 \text{ V}$$

There will be no current through the branch having the capacitor

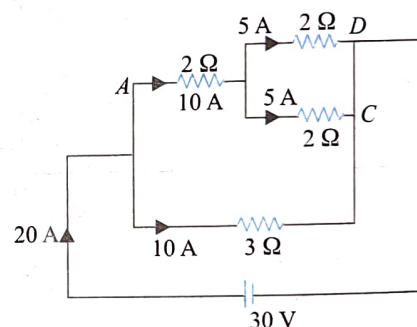


Hence, $V_a - V_b = 12$ volt.

It means the potential difference across the capacitor also should be 12 volt.

Hence the charge in capacitor, $q = CV = 5 \times 12 = 60 \mu\text{C}$

5. (15) Circuit can be redrawn as shown in figure below.



The equivalent resistance across the battery is given as

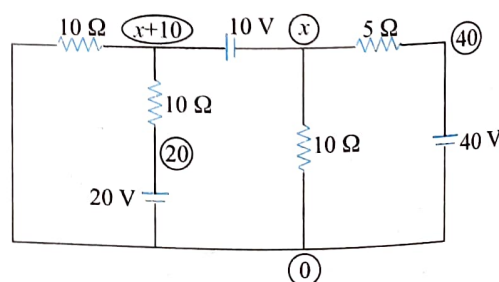
$$R_{eq} = \frac{3}{2} \Omega$$

$$\Rightarrow I = \frac{V}{R_{eq}} = 20 \text{ A}$$

Current in $I_{CD} = I_{AC} + I_B$

$$\Rightarrow I_{CD} = 15 \text{ A}$$

6. (4.80) Distributing potentials in circuit as shown in figure. Writing KCL equation for x , as



$$\frac{x-40}{5} + \frac{x}{10} + \frac{x+10-20}{10} = 0$$

$$5x = 80 \Rightarrow x = 16 \text{ V}$$

Current in $5\ \Omega$ resistance is given as

$$I_{5\Omega} = \frac{40-x}{5} = \frac{40-16}{5} = \frac{24}{5} = 4.8\text{ A}$$

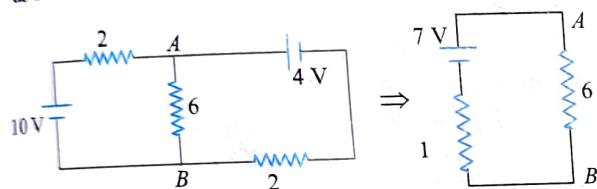
(9) By symmetry, points D and E have the same potential. Similarly, points C and F have the same potential.

$$r_{eq} = \frac{\frac{(3r)}{2}}{\frac{3r}{2} + r} = \frac{3r}{5} = \frac{3}{5} \times 5 = 3\ \Omega$$

(11) Current through the battery will be minimum when all the three switches are open. The resulting circuit is shown in figure. Hence,

$$V_{AB} = (0.5)V_{AB} \times \frac{24}{12} = 1\text{ V}$$

(12) 9 V and $16\ \Omega$ can be ignored because potential difference between A and C is fixed, which is 4 V. The circuit can be drawn as follows:

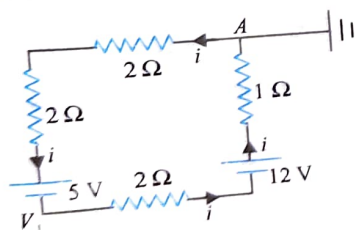


$$\text{Equivalent emf is } e = \frac{\Sigma(E/r)}{\Sigma(1/r)} = \frac{10/2 + 4/2}{1/2 + 1/2} = 7\text{ V}$$

$$\text{Equivalent internal resistance is } r_0 = \frac{1}{\Sigma(1/r)} = \frac{1}{1/2 + 1/2} = 1\ \Omega$$

$$\text{Now } V_{AB} = \frac{7}{(6+1)} \times 6 = 6\text{ V}$$

(13) The resulting circuit can be drawn as shown. No current will flow into the earth.



$$i = (12-5)/7 = 1\text{ A}, V_A - 2i - 2i - 5 = V_1; \text{ but } V_A = 0,$$

$$\text{so } V_1 = -9\text{ V}$$

(14) (2) As according to Avogadro's hypothesis,

$$\frac{N}{N_A} = \frac{m}{M} \text{ or } n = \frac{N}{V} = N_A \frac{m}{VM} = \frac{N_A d}{M} \left[\text{as } d = \frac{m}{V} \right]$$

$$\text{Electron density } n = \frac{6 \times 10^{23} \times (5 \times 10^{-3})}{(60 \times 10^{-3})} = 5 \times 10^{28}\text{ m}^{-3}$$

Now as each atom contributes one electron, the number of electrons per unit volume is

$$n_e = 1 \times n = 1 \times 5 \times 10^{28} = 5 \times 10^{28}\text{ m}^{-3}$$

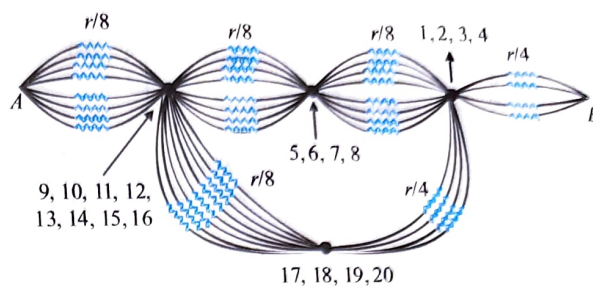
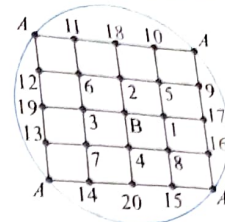
$$\text{Further } J = \frac{i}{A} = \frac{16}{10^{-6}} = 16 \times 10^6\text{ Am}^{-2}$$

$$J = nev_d \text{ or } v_d = \frac{J}{ne} = \frac{16 \times 10^6}{2 \times 10^{28}} = 8 \times 10^{-3}\text{ m s}^{-1}$$

12. (6) If we connect battery between A and B, then different points will be at the same potentials.

(1, 2, 3, 4), (5, 6, 7, 8),
(9, 10, 11, 12, 13, 14, 15, 16),
and (17, 18, 19, 20)

The resulting circuit can be drawn as follows:



$$R_{AB} = \frac{21}{40} r = \frac{21}{40} \times \frac{80}{7} = 6\ \Omega$$

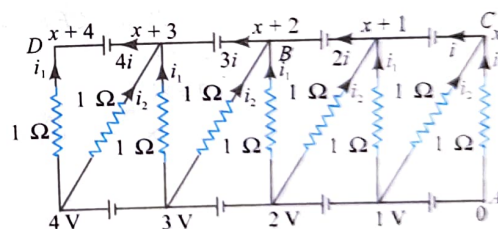
13. (8) Due to symmetry, the line of resistors passing through b will be useless. So

$$r_{eq} = 2r + \frac{2rr_{eq}}{2r + r_{eq}} = (1 + \sqrt{5})r = (\sqrt{5} + 1)2(\sqrt{5} - 1) = 8\ \Omega$$

14. (9) At junction C: $i_1 + i_2 = i$,

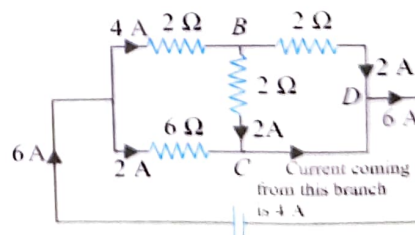
$$\text{where } i_1 = \frac{0-x}{1}, i_2 = \frac{1-x}{1}$$

$$\text{At D: } 4i + i_1 = 0, \text{ solve to get } x = \frac{4}{9}$$



$$\text{Now } V_A - i_1 + 1 + 1 = V_B \text{ or } V_A - V_B = -\left(2 + \frac{4}{9}\right) = -\frac{22}{9}\text{ V}$$

15. (4) $R_{eq} = 2\ \Omega$; $i = 12/2 = 6\text{ A}$



Current through branch CD = 4 A

16. (10) Potential difference across each of the resistors will be zero; hence, no current in any resistor.

$$17. (2) x = \frac{(2R+x)R}{3R+x}$$

$$3Rx + x^2 = 2R^2 + Rx$$

$$x^2 + 2Rx - 2R^2 = 0$$

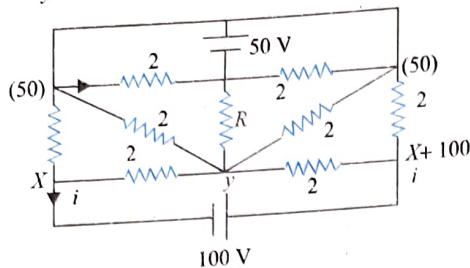
$$x = \frac{-2R \pm \sqrt{4R^2 + 8R^2}}{2} = R(\sqrt{3} - 1)$$

$$= (\sqrt{3} + 1)(\sqrt{3} - 1) = 3 - 1 = 2\ \Omega$$

18. (2) At y according to Kirchhoff's junction law,

$$\frac{y-x}{r} + \frac{y-x-100}{r} + \frac{y-50}{r} + \frac{y}{r} + \frac{y-50}{r} = 0 \quad \dots(i)$$

$$5y - 2x = 200$$



Similarly at x , $\dots(ii)$

$$i = \frac{50-x}{r} + \frac{y-x}{r}$$

At $x+100$, $\dots(iii)$

$$i = \frac{x+100-50}{r} + \frac{x+100-y}{r}$$

From Eqs. (ii) and (iii), we get $\dots(iv)$

$$y - 2x = 50$$

From Eqs. (i) and (iv), $y = 37.5$ V.

So, current through R is $37.5/18.75 = 2$ A.

19. (4) In steady state current flows in upper and lower loops of the circuit which are i_1 and i_2 respectively then we have

$$i_1 = \frac{10}{3+2} = 2A \text{ and } i_2 = \frac{20}{4+6} = 2A$$

Potential differences across points BG and CF are given as

$$V_{BG} = V_B - V_G = 3i_1 = 3 \times 2 = 6V$$

and $V_{CF} = V_C - V_F = 4i_2 = 4 \times 2 = 8V$

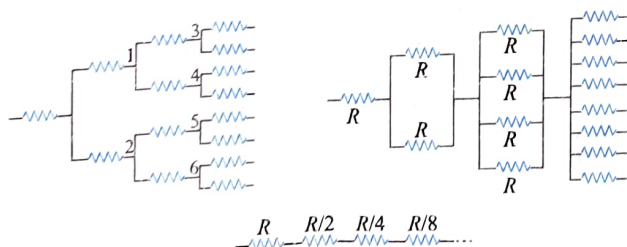
Charge on both the capacitors will be same as sum of charges on top and bottom plates of both the capacitors must be zero as initially these are uncharged. If charge on capacitors is q then we write the equation of potential drop in loop $BGFC$ as

$$-6 - \frac{q}{6 \times 10^{-6}} + 8 - \frac{q}{3 \times 10^{-6}} = 0$$

$$\Rightarrow \frac{(10^6)q}{2} = 2 \Rightarrow q = 4 \times 10^{-6} C$$

$$\Rightarrow q = 4 \mu C$$

20. (4) The points 1 and 2 are equipotential. They can be connected together. Similarly, the points 3, 4, 5 and 6 are equipotential. We can join them together. And so on. Finally the circuit reduce to a simple circuit



$$\therefore \text{Equivalent is } R_{eq} = R \left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \right]$$

$$= R \left[\frac{1}{1 - \frac{1}{2}} \right] = 2R = 2 \times 2 = 4\Omega$$

Single Correct Answer Type

$$1. (3) U = \frac{1}{2} \frac{q^2}{C} = \frac{1}{2C} (q_0 e^{-t/T})^2 = \frac{q_0^2}{2C} e^{-2t/T} \text{ (where } \tau = CR)$$

$$= U_i e^{-2t/\tau}$$

$$\Rightarrow \frac{1}{2} U_i = U_i e^{-2t_1/\tau} \Rightarrow \frac{1}{2} = e^{-2t_1/\tau} \Rightarrow t_1 = \frac{\tau}{2} \log_e 2$$

$$\text{Now } q = q_0 e^{-t/T} \Rightarrow \frac{1}{4} q_0 = q_0 e^{-t/2T}$$

$$\Rightarrow t_2 = T \log_e 4 = 2T \log_e 2$$

$$\therefore \frac{t_1}{t_2} = \frac{1}{4}$$

2. (4) Let R_0 be the initial resistance of both conductors.

At temperature θ , their resistance will be

$$R_1 = R_0(1 + \alpha_1\theta) \text{ and } R_2 = R_0(1 + \alpha_2\theta)$$

For series combination,

$$R_s = R_1 + R_2$$

$$R_{s0}(1 + \alpha_s\theta) = R_0(1 + \alpha_1\theta) + R_0(1 + \alpha_2\theta),$$

where $R_{s0} = R_0 + R_0 = 2R_0$

$$2R_0(1 + \alpha_s\theta) = 2R_0 + R_0\theta(\alpha_1 + \alpha_2)$$

$$\Rightarrow \alpha_s = \frac{\alpha_1 + \alpha_2}{2}$$

For parallel combination,

$$R_p = \frac{R_1 R_2}{R_1 + R_2}$$

$$R_{p0}(1 + \alpha_p\theta) = \frac{R_0(1 + \alpha_1\theta) R_0(1 + \alpha_2\theta)}{R_0(1 + \alpha_1\theta) + R_0(1 + \alpha_2\theta)} = \frac{R_0 R_0}{R_0 + R_0} = \frac{R_0}{2}$$

where

$$R_{p0} = \frac{R_0 R_0}{R_0 + R_0} = \frac{R_0}{2}$$

$$\frac{R_0}{2}(1 + \alpha_p\theta) = \frac{R_0^2(1 + \alpha_1\theta + \alpha_2\theta + \alpha_1\alpha_2\theta)}{R_0(2 + \alpha_1\theta + \alpha_2\theta)}$$

α_1 and α_2 are small quantities. So $\alpha_1\alpha_2$ is negligible. Hence.

$$\alpha_p = \frac{\alpha_1 + \alpha_2}{2 + (\alpha_1 + \alpha_2)\theta} = \frac{\alpha_1 + \alpha_2}{2} [1 - (\alpha_1 + \alpha_2)\theta]$$

$(\alpha_1 + \alpha_2)^2$ is negligible. So $\alpha_p = \frac{\alpha_1 + \alpha_2}{2}$

$$3. (3) V = 200(1 - e^{-t/\tau})$$

$$120 = 200(1 - e^{-t/\tau})$$

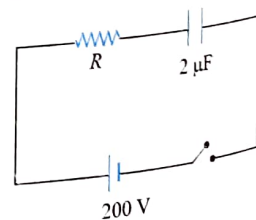
$$\Rightarrow e^{-t/\tau} = \frac{200 - 120}{200} = \frac{80}{200}$$

$$\frac{t}{\tau} = \log_e 2.5$$

$$= 2.302 \times \log_{10} 2.5 = 2.302 \times 0.4 = 0.9210$$

$$5 = (0.9210) \times R \times 2 \times 10^{-6}$$

$$\Rightarrow R = 2.7 \times 10^6 \Omega$$



$$4. (2) R = \frac{\rho l}{A}$$

By differentiation

$$0 = l dA + A dl$$

By differentiation

($\because V = AI$ constant)

$\dots(i)$

$$dR = \frac{\rho(Adl - IdA)}{A^2}$$

$$dR = \rho \frac{2Adl}{A^2}$$

$$dR = \frac{2\rho dl}{A}$$

$$\text{or } \frac{dR}{R} = 2 \cdot \frac{dl}{l}$$

$$\text{So, } \frac{dR}{R} \% = 2 \cdot \frac{dl}{l} \% = 2 \times 0.1\%$$

$$\frac{dR}{R} \% = 0.2\%$$

$$(4) Q = c\epsilon_0 e^{-t/\tau}$$

$$4\epsilon = 4\epsilon_0 e^{-t/\tau}$$

$$\text{When } t = 0 \Rightarrow \epsilon_0 = 25$$

$$\epsilon = \epsilon_0 = 25$$

$$\text{When } t = 200 \Rightarrow \epsilon = 5$$

$$5 = 25 e^{-\frac{200}{\tau}}$$

$$\ln 5 = \frac{200}{\tau}$$

$$\tau = \frac{200}{\ln 5} = \frac{200}{\ln 10 - \ln 2} = \frac{200}{\ln 10 - 0.693}$$

Alternative

Time constant is the time in which 63% discharging is completed.

$$\text{So remaining charge} = 0.37 \times 25 = 9.25 \text{ V}$$

Which time in $100 < t < 150 \text{ sec}$.

6. (2) Charge on the capacitor at any time 't' is

$$q = CV(1 - e^{-t/\tau})$$

$$\text{At } t = 2\tau$$

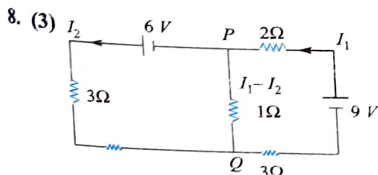
$$q = CV(1 - e^{-2})$$

$$7. (4) V = IR = I\rho \frac{l}{A}$$

$$\Rightarrow \rho = \frac{VA}{Il} = \frac{VA}{\ln e A v_d} = \frac{V}{l \times n \times e \times v_d}$$

$$\Rightarrow \rho = \frac{5}{0.1 \times 2.5 \times 10^{-19} \times 1.6 \times 10^{-19} \times 8 \times 10^{28}}$$

$$= 1.6 \times 10^{-5} \Omega \text{ m}$$



From KVL,

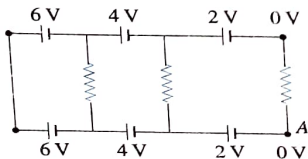
$$9 = 6I_1 - I_2$$

$$6 = 4I_2 - I_1$$

$$\text{Solving, } I_1 - I_2 = -0.13 \text{ A}$$

Therefore, 0.13A current will flow from Q to P.

9. (2)



...(ii)

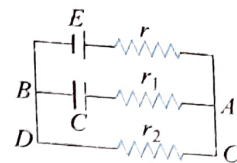
Taking voltage of point A as = 0. Then voltage at other points can be written as shown in the figure. Hence voltage across all resistance is zero. Hence current = 0.

10. (1) In steady state, current through AB = 0

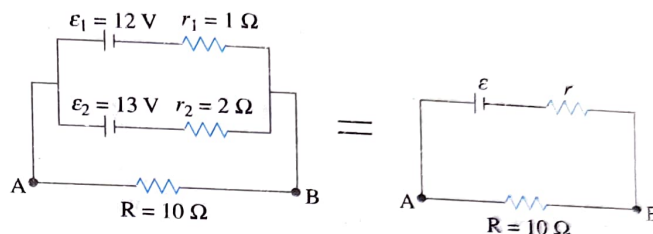
$$\Rightarrow V_{AB} = V_{CD}$$

$$\Rightarrow V_{AB} = \left(\frac{\epsilon}{r + r_2} \times r_2 \right) - V_{CD}$$

$$\Rightarrow Q_C = CV_{AB} = CE \left(\frac{r_2}{r + r_2} \right)$$



11. (3)



Electric emf of the batteries connected in parallel

$$\epsilon = \frac{\frac{\epsilon_1}{r_1} + \frac{\epsilon_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}} = \frac{\frac{12}{1} + \frac{13}{2}}{\frac{1}{1} + \frac{1}{2}} = \frac{37}{3} \text{ V}$$

and effective internal resistance of the combination

$$r = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2}} = \frac{2}{3} \Omega$$

Potential difference across load resistance

$$V_{AB} = IR = \frac{\epsilon}{(r + R)} \times R$$

$$\text{or } V_{AB} = \left(\frac{37/3}{2/3 + 10} \right) \times 10 = \frac{37}{32} \times 10 = 11.56 \text{ V}$$

Hence, option (3) should be correct.

JEE Advanced

Single Correct Answer Type

$$1. (3) \text{ We see that } R = \frac{\rho L}{Lt}$$

$$2. (2) R_{Al} = \frac{\rho l}{A} = \frac{2.7 \times 10^{-8} \times 50 \times 10^{-3}}{(7^2 - 2^2) \times 10^{-6}} = 30 \mu\Omega$$

$$R_{Fe} = \frac{1 \times 10^{-7} \times 50 \times 10^{-3}}{4 \times 10^{-6}} = \frac{5000}{4} \mu\Omega$$

Both the bars are in parallel.

$$R_{eq} = \frac{R_{Al} \times R_{Fe}}{R_{Al} + R_{Fe}} = \frac{1875}{64} \mu\Omega$$

3. (1) This is the problem of RC circuit where the product RC is a constant.

So due to leakage current, charge and current density will exponentially decay and will become zero at infinite time. So correct answer is (1) for any small element.

$$\text{Resistance } R = \frac{dr}{\sigma(2\pi rl)}$$

$$\text{Capacitance } C = \frac{\epsilon 2\pi rl}{dr}$$

$$\text{Product } R \times C = \frac{\epsilon}{\sigma} = \text{constant}$$

$$q = q_0 e^{-\left(\frac{r\sigma}{\epsilon}\right)}$$

$$I = \frac{dq}{dt} = \frac{q_0 \sigma}{\epsilon} e^{-\left(\frac{r\sigma}{\epsilon}\right)}$$

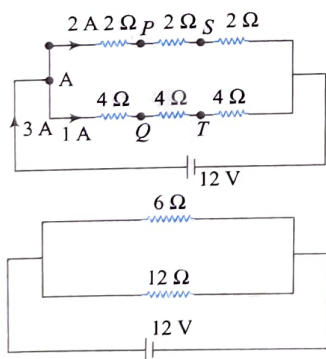
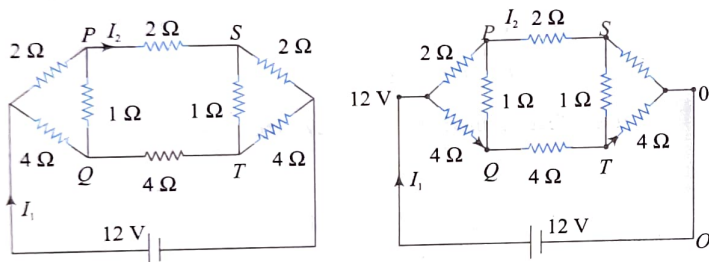
$$\text{Current density } = \frac{I}{A} = \frac{q_0 \sigma}{2\pi r l} e^{-\frac{r\sigma}{\epsilon}}$$

$$j \propto e^{-\frac{r\sigma}{\epsilon}}$$

Multiple Correct Answers Type

1. (1), (2), (3), (4)

Due to input and output symmetry P and Q and S and T have same potential.



$$R_{eq} = \frac{6 \times 12}{18} = 4 \Omega$$

$$I_1 = \frac{12}{4} = 3 \text{ A}$$

$$I_2 = \left(\frac{12}{6+12} \right) \times 3$$

$$I_2 = 2 \text{ A}$$

$$V_A = V_S = 2 \times 4 = 8 \text{ V}$$

$$V_A = V_T = 1 \times 8 = 8 \text{ V}$$

$$V_P = V_Q \Rightarrow \text{Current through } PQ = 0$$

$$V_P = V_Q \Rightarrow V_Q > V_S$$

$$I_1 = 3 \text{ A}$$

$$I_2 = 2 \text{ A}$$

...(i)

...(ii)

...(iii)

...(iv)

2. (1), (2), (4)

From loop (1)

$$-iR_1 - i_1 R_2 + V_1 = 0$$

$$\Rightarrow iR_1 + i_1 R_2 = V_1$$

...(i)

From loop (2)

$$i_1 R_2 - (i - i_1) R_3 = -V_2$$

$$i_1 (R_2 + R_3) - i R_3 = V_2$$

...(ii)

Solving equation (i) and (ii)

$$i_1 (R_2 + R_3) - i R_3 = -V_2 \times R_1$$

$$i_1 R_2 + i R_1 = V_1 \times R_3$$

$$i_1 (R_1 R_2 + R_1 R_3 + R_2 R_3)$$

$$= V_1 R_3 - V_2 R_1$$

$$\text{We get } i_1 = \frac{V_1 R_3 - V_2 R_1}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

If i_1 is zero

$$V_1 R_3 = V_2 R_1$$

It is possible in following options

(a) $V_1 = V_2 \Rightarrow R_1 = R_3$

(b) $V_1 = V_2 \Rightarrow R_1 = R_3$

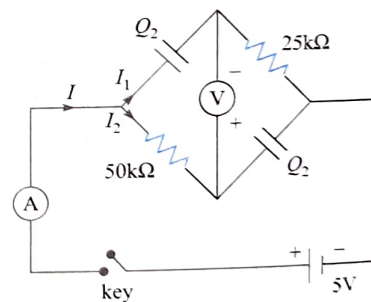
(d) $2V_1 = V_2 \Rightarrow R_3 = 2R_1$

3. (1), (2), (3), (4)

$$q_1 = (200 \times 10^{-3}) \left[1 - e^{-\frac{t}{1}} \right]; q_2 = (100 \times 10^{-3}) \left[1 - e^{-\frac{t}{1}} \right]$$

$$\frac{q_1}{C} = (50 \times 10^3) \frac{dq_2}{dt}$$

$$\frac{(200 \times 10^{-3})(1 - e^{-t})}{40 \times 10^{-6}} = (50 \times 10^3)(100 \times 10^{-3})[e^{-t}]$$



$$(1 - e^{-t}) \frac{10^{-6}}{20} = 50 \times 10^3 (e^{-t})$$

$$\frac{1}{2} = e^{-t}$$

$$t = \ln 2$$

$$I = I_1 + I_2 = (200 \times 10^{-3})(e^{-t}) + (100 \times 10^{-3}) e^{-t}$$

$$= 100 \times 10^{-3} [2e^{-t} + e^{-t}]$$

$$= (300 \times 10^{-3}) e^{-1}$$

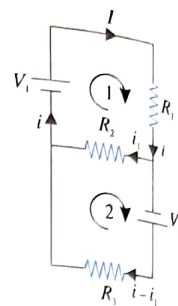
$$= \left(\frac{300 \times 10^{-3}}{e} \right)$$

At $t = \infty$, $I = 0$.

Linked Comprehension Type

1. (2), 2. (1)

$$\frac{kq}{r} = V_0 = q = \frac{V_0 r}{k}$$



$$\frac{1}{2} \left(\frac{qE}{m} \right) t^2 = h$$

$$\frac{1}{2} \frac{V_0 r}{k} \frac{2V_0}{hm} t^2 = h$$

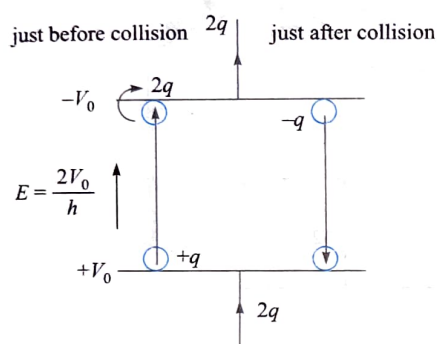
$$t^2 = \frac{h^2}{V_0^2} \frac{mk}{r}$$

$$t = \frac{h}{V_0} \sqrt{\frac{mk}{r}}$$

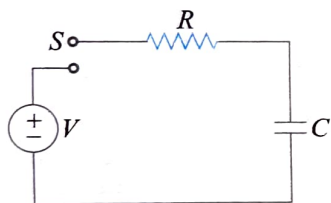
During every collision $2q$ charge will flow from circuit.

$$\text{Average current } I_{\text{avg}} = \frac{2q}{t} = \frac{2V_0^2}{h} \frac{r\sqrt{r}}{mk\sqrt{k}}$$

Hence the balls will bounce back to the bottom plate carrying the opposite charge they went up with $I_{\text{avg}} \propto V_0^2$.



3. (1)



When switch is closed for a very long time capacitor will get fully charged and charge on capacitor will be $q = CV$.

$$\text{Energy stored in capacitor } E_C = \frac{1}{2} CV^2 \quad \dots(i)$$

Work done by battery (W) = $Vq = VCV = CV^2$
dissipated across resistance $e_D = (\text{work done by battery}) - (\text{energy store})$

$$E_D = CV^2 - \frac{1}{2} CV^2 = \frac{1}{2} CV^2 \quad \dots(ii)$$

from (i) and (ii), $E_D = E_C$

4. (1) For time $t = 0$ to $t = T$ (say step-1)

$$\text{Charge on capacitor} = \frac{CV_0}{3}$$

$$\text{Energy stored in capacitor} = \frac{1}{2} C \frac{V_0^2}{9} = \frac{CV_0^2}{18}$$

$$\text{Work done by battery} = \frac{CV_0}{3} \times \frac{V}{3} = \frac{CV_0^2}{9}$$

$$\text{Heat loss} = \frac{CV_0}{9} - \frac{CV_0^2}{18} = \frac{CV_0^2}{18}$$

For time $t = T$ to $t = 2T$ (say step-2)

$$\text{Charge on capacitor} = \frac{2CV_0}{3}$$

$$\text{Extra charge flow through battery} = \frac{CV_0}{3}$$

$$\text{Work done by battery: } \frac{CV_0}{3} \cdot \frac{2V_0}{3} = \frac{2CV_0^2}{9}$$

$$\text{Final energy stored in capacitor: } \frac{1}{2} C \left(\frac{2V_0}{3} \right)^2 = \frac{4CV_0^2}{18}$$

$$\text{Energy stored in step-2: } \frac{4CV_0^2}{18} - \frac{CV_0^2}{18} = \frac{3CV_0^2}{18}$$

Heat loss in step-2 = work done by battery in step-2 - energy stored in capacitor in step-2

$$= \frac{2CV_0^2}{9} - \frac{3CV_0^2}{18} = \frac{CV_0^2}{18}$$

For time $2T$ onwards (say step-3)

$$\text{Charge on capacitor} = CV_0$$

$$\text{Extra charge flow through battery: } CV_0 - \frac{2CV_0}{3} = \frac{CV_0}{3}$$

$$\text{Work done by battery in this step: } \left(\frac{CV_0}{3} \right) (V_0) = \frac{CV_0^2}{3}$$

$$\text{And energy stored in capacitor: } \frac{1}{2} CV_0^2$$

$$\text{Energy stored in this step: } \frac{1}{2} CV_0^2 - \frac{4CV_0^2}{18} = \frac{5CV_0^2}{18}$$

$$\text{Heat loss in step-3: } \frac{CV_0^2}{3} - \frac{5CV_0^2}{18} = \frac{CV_0^2}{18}$$

$$\text{Now total heat loss } (E_D): \frac{CV_0^2}{18} + \frac{CV_0^2}{18} + \frac{CV_0^2}{18} = \frac{CV_0^2}{6}$$

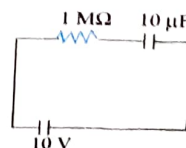
$$\text{Final energy stored in capacitor: } \frac{1}{2} CV_0^2$$

$$\text{So we can say that } E_D = \frac{1}{3} \left(\frac{1}{2} CV_0^2 \right)$$

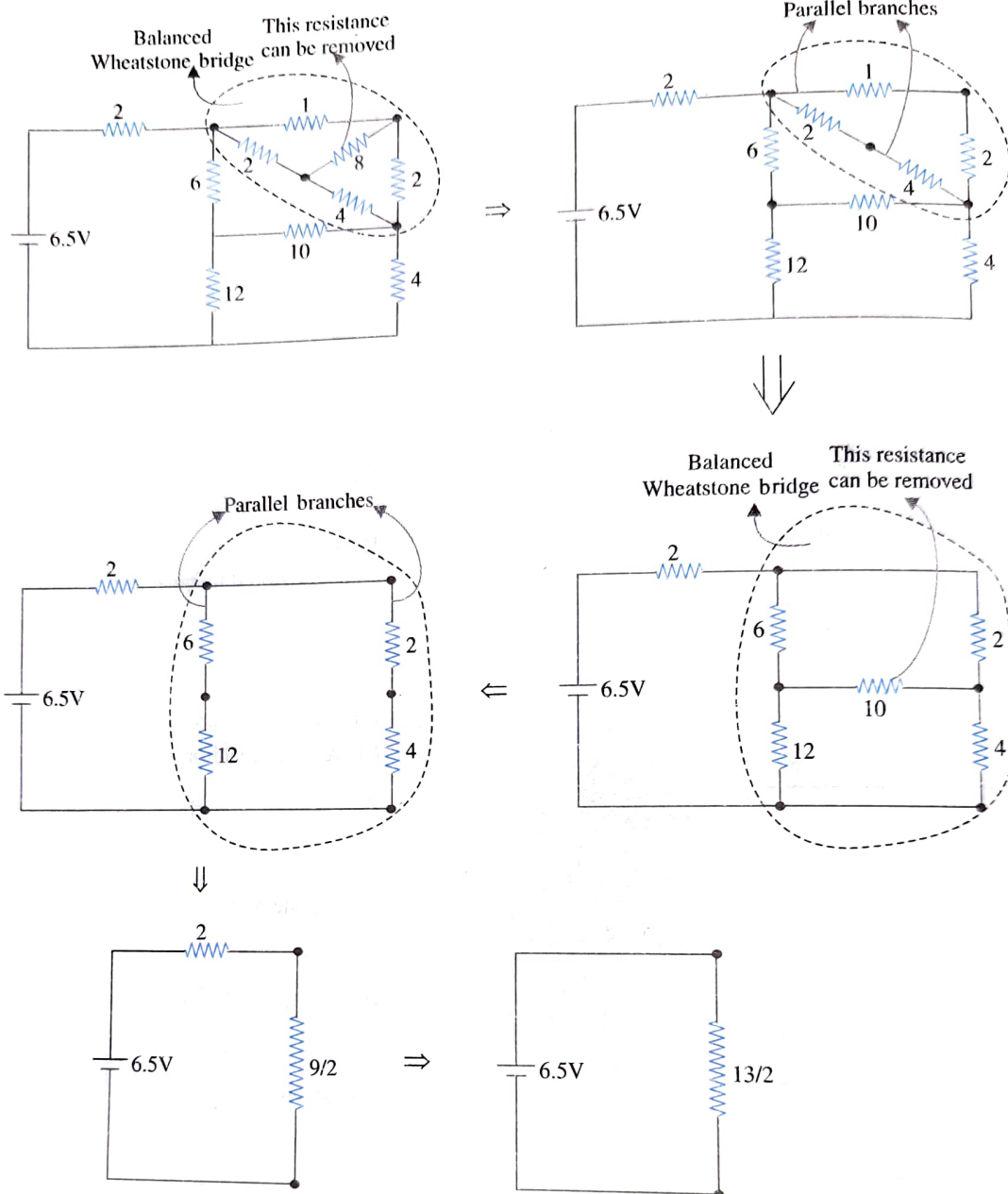
Numerical Value Type

$$1. (2) 4 = 10 (1 - e^{-t/4})$$

$$\therefore t = 2 \text{ s}$$



2. (1)



$\therefore$ Hence current through battery $I = \frac{6.5}{(13/2)} = 1$ amp

Chapter 6

Concept Application Exercises

Exercise 6.1

1. As shown in figure, the voltmeter is in series with R , so $V = V_1 + V_R$, i.e.,
 $110 = 5 + V_R$

i.e., $V_R = 110 - 5 = 105 \text{ V}$

And as in series, potential divides in proportion to resistance, i.e.,

$$\frac{V_R}{V_1} = \frac{R}{R_1} \text{ or } \frac{105}{5} = \frac{R}{20 \text{ k}\Omega} \text{ or } R = 420 \text{ k}\Omega$$

2. Given, $G = 12.0 \Omega$, $I_g = 2.5 \text{ mA} = 2.5 \times 10^{-3} \text{ A}$

(a) $I = 7.5 \text{ A}$, $S = ?$

$$S = G \times \frac{I_g}{I - I_g} = 12 \times \frac{2.5 \times 10^{-3}}{7.5 - 2.5 \times 10^{-3}} \Omega \approx 4.0 \times 10^{-3} \Omega$$

(b) $V = 10.0 \text{ V}$, $R = ?$

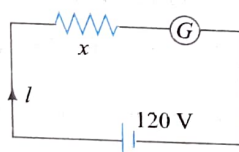
$$\therefore R = \frac{V}{I_g} - G = \frac{10.0}{2.5 \times 10^{-3}} - 12.0 = 3988 \Omega$$

3. We have $I_2 = 1.00 \text{ mA} = 1.00 \times 10^{-3} \text{ A}$ and $R_2 = 20.0 \Omega$,
 and the ammeter should be able to handle maximum current,
 $I = 50.0 \times 10^{-3} \text{ A}$.

Solving equation $I_g R_g = (I - I_g) R_s$ for R_s , we get

$$R_s = \frac{I_g R_g}{I - I_g} = \frac{(1.00 \times 10^{-3} \text{ A})(20.0 \Omega)}{50.0 \times 10^{-3} \text{ A} - 1.00 \times 10^{-3} \text{ A}} = 0.408 \Omega$$

4. We should connect this galvanometer in series with the given resistor. In general, resistance of galvanometer is small, so it can be neglected. If I is the current in circuit, then $R = 120/I$.



$$I_{\text{max}} = 40 \times 10^{-6} \text{ A}, R_{\text{min}} = \frac{120}{40 \times 10^{-6}} = 3 \text{ M}\Omega$$

5. No current will flow through voltmeter. As it is ideal (infinite resistance). Current through two batteries

$$i = \frac{1.5 - 1.3}{r_1 + r_2} = \frac{0.2}{r_1 + r_2}$$

$$\text{Now, } V = E_2 - ir_2$$

$$1.45 = 1.5 - \left(\frac{0.2}{r_1 + r_2} \right) (r_2)$$

Solving this equation, we get, $r_1 = 3r_2$

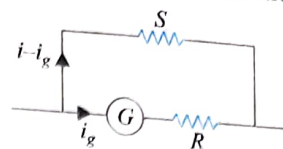
6. In series, PD distributes in direct ratio of resistance
 In first case,

$$\frac{198}{V_{AB} - 198} = \frac{900}{R_1} \quad \dots (i)$$

$$\text{In second case, } \frac{180}{V_{AB} - 180} = \frac{900}{2R_1} \quad \dots (ii)$$

Solving these two equations, we get $V_{AB} = 220 \text{ V}$

7. In parallel current distributes in inverse ratio of resistance.



$$\therefore \left(\frac{i - i_g}{i_g} \right) = \frac{G + R}{S}$$

$$\therefore R = \left(\frac{i - i_g}{i_g} \right) S - G = \left(\frac{20}{10^{-3}} \right) (0.005) - 20 = 80 \Omega$$

8. All these resistances are in parallel.

$$\therefore R_{\text{net}} = \frac{R}{3} + r = 4 \Omega$$

$$\text{Hence, the main current } i = \frac{E}{R_{\text{net}}} = 1 \text{ A}$$

Current through either of the resistance is

$$\frac{i}{3} \text{ or } \frac{1}{3} \text{ A}$$

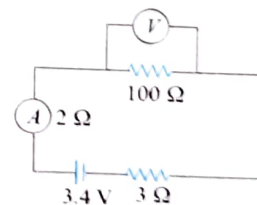
$$\therefore V = iR = \left(\frac{1}{3} \right) (9) = 3 \text{ V}$$

9. Total resistance of the circuit, $R_{AB} = 60 + \frac{(60)(120)}{60 + 120} = 100 \Omega$

$$\text{The current through the circuit, } i = \frac{120}{100} = 1.2 \text{ A}$$

$$\text{Now, reading of voltmeter} = 120 - \text{potential drop across } R_1 = 120 - (60)(1.2) = 48 \text{ V}$$

10. Reading of voltmeter = 3.4 - Voltage drop across ammeter and 3Ω resistance
 $= (3.4) - 0.04 \times 2 - 0.04 \times 3 = 3.2 \text{ V}$



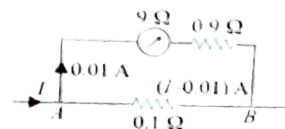
$$\text{Now, } 3.2 \text{ V} = \frac{(0.04)(100 R)}{(100 + R)} \text{ where, } R = \text{resistance of voltmeter}$$

$$\therefore R = 400 \Omega$$

$$\text{If voltmeter is ideal, then } i = \frac{3.4}{2 + 100 + 3} = 0.03238 \text{ A}$$

$$\text{Reading of voltmeter} = 100i = 3.238 \text{ V}$$

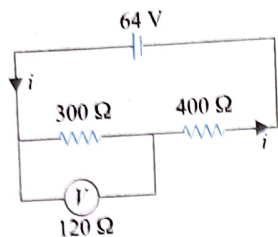
11. In parallel, current distributes in inverse ratio of resistance.



$$\frac{0.01}{I - 0.01} = \frac{0.1}{9 + 0.9}$$

Solving we get, $I = 1 \text{ A}$

2. Voltmeter reads 32 V, half of 64 V. Hence, resistance of 400Ω and voltmeter is also equal to 300Ω .



$$300 = \left(\frac{400 \times R}{400 + R} \right)$$

where R = resistance of voltmeter.

Solving the above equation, we get $R = 1200 \Omega$

In the new situation,

$$R_{\text{net}} = 400 + \frac{(300)(1200)}{300 + 1200} = 640 \Omega$$

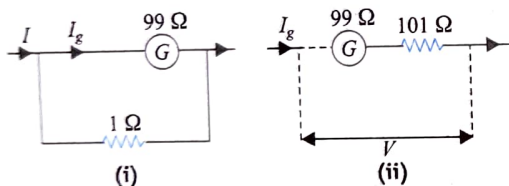
$$\therefore i = \frac{64}{640} = 0.1 \text{ A}$$

The voltage drop across voltmeter = $64 - \text{potential drop across } 400 \Omega \text{ resistor}$

$$= 64 - (400)i = 64 - (400)(0.1) = 24 \text{ V}$$

13. For ammeter $99I_g = (I - I_g)l$

$$\Rightarrow I = 100I_g \quad \dots(i)$$



I_g is the full-scale deflection current of the galvanometer and I the range of ammeter.

For the circuit in Fig. (i), given in the question

$$\frac{12V}{2 + r + \frac{99 \times 1}{99 + 1}} = 3 \text{ A} \Rightarrow r = 1.01 \Omega$$

For voltmeter, range

$$V = I_g(99 + 101) \Rightarrow V = 200I_g \quad \dots(ii)$$

Also resistance of the voltmeter = $99 + 101 = 200 \Omega$

In Fig. (ii), resistance across the terminals of the battery

$$R_1 = r + \frac{200 \times 2}{200 + 2} = 2.99 \Omega$$

$$\therefore \text{Current drawn from the battery, } I_1 = \frac{12}{2.99} = 4.01 \text{ A}$$

$\therefore$ Voltmeter reading

$$\frac{4}{5} V = 12 - I_1 r = 12 - 4.01 \times 1.01$$

$$V = 7.96 \times \frac{5}{4} = 9.95 \text{ V}$$

$$\text{Using Eq. (ii), } I_g = \frac{9.95}{200} = 0.05 \text{ A}$$

Using Eq. (i), range of the ammeter, $I = 100I_g = 5 \text{ A}$

Range of voltmeter $V = 200 \times 0.05 = 10 \text{ V}$

14. (i) Let resistance of the voltmeter be $R \text{ ohm}$. The equivalent resistance of voltmeter ($R \text{ ohm}$) and 100Ω in parallel is

$$R_{BC} = \frac{100 \times R}{100 + R} = \frac{100R}{100 + R}$$

$$\text{Resistance of the ammeter} = \frac{4}{3} \Omega$$

$$\text{Total resistance of the circuit, } R_{\text{total}} = \frac{100R}{100 + R} + \frac{4}{3} + 2 \Omega$$

Current in the circuit as read by the ammeter = 0.02 A

$$\text{Now, } 0.002 = \frac{1.4}{\frac{100R}{100 + R} + \frac{4}{3} + 2} \quad \left(\because I = \frac{V}{R} \right)$$

$$\text{or } R = 200 \Omega$$

$\therefore$ Resistance of the voltmeter = 200Ω

$$(ii) \text{ Effective resistance between B and C, } R_{BC} = \frac{100 \times 200}{100 + 200} = \frac{200}{3} \Omega$$

The potential drop across this resistance

$$= V_{BC} = i \times R_{BC} = 0.02 \times \frac{200}{3} = \frac{4}{3} \text{ V} = 1.333 \text{ V}$$

Reading of the voltmeter = 1.1 V

$$\text{Error in the reading of the voltmeter} = 1.1 - 1.333 = -0.233 \text{ V}$$

Exercise 6.2

1. Let the length of the potentiometer wire be l , and V be the potential difference across it, then potential gradient is $K = V/l$.

Let balance point be obtained at l_1 , then measured potential difference is $Kl_1 = V_1/l_1$. Let us make error Δl in measuring l_1 , then the percentage error is

$$\frac{k\Delta l}{kl_1} \times 100 = \frac{\Delta l}{l_1} \times 100$$

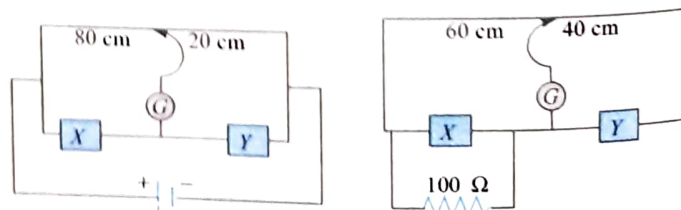
If l is increased, then l_1 also increases and hence the percentage error will decrease. So accuracy increases.

2. For balanced Wheatstone bridge, $\frac{X}{40} = \frac{3}{60}$ or $X = 2 \Omega$

3. A potentiometer is said to be sensitive if fall of potential per unit length, i.e., potential gradient (dV/dl) is small. The slope of $V-l$ graph gives potential gradient, which is smaller for potentiometer wire Y than for potentiometer wire X . Therefore, potentiometer Y will be preferred for comparing emfs of the two cells.

4. From the first null point: $\frac{X}{Y} = \frac{80}{20} \quad \dots(i)$

$$\text{From the second null point: } \frac{\left(\frac{100X}{100 + X} \right)}{Y} = \frac{60}{40} \quad \dots(ii)$$



From Eqs. (i) and (ii), we get

$$X = \frac{500}{3} \Omega \text{ and } Y = \frac{125}{3} \Omega$$

5. Internal resistance of the cell is given by

$$r = R \left(\frac{l_1 - l_2}{l_2} \right) = 1 \times \frac{60 - 30}{30} \Omega = 1 \Omega$$

$$6. k = \frac{E_B}{L} = \frac{1.1}{440} = 0.0025 \text{ V cm}^{-1}$$

Potential difference across R is $0.0025 \times 220 = 0.550 \text{ V}$

Error in the reading of voltmeter is

$$\text{reading of voltmeter} - \text{reading of potentiometer} \\ = 0.5 - 0.55 = -0.05 \text{ V}$$

7. In a meter bridge null deflection occurs at balancing state of Wheatstone bridge for which we use

$$\frac{10}{R} = \frac{AC}{CB}$$

$$\Rightarrow R = 10 \left(\frac{CB}{AC} \right) = (10) \left(\frac{100 - 40}{40} \right) = 10 \left(\frac{60}{40} \right)$$

$$\Rightarrow R = 15 \Omega$$

$$8. E_0 = V_{AC} = (i)_{AC} (R)_{AC} = \left(\frac{E}{10} \right) \left(\frac{10}{1} \times 0.2 \right) = \frac{E}{5}$$

$$\text{In second case, } E_0 = \left(\frac{E}{10+x} \right) \left(\frac{10}{1} \times 0.3 \right)$$

Solving Eqs. (i) and (ii), we get $x = 5 \Omega$

$$9. (a) V_A - V_B = 6 \text{ V}$$

Since, $V_B = 0$

$$\therefore V_A = 6 \text{ V}$$

$$V_A - V_C = 4 \text{ V} \Rightarrow V_C = V_A - 4 = 2 \text{ V}$$

$$(b) V_A - V_D = V_A - V_C = 4 \text{ V}$$

From unitary method, we can find that,

$$AD = \left(\frac{100}{6} \right) (4) = 66.67 \text{ cm}$$

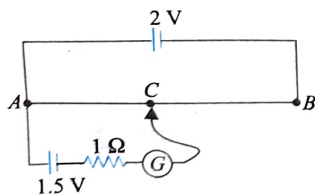
(c) Since, they are at same potential, no current will flow through it.

(d) $V_A - V_B$ is still $6 \text{ V} \therefore V_A = 6 \text{ V}$

Further, $V_A - V_C = 7.5 \text{ V} \therefore V_C = -1.5 \text{ V}$

Since, EMF of the battery in lower circuit is more than the EMF of the battery in upper circuit. No such point will exist.

10.

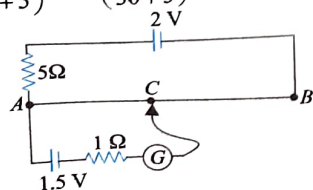


$$\text{Potential gradient across wire} = AB = \frac{2}{10} = 0.2 \text{ V/m}$$

$$\text{Now, } V_{AC} = 1.5 \text{ V or } (0.2)(AC) = 1.5$$

$$\therefore AC = 7.5 \text{ m}$$

$$(a) V_{AB} = \left(\frac{R_{AB}}{R_{AB} + 5} \right) \times 2 = \left(\frac{30}{30 + 5} \right) \times 2 = \frac{12}{7} \text{ V}$$

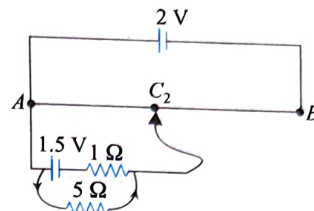


$$\therefore \text{Potential gradient across } AB = \frac{12}{70} \text{ V/m}$$

$$\text{Now, } V_{AC} = 1.5 \text{ V}$$

$$\therefore \left(\frac{12}{70} \right) (AC_1) = 1.5$$

$$\therefore AC_1 = 8.75 \text{ m}$$



$$(b) V_{AC_2} = V$$

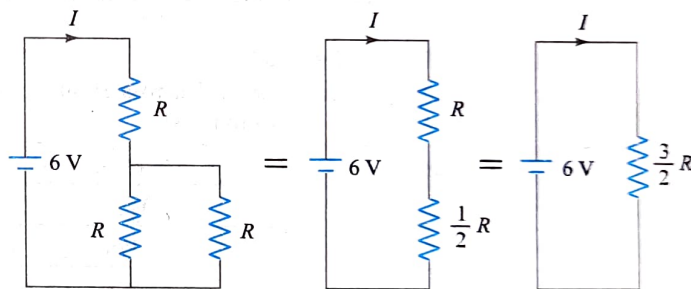
$$\text{or } (0.2)(AC_2) = \left(\frac{5}{5+1} \right) (1.5)$$

$$\text{or } AC_2 = 6.25 \text{ m}$$

Exercises

Single Correct Answer Type

1. (2) The circuit may be redrawn as follows:



$$\text{Current is given by } I = \frac{6}{\frac{3}{2}R} = \frac{4}{R} \text{ A}$$

Therefore, current through the voltmeter is $I/2$ or $2/R \text{ A}$. Hence, the reading of the voltmeter is $(2/R)(R)$ or 2 V .

2. (1) At null point, $R_1/R_2 = R_3/R_4 = x/(100 - x)$. If radius of the wire is doubled, then the resistance of AC will change and the resistance of CB will also change. But since R_1/R_2 does not change, so R_3/R_4 should also not change at null point. Therefore, point C does not change.

3. (1) Using the principle of potentiometer, $V \propto l$. So

$$\frac{V}{E} = \frac{l}{L} \text{ or } V = \frac{l}{L} E = \frac{30}{100} E = \frac{30E}{100}$$

$$4. (1) \frac{X}{Y} = \frac{20}{80} = \frac{1}{4} \text{ or } Y = 4X$$

$$\frac{4X}{Y} = \frac{l}{100 - l} \text{ or } \frac{4X}{4X} = \frac{l}{100 - l} \text{ or } l = 50 \text{ cm}$$

5. (3) Current through R is $12/(500 + R)$

$$\text{Voltage across } R \text{ is } \frac{12R}{500 + R}$$

Since galvanometer shows zero deflection, so

$$\frac{12R}{500 + R} = 2 \text{ or } R = 100 \Omega$$

6. (3) $i_g = \frac{V}{G + R}$ or $10^{-3} = \frac{2.5}{10 + R}$ or $R = 2490 \Omega$

7. (2) Total resistance is

$$\left(\frac{5 \times 15}{5 + 15} + 1.25 \right) = \left(\frac{75}{20} + 1.25 \right) \Omega = (3.75 + 1.25) \Omega = 5 \Omega$$

$$I = \frac{20}{5} \text{ A} = 4 \text{ A}$$

Current through 5Ω is $\frac{15}{20} \times 4 \text{ A} = 3 \text{ A}$

Voltmeter reading = Potential drop across 1.25Ω
 $= 4 \times 1.25 \text{ V} = 5 \text{ V}$

8. (1) Effective resistance across the voltmeter is $200/2 = k\Omega$.

Total resistance across dc supply is $400 + 100 = 500 k\Omega$.

Thus, the voltage across the voltmeter is $100/500(60 \text{ V}) = 12 \text{ V}$.

9. (2) When X is connected to Y , the balance length l is proportional to the potential difference across the 100Ω resistor. When X is connected to Z , the balance length is proportional to the potential difference across the 100Ω resistor and resistor R .

$$\frac{100 + R}{100} = \frac{588}{400} \text{ or } R = 47 \Omega$$

10. (2) A careful analysis would show that the voltage along R is 1.03 V . So

$$1.03 = 1 \times R \text{ or } R = 1.03 \Omega$$

11. (3) If V is the potential difference applied across P and Q , the current through M is determined as follows:

Circuit	Current
(a)	$V/4$
(b)	$3 V/8$
(c)	$V/2$
(d)	$V/3$

Hence, circuit arrangement (c) gives the largest reading in ammeter M .

12. (2) The voltage per unit length on the meter wire PQ is

$$\frac{6.00 \text{ mV}}{0.60 \text{ m}} \text{ or } 10 \text{ mVm}^{-1}$$

Hence, potential across the meter wire PQ is $10 \text{ mVm}^{-1}(1 \text{ m}) = 10 \text{ mV}$. Current drawn from the driver cell is

$$I = \frac{10 \text{ mV}}{5 \Omega} = 2 \text{ mA}$$

Resistance of the resistor R is

$$R = \frac{2 \text{ V} - 10 \text{ mV}}{2 \text{ mA}} = \frac{1990 \text{ mV}}{2 \text{ mV}} = 995 \Omega$$

13. (2) $P/Q = R/S$. If P is increased, then either R or Q should be increased or S should be decreased.

14. (2) In case of internal resistance measurement by potentiometer,

$$\frac{V_1}{V_2} = \frac{I_1}{I_2} = \frac{[ER_1/(R_1 + r)]}{[ER_2/(R_2 + r)]} = \frac{R_1(R_2 + r)}{R_2(R_1 + r)}$$

Here $I_1 = 2 \text{ m}$, $I_2 = 3 \text{ m}$, $R_1 = 5 \Omega$, and $R_2 = 10 \Omega$. So

$$\frac{2}{3} = \frac{5(10 + r)}{10(5 + r)} \text{ or } r = 10 \Omega$$

15. (1) $3 = IR_1$ or $3 = 1 \times R_1$ or $R_1 = 3 \Omega$

When points A and B are connected by a conducting wire, R_2 is short-circuited. Therefore,

$$10.5 = I'R_1 \text{ or } 10.5 = I' \times 3$$

$$\text{or } I' = \frac{10.5}{3} = 3.5 \text{ A}$$

$$\text{But } 10.5 = E - I'r \text{ or } 10.5 = 12 - 3.5r$$

$$\therefore r = \frac{1.5}{3.5} = \frac{3}{7} \Omega$$

16. (2) The current through the galvanometer producing full-scale deflection is

$$I = \frac{V}{R} = \frac{20 \times 10^{-3}}{80} = 2.5 \times 10^{-4} \text{ A}$$

To convert the galvanometer into a voltmeter, a high resistance is connected in series with the galvanometer. Therefore,

$$5 \text{ V} = (2.5 \times 10^{-4})(R + 80) \text{ or } R = 19.92 k\Omega$$

17. (1) $V_1 = E - ir = 50 - \frac{50}{220} \times 20$

$$= 50 - 4.6 = 45.4 \text{ V}$$

$$\text{Now, } V_2 = 50 - \frac{50}{180} \times 20 = 44.4 \text{ V}$$

$$\text{Percentage change} = \frac{V_1 - V_2}{V_1} \times 100$$

$$= 2.27 \text{ (also see the question)}$$

18. (1) The electric current through ideal voltmeter is zero. According to the loop rule,

$$E - 1 \times I - 1 \times I = 0 \text{ or } I = \frac{E}{2} = \frac{2}{2} = 1 \text{ A}$$

Reading of the voltmeter is

$$V_A - V_B = [1 \times I] = [1 \times 1] = 1 \text{ V}$$

19. (3) $k = 0.05 \text{ mVcm}^{-1} = 5 \text{ mVm}^{-1}$

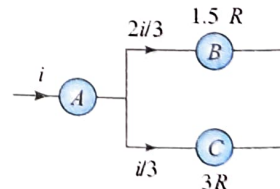
$$I = \frac{kl}{r} = \frac{5 \times 10^{-3} \times 1}{5} = 10^{-3} \text{ A}$$

$$\text{Now, } 2 = I(R + r) = 10^{-3}(R + 5) \text{ or } R = 1995 \Omega$$

20. (1) The value of $E = \text{Potential gradient} \times \text{Length}$
 $= (5 \text{ mVm}^{-1})(60 \text{ cm}) = 3 \text{ mV}$

21. (1) $V_A = iR$,

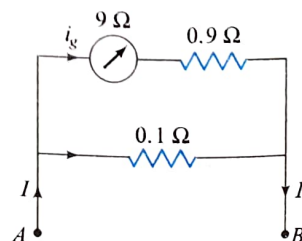
$$V_B = \frac{2i}{3} \times 1.5R = iR, V_C = (i/3)(3R) = iR$$



22. (3) $i_g = 10 \text{ mA} = 0.01 \text{ A}$

$$V_A - V_B = (I - i_g) 0.1 = i_g(9 + 0.9)$$

$$\text{or } I = \frac{10 \times 0.01}{0.1} = 1 \text{ A}$$



32. (2) Let a current of x ampere pass through the voltmeter; then $(4-x)$ ampere passes through the resistance R . Therefore, voltmeter reading is

$$20 = (4-x)R, \text{ i.e., } R = \frac{20}{4-x}, \text{ i.e., } R > 5 \Omega$$

$$33. (4) \frac{I_g}{I} = \frac{S}{S+G} = \frac{(G/10)}{(G/10)+G} = \frac{1}{11}$$

$$\text{Initially, } \alpha_i = \theta/I_g \quad \dots(i)$$

Finally, after the shunt is used,

$$\alpha_f = \theta/I \quad \dots(ii)$$

$$\therefore \frac{\alpha_f}{\alpha_i} = \frac{\theta/I}{\theta/I_g} = \frac{I_g}{I} = \frac{1}{11}$$

So current sensitivity becomes 1/11-fold.

34. (2) Let I be the current in the circuit, then $I \times 4000 = 30$ V.
If voltmeter is put across 3000Ω resistance, then reading of voltmeter is $I \times 3000 = \frac{30}{4000} \times 3000 = 22.5$ V

35. (2) We know that

$$R = \frac{V}{I_g} - G$$

The voltmeter gives the full-scale deflection for potential difference V . Its resistance is G . Hence, $I_g = (V/G)$. Given that $V = nV$. Therefore,

$$R = \frac{nV}{(V/G)} - G = (n-1)G$$

$$36. (2) \frac{I_g}{I} = \frac{S}{S+G} \text{ or } \frac{1}{34} = \frac{S}{S+G}$$

$$\therefore S = (G/33) = (3663/33) = 111 \Omega$$

$$37. (3) R_s = \frac{SG}{S+G} = \frac{111 \times 3663}{111 + 3663} = 107.7 \Omega$$

38. (4) Compensation external resistance

$$G - \frac{SG}{S+G} = 3663 - 107.7 = 3555.3 \Omega$$

39. (1) Total range is doubled, i.e., $4I_g$

$$\text{Now shunt required is } S = \frac{I_g}{4I_g - I_g} \times G = 10 \Omega$$

$$\frac{1}{30} + \frac{1}{x} = \frac{1}{10} \text{ or } x = 15 \Omega$$

40. (3) Initially, when switch is open, V_2 and V_3 will be out of circuit or they are short-circuited. So no potential difference is across V_2 and V_3 . On closing the switch, all three voltmeters will be in parallel. Now, there is some potential difference across V_2 and V_3 . So potential difference across V_2 and V_3 increases and potential difference across V_1 remains the same.

41. (2) Resistors 20Ω , 100Ω , and 25Ω will be in parallel. Their equivalent is 10Ω .

$$I = \frac{200}{5+10+5} = 10 \text{ A}$$

Potential difference across 10Ω is $10I = 10 \times 10 = 100$ V
This will be the voltmeter reading. Also, this will be the potential difference across each of 20Ω , 100Ω , and 25Ω resistors.

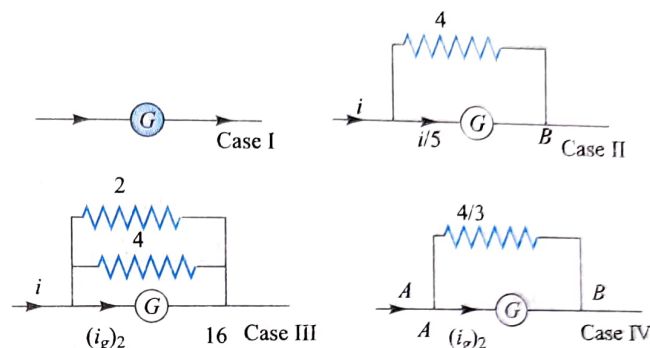
$$\text{Ammeter reading} = \text{current through } 25 \Omega = \frac{100}{25} = 4 \text{ A}$$

$$33. (1) \text{ Current in } AB = I = \frac{e}{R+r}$$

$$\text{potential difference across } AB = IR = \frac{eR}{R+r}$$

$$\text{To obtain balanced point, } \frac{eR}{R+r} > \frac{e}{3} \text{ or } R > r/2$$

34. (1) When only 4Ω resistance is shunted $i_g = i/5$
 $G \times i/5 = 4 \times (4/5)$ or $G = 16 \Omega$



$$35. (2) 1.45 = 1.3 + Ir_A$$

$$1.45 = 1.5 - Ir_B$$

$$Ir_B = 0.05$$

$$Ir_A = 0.15$$

$$\text{Dividing, we get } \frac{r_B}{r_A} = \frac{1}{3}$$

$$\text{or } r_A = 3r_B$$

36. (2) Clearly, $E_2 = IY$

$$2 = \frac{12}{500+Y} Y \text{ or } 500+Y = 6Y$$

$$\text{or } 5Y = 500 \text{ or } Y = 100 \Omega$$

37. (2) Potential difference V_p across P as determined from E_1 is given

$$\text{by } V_p = \left(\frac{E_1}{P+Q} \right) P.$$

Potential V_p across P as determined from E_2 is same as E_2 because no current is drawn, i.e., $V_p = E_2$.

Therefore,

$$E_2 = E_1 \left(\frac{P}{P+Q} \right) \text{ or } \frac{E_2}{E_1} = \frac{P}{P+Q}$$

38. (1) In a Wheatstone bridge, the deflection in the galvanometer does not change if the battery and galvanometer are interchanged.

39. (1) $V = E - Ir_1$, $V = 0$

$$\therefore E = Ir_1$$

$$\text{Total emf} = Ir_1 = Ir_1 = 2Ir_1$$

$$\text{Total resistance} = R + r_1 + r_2$$

$$\text{Now, } I = \frac{2Ir_1}{R + r_1 + r_2}$$

$$\text{or } R + r_1 + r_2 = 2r_1$$

$$\text{or } R = 2r_1 - r_1 - r_2 \text{ or } R = r_1 - r_2$$

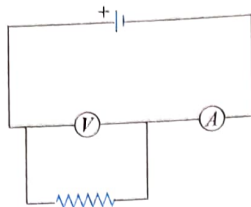
40. (1) Effective resistance across voltmeter is $50 \text{ k}\Omega$. Total resistance across the dc supply is $450 \text{ k}\Omega$.

Current drawn from supply $1000 \text{ V}/450 \text{ k}\Omega = 1/450 \text{ A}$. Potential difference across voltmeter is

$$\frac{50 \times 1000}{450} \text{ V} = \frac{1000}{9} \text{ V} = 111 \text{ V}$$

41. (4) When $r = 0$ the terminal potential difference in the arm containing R_1 remains unchanged, so the ammeter reading does not change.

42. (4) When a resistance is joined in parallel with the voltmeter, the total resistance of the circuit decreases. Current will increase, so ammeter reading will increase. The equivalent resistance across the voltmeter decreases and hence its reading will decrease.



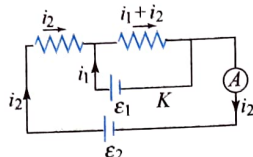
43. (1) A and B are effectively in parallel and hence give the same reading every time.

44. (2) $R(i_1 + i_2) = E_1$

$$i_2 R + (i_1 + i_2)R = E_2$$

$$i_2 = \frac{E_2 - E_1}{R}$$

$$[\text{Initially, } E = E_2/R]$$



45. (1) $R_0 = 2500 \Omega$

$$E = 125 \text{ V}$$

When r is connected in series,

$$i = \frac{E}{R_0 + r}$$

Reading in voltmeter $= E - ir = 100$

$$100 = 125 - \frac{125}{2500 + r} \times r$$

$$\text{or } r = 625 \Omega$$

46. (4) $(0.1 + 0.3 + 0.6)i_1 = (9 + 0.9) \times 10$

$$i = 99 \text{ mA}$$

$$I = (99 + 10) \text{ mA} = 0.109 \text{ A}$$

47. (2) When switch S_1 is open

$$\frac{6}{E} = \frac{L}{2} / L = \frac{1}{2}$$

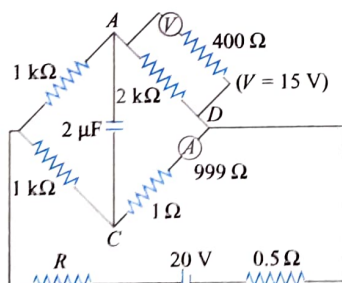
$$\therefore E = 12 \text{ V}$$

When switch S_2 is closed, we get

$$\frac{6 \times 10}{10 + r} = \frac{5L}{12L} \quad E = \frac{5}{12} \times 12 = 5$$

$$\therefore 10 + r = 12 \quad \text{or } r = 2 \Omega$$

48. (4) From figure



$$V_A - V_D = 15 \text{ V}$$

$$\text{and } V_C - V_D = iR = 15 \times 10^{-3} \times (1 + 999) = 15 \text{ V}$$

$$V_A - V_C = 15 - 15 = 0$$

$$\text{Energy stored} = 0$$

49. (1) This is a case of balanced Wheatstone bridge.

50. (1) If V_1 decreases, then potential difference across xz will decrease. So to have same potential difference, i.e., V_2 across xz , length xz has to be increased or z should be moved toward y .

- (2) is incorrect because for this, z should have been moved toward x .

- (3) will be incorrect because xz can get warm up if current through it increases which is not possible.

- (4) is incorrect because there is no current in R_1 .

51. (4) As $V = \epsilon - Ir$

From graph it is clear that slope $= -r = -y/x$

$$\text{or } r = y/x$$

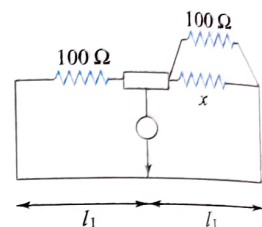
52. (2) As potential gradient k in the potentiometer wire will be less than $(\epsilon/100) \text{ (V cm}^{-1}\text{)}$ because of the presence of r , so for the battery of emf $\epsilon/2$, the balance point l will be greater than 50 cm as shown.

$$l = \frac{\epsilon/2}{k(\epsilon/100)} \quad \text{or } l > 50 \text{ cm}$$

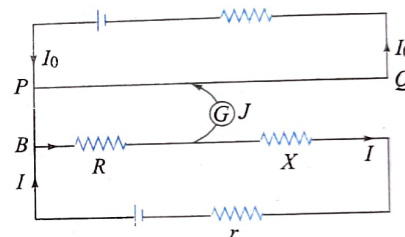
53. (2) The Wheatstone bridge is in balanced condition, so

$$\frac{100}{l_1} = \frac{100x}{l_2}$$

$$\therefore \frac{l_1}{l_2} = 2 \quad \text{or } x = 100 \Omega$$



54. (1) In potentiometer wire, potential difference is directly proportional to length. Let the potential drop of unit length of a potentiometer wire be K .



For zero deflection, the current will flow independently in two circles:

$$IR = K \times 10 \quad \dots(i)$$

$$IR + IX = K \times 30 \quad \dots(ii)$$

Subtracting Eq. (ii) from Eq. (i), we get

$$IX = k \times 20 \quad \dots(iii)$$

Dividing Eq. (i) by Eq. (ii), we have

$$\frac{R}{X} = \frac{1}{2}$$

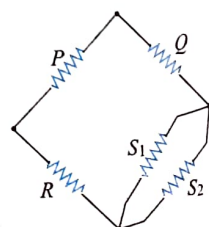
55. (4) In first two circuit diagrams, corrections have to be made while third one is absolutely wrong. So, none of the circuit diagrams is giving correct value of R .

56. (2) The ammeter is not showing any reading. It means the potential drop across CB due to E_1 is equal to that of the emf E_2 . And this is the reading of the voltmeter.

57. (4) Using the condition for Wheatstone bridge to be balanced, we get

$$\frac{P}{Q} = \frac{R}{\frac{S_1 S_2}{S_1 + S_2}}$$

$$\text{or } \frac{P}{Q} = \frac{R(S_1 + S_2)}{S_1 S_2}$$



58. (4) For balanced meter bridge (null deflection), we get

$$\frac{55}{R} = \frac{20}{80} \quad \text{or } R = 220 \Omega$$

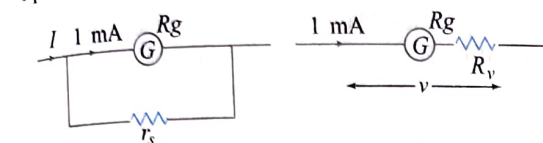
$$I = \frac{E - E_1}{R'} = \frac{E - E_2}{R + R'}$$

$$\frac{E - E_2}{E - E_1} = \frac{R'}{R + R'}$$

$$\frac{E - E_2}{E - E_1} = \frac{R}{R'}$$

$$I = \frac{E_1 - E_2}{R}$$

With these diagrams, we can easily find that the maximum equivalent resistance will be across voltmeter of range 10 V.



$$v_p = 2 \left[\frac{20 + x}{40} \right]$$

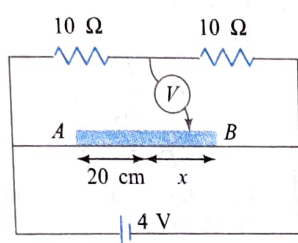
$$v_Q = 2$$

$$v_p - v_Q = \frac{x}{10}$$

$$2 \sin \pi t = \frac{x}{10}$$

$$x = 20 \sin \pi t$$

$$\frac{dx}{dt} = (20\pi \cos \pi t) \text{ cm s}^{-1}$$



(1) Potential difference across AC is zero.

$$5 - 2I = 0$$

$$\text{or } I = 2.5 \text{ A}$$

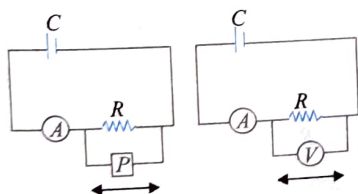
Let the resistance of part BC be r . Applying KVL, we get

$$10 + 5 - 2I - Ir - I = 0$$

$$\text{or } 2.5r = 7.5 \text{ or } r = 3 \Omega$$

As resistance of part AB = 9 Ω , length AC = 66.7 cm

(1) Effective resistance of R and V is less than effective resistance of R and P , hence pd across voltmeter (or R) is less than previous value.



Also overall resistance decreases, so overall current increases, hence reading of ammeter increases.

(4) E_1 could be balanced if positive terminal of E_1 would be connected to A.

$$(3) R_{AB} = 16 \Omega, V_{AB} = \frac{R_{AB} E_0}{R + R_{AB}} = \frac{16 \times 12}{8 + 16} = 8 \text{ V}$$

Current in the loop of E_1 and E_2 :

$$I = \frac{E_2 + E_1}{r_1 + r_2} = \frac{4 + 2}{2 + 6} = \frac{3}{4} \text{ A}$$

$$E_1 - Ir_1 = V_{AB} = \frac{AN}{AB} (V_{AB}) \quad \text{or} \quad 2 - \frac{3}{4} \times 2 = \frac{AN}{4} \times 8$$

$$\text{or } AN = \frac{1}{4} \text{ m} = 25 \text{ cm}$$

66. (2) t_1 is the time of capacitor to discharge from $2V/3$ to $V/3$. For

$$\text{this, } \frac{V}{3} = \frac{2V}{3} e^{-t_1/\tau}$$

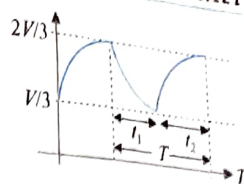
$$\text{or } t_1 = \tau \ln 2 = RC \ln 2$$

t_2 is time for charging of capacitor from $V/3$ to $2V/3$.

$$\text{So } \int_{VC/3}^{2VC/3} \frac{dq}{VC - q} = \int_0^{t_2} \frac{dt}{RC}$$

$$\text{or } t_2 = RC \ln 2$$

$$T = t_1 + t_2 = 2RC \ln 2$$



$$67. (1) \frac{\epsilon}{2} = \epsilon - ir \quad \text{or} \quad i = \frac{\epsilon}{2r}$$

$$2\epsilon = i(3 + r)$$

$$r = 1 \Omega$$

...(i)

...(ii)

$$68. (1) \text{ Case I: 10 cm from A } P_1 = \left(\frac{\epsilon}{r + 10} \right)^2 \times 10$$

$$\text{Case II 10 cm from B } P_2 = \left(\frac{\epsilon}{r + 90} \right)^2 \times 90$$

$$P_1 = P_2 \Rightarrow r = 30 \Omega$$

69. (1) Ohm's law $V = IR$ or $V \propto I$ (for ammeter)

$$\frac{V_2}{V_1} = \frac{I_2}{I_1} = 2$$

$$\text{or } 10 = 2 \times 10 \times \frac{R}{2 + R} \quad \text{or } R = 2 \Omega$$

70. (3) $I = 1 \text{ A}$

r is the internal resistance of the battery.

$$12 = (X + Y + r) (1)$$

$$\text{Also } 1 = (X) (1) \quad \text{or } X = 1 \Omega$$

Voltage across X (when A and B are shorted)

$$10 = \left(\frac{12}{X + r} \right) X \quad \text{or } 10 = \frac{12}{1 + r} \quad \text{or } 1 + r = \frac{6}{5}$$

$$\text{or } r = \frac{1}{5} \Omega$$

...(i)

$$71. (3) \frac{R_1}{R_2} = \frac{30}{70} = \frac{3}{7}$$

$$\frac{R_1}{R_2} = \frac{50}{60} = 1 \quad \text{or } R_1 = R_2'$$

$$R_1 = \frac{R_2 \times 10}{R_2 + 10} = \frac{3}{7} R_2 \quad \text{or } R_2 = \frac{40}{3} \Omega \quad \text{or } R_1 = \frac{40}{7} \Omega$$

...(i)

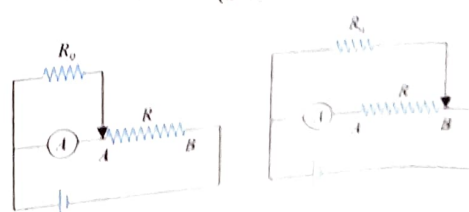
...(ii)

72. (4) When X is at A or B:

$$\text{Reading of ammeter } I_0 = \frac{\epsilon}{R}$$

$$R = x + y \quad \text{or } I_0 = \frac{\epsilon}{(x + y)}$$

(i)



When sliding key is at any other position

$$I_1 = \frac{\varepsilon}{\left(\frac{R_0 x}{R_0 + x} + x \right)}$$

Reading of ammeter $I' = I_1 \frac{R_0}{(R_0 + x)}$

$$= \frac{\varepsilon(R_0 + x)}{[R_0 x + (R_0 + x)](R_0 + x)}$$

$$= \frac{\varepsilon R_0}{R_0(x + x) + x^2} = \frac{\varepsilon}{(x + x) + \frac{x^2}{R_0}}$$

Hence $I' < I_0$

Hence the reading of ammeter first decreases then increases.

73. (1) For getting null point

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \text{ as } R_2 = R_4$$

$$R_1 = R_3$$

Let the pointer at point 5 is moved to left by distance x to get null point as shown in the figure. If resistance per unit length of wire 3 is r then that of wire 1 will be $2r$.

$$(2 - x)2r = x \times 2r + 2 \times r$$

or $4x = 2$

or $x = \frac{2}{4} = 0.5 \text{ m}$

74. (2) $V = \frac{R_1 E}{R + R_1} = \frac{E}{1 + R/R_1}$

If S_1 is closed: $R_1 = 3R$

or $V_1 = 3E/4 = 0.75 E$

If S_2 is closed: $R_1 = 6R$

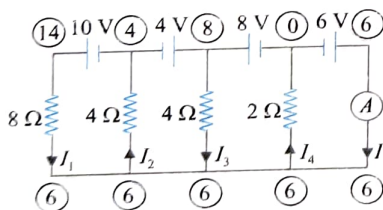
or $V_2 = 6E/7 = 0.85 E$

If S_1 and S_2 both are closed,

$$R_1 = 2R \text{ or } V_3 = 2E/3 = 0.67E$$

or $V_2 > V_1 > V_3$

75. (3)



The current through ammeter

$$I = I_4 - I_3 + I_2 - I_1$$

$$= \left(\frac{6-0}{2} \right) - \left(\frac{8-6}{4} \right) + \frac{(6-4)}{4} - \frac{(14-6)}{8}$$

$$= 3 - \frac{1}{2} + \frac{1}{2} - 1 = 3 - 1 = 2 \text{ A}$$

Multiple Correct Answers Type

1. (2), (3)

The potential measures the exact value of emf of a battery. Therefore, $E = 1.55 \text{ V}$.

Also, $1.4 = I(280)$ or $I = 0.005 \text{ A}$

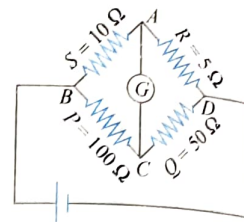
Also, $V = E - Ir$ or $r = \frac{(E - V)}{I} = \frac{1.55 - 1.40}{0.005} = 30 \Omega$

2. (2), (3)

If P is slightly increased, potential of C will decrease.

Hence, current will flow from A to C .

If Q is slightly increased, potential of C will increase, hence current will flow from C to A .



3. (2), (3)

When switch S is open, R_1 and R_2 are short-circuited. E will be divided between V_1 and V_2 only. After closing the switch, V_1 , R_1 , and R_2 will be in parallel. Their effective resistance will be less than the resistance of V_1 . Hence, potential difference across V_2 decreases and so across V increases.

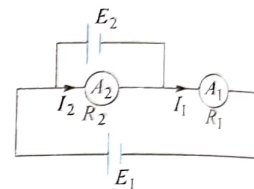
4. (1), (2), (4)

By applying Kirchhoff's law, we can obtain

$$I_2 = \frac{E_2}{R_2} \text{ and } I_1 = \frac{E_1 - E_2}{R_1}$$

If $E_1 > E_2$, I_1 and I_2 can be the same for some value of R_1 and R_2 , hence (1) is correct.

For (2), if $E_2 > E_1$, then I_1 will be in the direction opposite to that shown in the figure and magnitude of I_1 will be $I_1 = (E_2 - E_1)/R_1$. Now I_1 and I_2 can be same only if $R_1 < R_2$.



5. (2), (3)

If polarity of E_2 is reversed, then

$$I_2 = \frac{E_2}{R_2} \text{ and } I_1 = \frac{E_1 + E_2}{R_1}$$

for $I_1 = I_2$, $R_1 > R_2$

6. (1), (4)

Let resistance of voltmeter be R_V . Then the equivalent resistance of circuit is

$$R_{eq} = R_2 + \frac{R_1 R_V}{R_1 + R_V} = \frac{R_1 R_2 + R_2 R_V + R_1 R_V}{R_1 + R_V}$$

$$I = \frac{E}{R_{eq}} = \frac{E(R_1 + R_V)}{R_1 R_2 + R_2 R_V + R_1 R_V}$$

On putting voltmeter across R_2 , we can get current

$$I' = \frac{E(R_2 + R_V)}{R_1 R_2 + R_2 R_V + R_1 R_V} \text{ as } R_1 > R_2 \Rightarrow I > I'$$

If voltmeter is ideal, then in both cases R_1 and R_2 will be in series and current in both cases will be $I = E/(R_1 + R_2)$.

7. (1), (2), (4)

Since $R_1/R_2 = C_2/C_1$, the Wheatstone bridge is balanced. Hence, $V_C = V_D$. No current passes through the galvanometer. Hence, option (1) is correct.

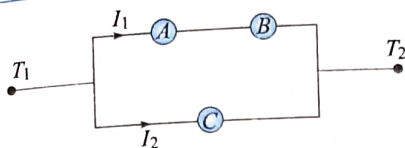
Potential difference across R_1 = potential difference across $C_1 = 4 \text{ V}$

Potential difference across R_2 = potential difference across $C_2 = 5 \text{ V}$

Potential difference across 5Ω is the potential difference across $8 \mu\text{F}$ capacitor. So, charge across $8 \mu\text{F}$ capacitor is $Q = CV = 8 \mu\text{F} \times 5 \text{ V} = 40 \mu\text{C}$. Hence, option (2) is correct and so is option (4).

8. (1), (2), (4)

As A and B are connected in series, hence $IA = IB$. The potential difference across both the branches will be the same.



$$I_A R_B + I_B R_B = I_C R_C \text{ or } \frac{I_B}{I_C} = \frac{R_C}{R_A + R_B}$$

- (1), (2)
For null point, current flows in the loop CD only.

$$i = \frac{3 \text{ V}}{2 \Omega + 1 \Omega} = 1 \text{ A}$$

$$V_{CD} = 1 \text{ V} - 1(1) = 0$$

Therefore, option (1) is correct. That is, $V_A > V_B$. When jockey touches B, current from A to B increases the PD across the secondary circuit. Therefore, option (2) is correct.

11. (1), (2)

$$\frac{R}{(R_1 + R_2)} = \frac{L/4}{3L/4}$$

$$3R = (R_1 + R_2)$$

$$\frac{R_1 + R_2}{R} = \frac{2}{3}$$

$$3R = R_2 + R_1$$

$$R = 2R_2 - R_1$$

$$4R = 3R_2$$

$$R_1 = \frac{5R}{3}$$

$$R_2 = \frac{4R}{3}$$

12. (1), (2), (4)

Let the currents be as shown in the figure.

Applying KVL along ABCDA, we get

$$-10i - 2 + (2-i)1 = 0$$

$$\therefore i = 0$$

$$\text{Potential difference across } = (2-i)1 = 2 \times 1 = 2 \text{ V}$$

13. (1), (2)

Reading of C = $V \{i \text{ in that branch} = 0\}$

$$\text{Reading of A} = \frac{V}{R} \Rightarrow R$$

$$V_B = iR_{\text{ammeter}} = 0 \Rightarrow V_{\text{cap}} = 0$$

14. (1), (3), (4)

As the ammeters are ideal, we can conclude that the three resistors R_1 , R_2 , and R are actually connected in parallel, the equivalent resistance being

$$R_{\text{eq}} = \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

In the figure, the electric currents on each resistor and battery are shown. The readings of the two ammeters are different and, consequently, by simple symmetry considerations, resistors R_1 and R_3 are necessarily different. In other words, if we interchange R_1 and R_3 , the readings of the ammeters will also be interchanged. Therefore, we have two possible alternatives. $R_2 = R_1$ (if resistors

R_2 and R_1 switched, the readings of the ammeters do not vary) or $R_2 = R_3$ (R_2 and R_3 can be switched and the ammeters do not change). The two solutions are:

If $R_2 = R_1$ we obtain $I_1 = I_2 = 0.15 \text{ A}$ and $I_3 = 0.05 \text{ A}$, and the battery current is $I = 0.35 \text{ A}$.

If $R_2 = R_3$ we obtain $I_2 = I_3 = 0.1 \text{ A}$ and $I_1 = 0.2 \text{ A}$, and the battery current is $I = 0.4 \text{ A}$.

15. (1), (3), (4)

When switch S is opened, parallel connection across battery decreases. Hence, equivalent resistance across the battery increases, power dissipated by left resistance R decreases, voltmeter reading decreases and ammeter reading decreases.

16. (1), (2), (4)

If $R_1/R_2 = R_3/R_4$, there will be no current through galvanometer. If emf of battery is double, there will not be any change in situation i.e., $(R_1/R_2 = R_3/R_4)$. Even if all the resistances are doubled, the ratio remains same. If the positions of the galvanometer and battery is interchanged, the ratio $R_1/R_3 = R_2/R_4$ will remain same.

Linked Comprehension Type

For Problems 1–2

1. (1), 2. (4)

Let R_1 and R_2 be the resistances of the ammeter and the voltmeter, respectively. Let the external resistance be denoted by R and the internal resistance of battery be r . The equivalent resistance of the parallel combination of R and R_2 is given by

$$R' = \frac{RR_2}{R + R_2}$$

The total resistance RT of circuit then becomes

$$RT = R_1 + r + \frac{RR_2}{R + R_2}$$

The current in the circuit is given by

$$I = \frac{E}{R_1 + r + \frac{RR_2}{R + R_2}}$$

This must be equal to 0.04 A , the reading indicated by the ammeter.

$$\frac{3.4}{2 + 3 + \frac{100R_2}{100 + R_2}} = 0.04 \text{ or } R_2 = 400 \Omega$$

In case of an ideal voltmeter, no current flows through it. In that case, current in the circuit is

$$I' = \frac{3.4}{2 + 3 + 100} = \frac{3.4}{105} = 0.0324 \text{ A}$$

Potential drop across the resistance R would be $100 \times 0.0324 = 3.24 \text{ V}$. This should be the reading indicated by an ideal voltmeter.

For Problems 3–4

3. (4), 4. (3)

$$r = \frac{l_1 - l_2}{l_2} R = \frac{84 - 70}{70} \times 5 = 1 \Omega$$

$$l = \frac{l_1 - l_2}{l_1} \times 4, \text{ put } l_1 = 84 \text{ cm and get } l_2 = 67.2 \text{ cm}$$

For Problems 5–7

5. (1), 6. (4), 7. (2)

$$\frac{e}{1.02} = \frac{88}{66} \text{ or } e = 1.36 \text{ V}$$

4 V is greater than applied emf 2 V, hence no balance point is obtained.

On connecting the resistance across e , current will flow in e due to which terminal potential difference will be less than emf and the balancing length will decrease.

For Problems 8–10

8. (3), 9. (1), 10. (1)

Just after closing, capacitor behaves as a short circuit and all current flows through it, hence ammeter reads zero.

After a long time, capacitor behaves like an open circuit and no current flows through it. Therefore,

$$i = \frac{V_0}{R_1 + R_2} = \frac{30}{10 + 5} = 2 \text{ mA}$$

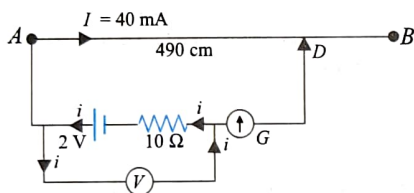
Just after reopening, potential difference across R_2 remains the same as charge on the capacitor does not change initially, hence current remains the same.

For Problems 11–13

11. (2), 12. (3), 13. (4)

Since 2 V is balanced across a length of 500 cm = 5 m, so the potential gradient is $k = 2/5 \text{ Vm}^{-1}$

Reading of voltmeter = potential difference across voltmeter
 $= V_A - V_D = k(4.9)$



$$\text{Reading of the voltmeter} = \frac{2}{5} \times 4.9 = 1.96 \text{ V}$$

Terminal potential difference of the battery will also be 1.96 V. So

$$1.96 = 2 - i \times 10 \text{ or } i = 4 \times 10^{-3} \text{ A}$$

So, resistance of the voltmeter is

$$R_V = 1.96 / (4 \times 10^{-3}) = 490 \Omega$$

For Problems 14–15

14. (4), 15. (2)

$$k = \frac{E_1}{100} = \frac{2}{100} = 0.02 \text{ Vcm}^{-1}$$

$$E_2 = k(75) = 0.02 \times 75 = 1.5 \text{ V}$$

For Problems 16–18

16. (3), 17. (1), 18. (1)

Just after closing, capacitor behaves as short circuit and all current flows through it; hence ammeter reads zero.

After a long time, capacitor behaves as an open circuit and no current flows through it. Therefore,

$$i = \frac{V_0}{R_1 + R_2} = \frac{30}{10 + 5} = 2 \text{ mA}$$

Just after reopening, potential difference across R_2 remains the same initially as charge on capacitor does not change initially. Hence, current remains the same.

For Problems 19–21

19. (1), 20. (1), 21. (4)

$$R_A = R_A = \frac{R \times R_V}{R + R_V} < R$$

$$R_B = R + R_0 > R$$

(Tough)% error in case A.

$$\frac{R_A - R}{R} \times 100 = \left(\frac{R_V}{R + R_V} - 1 \right) \times 100$$

$$\frac{R}{R + R_V} \times 100 = -1\%$$

Percentage error in case B

$$\frac{R_B - R}{R} \times 100 = \frac{R_G}{R} \times 100 = 10\%$$

Hence, percentage error in circuit B is more than that in A.

For Problems 22–24

22. (2), 23. (1), 24. (4)

When ends of ohm meter are shorted, the ohm meter will show zero reading at that time the galvanometer will show full scale deflection. Full scale current $i_g = 0.5 \times 10^{-5} \text{ A}$

$$i_g = \frac{10}{(G + R)} \text{ or } 0.5 \times 10^{-3} = \frac{10}{(100 + R)}$$

$$\text{or } (100 + R) = \frac{10}{0.5 \times 10^{-3}} = 20000$$

$$\text{or } R = 20000 - 100 = 19900 \Omega = 19.9 \text{ k}\Omega$$

If needle deflect to middle of scale the current through the galvanometer will be $(1/2) \times 0.5 \text{ mA}$ i.e., 0.25 mA.

$$\text{Hence } 0.25 \times 10^{-3} = \frac{10}{(100 + 19900 + R')}$$

$$20000 + R' = \frac{10}{0.25 \times 10^{-3}} = 40000$$

$$\text{or } R' = 20000 = 20 \text{ k}\Omega$$

If 5 Ω is measured with this ohm meter, the needle will be near 0 Ω mark, hence the needle will be between C and zero.

Matrix Match Type

1. i. → c.; ii. → d.; iii. → b., d.; iv. → a.

i. If deflection in a galvanometer is in the same direction for the position of jockey on one side of the null point, then for the position of jockey on the other side of the null point, the deflection in the galvanometer should be in the opposite direction. But if emf of the battery in the primary circuit is less than the emf of the cell to be measured, then for all positions of jockey on wire, deflection in the galvanometer will be in one direction only.

ii. Due to protective resistance, the galvanometer will show less deflection when away from the null point. Hence, uncertainty in the location of the null point increases.

iii. For a short potentiometer wire, accuracy is less.

iv. For a long potentiometer wire, accuracy is more.

2. i. → b.; ii. → c.; iii. → c.; iv. → c.

When the switch is closed, the equivalent resistance is R . After opening the switch, the equivalent resistance becomes $2R$. Hence, the equivalent resistance increases.

Also, current through the battery decreases, hence ammeter reading decreases. Current through the left R also decreases. So voltmeter reading decreases and power dissipated through the left R also decreases.

3. i. → c.; ii. → d.; iii. → a.; iv. → b.

For 2 V range: $I_g(R_G + R_1) = 2 \text{ V}$

$$\text{or } R_1 = \frac{2}{10^{-3}} - 40 = 1960 \Omega = 1.96 \text{ k}\Omega$$

For 10 V range: $I_g(R_G + R_1 + R_2) = 10 \text{ V}$

$$R_2 = \frac{10}{1 \times 10^{-3}} - 40 - 1960 = 8000 \Omega = 8 \text{ k}\Omega$$

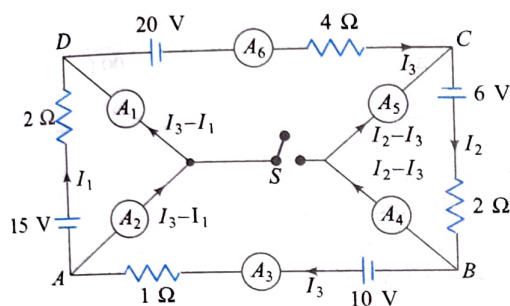
For 100 V range: $I_G(R_G + R_1 + R_2 + R_3) = 100 \text{ V}$

$$R_3 = \frac{100}{10^{-3}} - 40 - 1960 - 8000 = 90000 \Omega = 90 \text{ k}\Omega$$

The overall resistance of the meter on 100 V range is $R_G + R_1 + R_2 + R_3 = 100 \text{ k}\Omega$

The overall resistance of the meter on 2 V range is $R_G + R_1 = 2 \text{ k}\Omega$

5. i. $\rightarrow$ a., b., c., d.; ii. $\rightarrow$ a., b.; iii. $\rightarrow$ b., c.; iv. $\rightarrow$ b., c., d.
When S is open:



$$I_1 = \frac{15}{2} = 7.5 \text{ A}$$

$$I_2 = \frac{6}{2} = 3 \text{ A}$$

Loop ADCBA:

$$15 - 20 + 6 + 10 = 2I_1 + 4I_3 + 2I_2 + 1I_3$$

$$I_3 = -2 \text{ A}$$

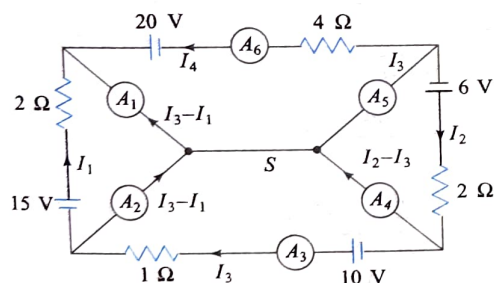
Current in A_1 and A_2 :

$$I_3 - I_1 = -2 - 7.5 = -9.5 \text{ A}$$

Current in A_4 and A_5 :

$$I_2 - I_3 = 3 - (-2) = 5 \text{ A}$$

After closing S:



$$I_1 = \frac{15}{2} = 7.5 \text{ A}$$

$$I_2 = \frac{6}{2} = 3 \text{ A}$$

$$I_3 = \frac{10}{1} = 10 \text{ A}$$

$$I_4 = \frac{20}{4} = 5 \text{ A}$$

Current in A_1 :

$$I_1 + I_4 = 7.5 + 5 = 12.5 \text{ A}$$

Current in A_2 :

$$I_3 - I_1 = 10 - 7.5 = 2.5 \text{ A}$$

Current in A_4 :

$$I_2 - I_3 = 3 - 10 = -7 \text{ A}$$

Current in A_5 :

$$I_4 - I_2 = 5 + 3 = 8 \text{ A}$$

5. i. $\rightarrow$ b., c.; ii. $\rightarrow$ d.; iii. $\rightarrow$ d.; iv. $\rightarrow$ a.

It is possible that small length of potentiometer wire is used for positive terminal of emf. to be measured may not be connected at extreme ends of the potentiometer wire.

Sensitivity of the potentiometer decreases.

Sensitivity of the potentiometer decreases

Accuracy of the potentiometer increases.

6. i. $\rightarrow$ d.; ii. $\rightarrow$ c.; iii. $\rightarrow$ a.; iv. $\rightarrow$ d.

Effective resistance of the circuit = 4Ω

Potential difference across $3 \Omega = 20 \text{ V} - 8 \text{ V} = 12 \text{ V}$

Find currents in resistors using Ohm's law and series parallel and then use Junction law to find current in ammeter.

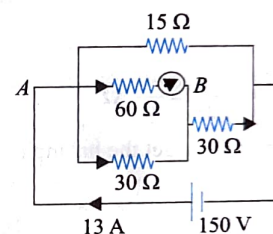
Effective resistance of the circuit = 10Ω

7. i. $\rightarrow$ a.; ii. $\rightarrow$ a.; iii. $\rightarrow$ a.; iv. $\rightarrow$ a.

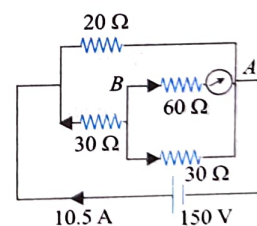
In each case there will be no current in the branch containing voltmeter or ammeter. Therefore, reading of all instruments will be zero.

8. i. $\rightarrow$ a., d.; ii. $\rightarrow$ b.; iii. $\rightarrow$ a., c.; iv. $\rightarrow$ a.

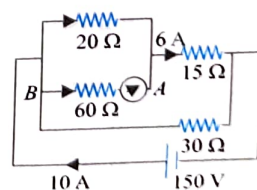
When switch S_1 is closed



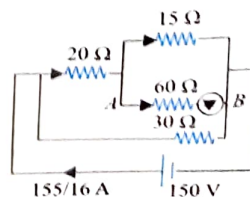
When switch S_2 is closed



When switch S_3 is closed



When switch S_4 is closed



Numerical Value Type

1. (3) Let R be the combined resistance of galvanometer and an unknown resistance and r the internal resistance of each battery. When the batteries, each of emf E are connected in series, the net emf = $2E$ and net internal resistance = $2r$.

$$\text{Current } i_1 = \frac{2E}{R+2r} \text{ or } 1.0 = \frac{2 \times 1.5}{R+2r}$$

$$\text{or } R+2r=3 \quad \dots(i)$$

When the batteries are connected in parallel, the emf remains E and net internal resistance become $r/2$. Therefore, current is

$$i_2 = \frac{E}{R+\frac{r}{2}} = \frac{2E}{2R+r}$$

$$\text{or } 2R+r = \frac{2E}{i_2} = \frac{2 \times 1.5}{0.6} = 5.0 \quad \dots(ii)$$

Solving Eqs. (i) and (ii), we get $r = \frac{1}{3} \Omega$

$$2. (9) \frac{V-2}{40} = \frac{2-0}{40} \text{ or } V=4$$

$$I_1 = \frac{2}{40} = 0.05 \text{ A}$$

$$\therefore \frac{V-2}{R} = 0.45$$

$$\text{or } R = \frac{4-2}{0.45} = \frac{2}{0.45} = \frac{40}{9} \Omega$$

3. (7) This means you will not get the balancing point.

$$i = \frac{2}{1+15} = \frac{1}{8}, E_p = \frac{1}{8} \times 3 \times \frac{7}{2} = \frac{21}{16}$$

$$\frac{21}{16} = i \times 3 \times \frac{9}{2} \text{ or } i = \frac{7}{72} \text{ A}$$

$$\frac{7}{72} = \frac{2}{16+R} \text{ or } R = \frac{32}{7} \Omega$$

4. (0.125) Equivalent resistance of the ammeter is

$$\frac{(480 \Omega)(20 \Omega)}{480 \Omega + 20 \Omega} = 19.2 \Omega$$

The equivalent resistance of the circuit is $140.8 \Omega + 19.2 \Omega = 160 \Omega$. Therefore, current, $i = 20 \text{ V}/160 \Omega = (1/8) \text{ A}$

$$5. (115) \text{ Potential gradient, } k = \frac{IR_p}{L} = \frac{ER_p}{(R+R_p+r)L}$$

(where R_p is resistance of the wire)

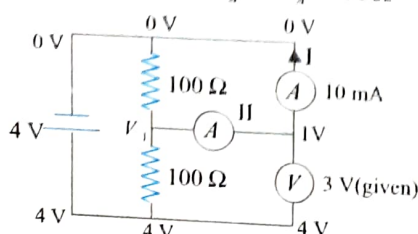
$$\text{or } 50 \times 10^{-3} = \frac{2.5 \times 30}{(R+30+5) \times 10} \text{ or } R = 115 \Omega$$

6. (2) Here all the resistances will be in parallel.

Current through 12Ω is $I_1 = 24/12 = 2 \text{ A}$. Hence, reading of $A_1 = I_1 = 2 \text{ A}$.

7. (1) R_A = resistance of ammeter

$$1 \text{ V} - 0 \text{ V} = (10 \text{ mA}) R_A \text{ or } R_A = 100 \Omega$$



8. (1) $I_G = 10 \text{ mA}$, $G = 10 \Omega$

$S(I - I_G) = I_G G$ where S is shunt in parallel, solve to $S = 0.1 \Omega$

9. (4) Let the voltmeter reading when the voltmeter has zero error be

$$\frac{I_1 R}{I_2 R} = \frac{V_1}{V_2} \text{ or } \frac{1.75}{2.75} = \frac{14.4 - V}{22.4 - V} \text{ or } \frac{7}{11} = \frac{14.4 - V}{22.4 - V}$$

$$\text{or } 7 \times (22.4 - V) = 11(14.4 - V)$$

$$\text{or } 156.8 - 7V = 158.4 - 11V \text{ or } V = 0.4 \text{ V}$$

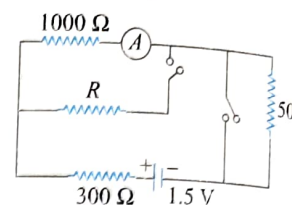
10. (6) Step I: Discuss the circuit when both switches are open:

According to loop rule:

$$1.5 - 300I - 100I - 50I = 0$$

$$\therefore I = \frac{1.5}{450} = \frac{15}{4500}$$

$$= \frac{1}{300} \text{ A}$$



Step II: Discuss the circuit after closing the switch.

In loop $ABCDEA$,

$$-IR + 1.5 - 300I_1 = 0$$

$$\text{or } 300I_1 + IR = 1.5$$

In loop $BCGFB$,

$$-100I + (I_1 - I)R = 0$$

$$\text{or } (I_1 - I)R = 100I$$

$$\text{or } I_1 R = (100 + R)I$$

$$\therefore I_1 = \frac{(100 + R)I}{R}$$

From Eqs. (i) and (ii),

$$300 \frac{(100 + R)I}{R} + IR = 1.5$$

$$\text{or } 300 \frac{(100 + R)}{R} \frac{1}{300} + \frac{R}{300} = 1.5$$

$$\therefore R = 6000 \Omega$$

$$11. (250) \frac{R}{X} = \frac{l}{100 - l}$$

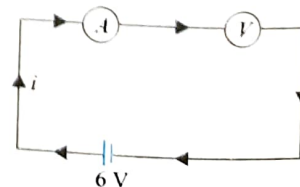
$$\Rightarrow X = \left(\frac{100 - l}{l} \right) R = \left(\frac{100 - 75}{75} \right) (300) = 100 \Omega$$

$$\text{Now, } X = \frac{\rho l}{A} = \frac{\rho l}{(\pi d^2/4)}$$

$$\therefore \rho = \frac{\pi d^2 X}{4l} = \frac{(22/7)(10^{-3})^2 (100)}{(4)(0.314)}$$

$$= 2.5 \times 10^{-4} \Omega - \text{m} = 250 \times 10^{-6} \Omega - \text{m}$$

12. (2) Let R = resistance of ammeter



Potential difference across voltmeter = $6V$ - potential difference across ammeter

$$\text{In first case, } V = 6 - iR$$

$$\text{In second case, } \frac{V}{2} = 6 - (2i)R$$

Solving these two equations, we get

$$V = 4 \text{ volt}$$

$$\therefore V/2 = 2 \text{ volt}$$

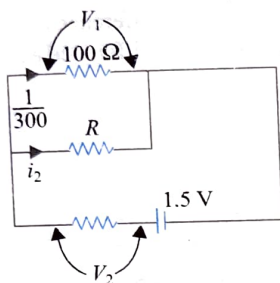
(600) Current with both switches opened is

$$\frac{V}{R_{eq}} = \frac{1.5}{450} = \frac{1}{300} = i$$

After closing the switch, if V_1 and V_2 are potential differences as shown in figure then we have

$$V_1 + V_2 = V \Rightarrow \frac{1}{3} + V_2 = \frac{3}{2}$$

$$V_2 = \frac{9-2}{6} = \frac{7}{6}V$$



Current through battery is

$$i = \frac{7}{6} \times \frac{1}{300} = \frac{7}{1800} \text{ A}$$

By KCL, we have

$$i_2 = \frac{7}{1800} - \frac{1}{300} = \frac{1}{1800}; i_2 R = i_1 R_1$$

$$= R = 180 \left(\frac{1}{3} \right) = 600 \Omega$$

$$14. (4) \frac{V}{l} = \frac{I_1 R}{30} = \frac{I_2 R}{10}$$

$$\varepsilon_{S_1} = I_1(r_1 + R)$$

$$10 = I_1(1 + R) \Rightarrow I_1 = \frac{10}{1 + R}$$

$$\varepsilon_{S_2} = I_2(r_2 + R)$$

$$\Rightarrow 5 = I_2(r_2 + R)$$

$$\Rightarrow I_2 = \frac{5}{2 + R}$$

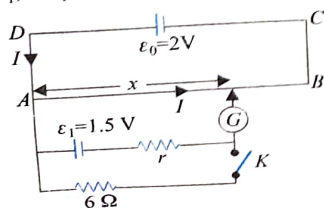
$$\frac{10R}{(1 + R) \times 30} = \frac{5R}{10 \times (2 + R)}$$

$$\Rightarrow 4 + 2R = 3 + 3R$$

$$\Rightarrow R = 4 \Omega$$

15. (1.5) Let λ is resistance per unit length of wire AB. When k is opened

$$I(\lambda x_1) = \varepsilon_1$$



k is closed

$$I\lambda x_1 = \varepsilon_1 - Ir$$

$$I = \frac{\varepsilon_1}{R + r}$$

$$\Rightarrow r = \left(\frac{x_1}{x_2} - 1 \right) R = \left(\frac{0.75}{0.60} - 1 \right) 6$$

$$r = 1.5 \Omega$$

16. (10) When 'S' is open, current through ammeter is

$$i_1 = \frac{100}{10 + 10 + r} = \frac{100}{20 + r}$$

$$\text{When 'S' is closed, } i_2 = \frac{100}{r}$$

$$\text{Given } i_2 = 2i_1 \Rightarrow \frac{100}{r} = 3 \times \frac{100}{20 + r}$$

$$20 + r = 3r$$

$$\Rightarrow r = 10 \Omega$$

17. (20) Let current be I_0 in the original circuit. Potential difference across ammeter is $= E - V_0$

$$\therefore \text{Resistance of ammeter } R_A = \frac{E - V_0}{I_0} \quad \dots(i)$$

After the resistance is added to the voltmeter, the potential difference across voltmeter becomes $\frac{V_0}{10}$

$$\therefore \text{Potential difference across ammeter} = E - \frac{V_0}{10}$$

$$R_A = \frac{E - V_0/10}{10I_0} \quad \dots(ii)$$

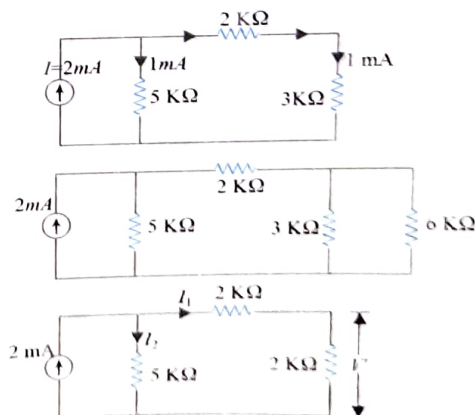
[the new current is $10I_0$]

From (i) and (ii)

$$\frac{10E - V_0}{100I_0} = \frac{E - V_0}{I_0}$$

$$\Rightarrow V_0 = \frac{90E}{99} = \frac{10E}{11} = \frac{10 \times 22}{11} = 20 \text{ V}$$

18. (2) Let the potential gradient along the potentiometer wire PQ be K .



When galvanometer shows zero deflection the current in two loops are independent of each other. If current in loop having R and R_x is i , then

$$iR = Kx$$

$$\text{and } i(R + R_x) = K(3x)$$

$$\dots(i)$$

$$\dots(ii)$$

(ii) - (i)

$$iR_x = 2K_x$$

(iii) ÷ (i)

$$= \frac{R_x}{R} = 2$$

$$\Rightarrow R_x = 2R$$

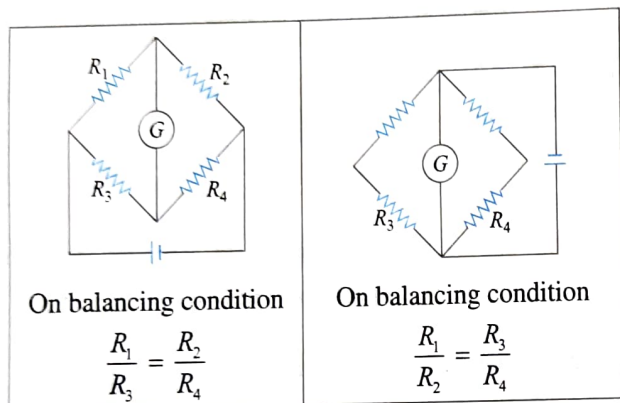
...(iii)

Archives

JEE Main

Single Correct Answer Type

1. (4) Options (1), (2) and (3) are theoretically correct statements.
Now let us check, if the cell and the galvanometer are exchanged.



We observe here that in both cases we are getting same equations, hence the null point does not disturb. So (4) statement is false.

2. (3) $i_g = 5 \times 10^{-3} \text{ A}$
 $G = 15 \Omega$

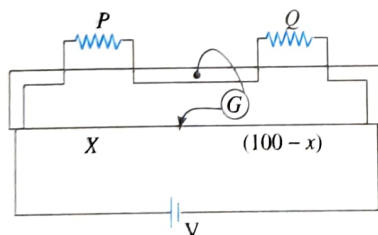
Let series resistance be R .

$$V = i_g (R + G)$$

$$10 = 5 \times 10^{-3} (R + 15)$$

$$R = 2000 - 15 = 1985 = 1.985 \times 10^3 \Omega$$

3. (4)



$$\text{Initially } \frac{P}{Q} = \frac{x}{100-x}$$

...(i)

When resistances are interchanged

$$\frac{Q}{P} = \frac{x-10}{110-x}$$

...(ii)

Also given $P + Q = 1000$

...(iii)

$$\text{From (i)} \frac{P}{P+Q} = \frac{x}{x+100-x} = \frac{x}{100}$$

...(iv)

$$\text{or } \frac{P}{1000} = \frac{x}{100} \text{ or } P = 10x$$

...(v)

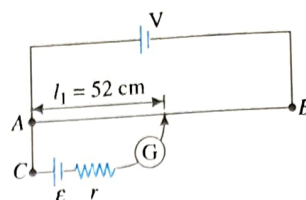
$$\text{From (ii)} \frac{Q+P}{P} = \frac{(x-10)+(110-x)}{(110-x)}$$

$$\text{or } \frac{1000}{P} = \frac{100}{(110-x)}$$

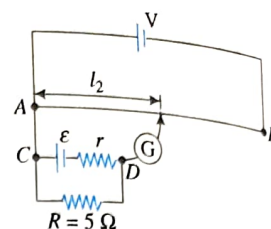
...(vi)

From (v) and (vi), $P = 550 \Omega$

4. (3)



Case I: Initially



Case II: where R is shunted

In case I: The potential difference across the length l_1 will be balanced by the emf of the cell

$$\text{or } \varepsilon = K l_1$$

...(i)

In case II: The potential difference across the length l_2 will be balanced by the potential difference across the shunt resistance ' R '

$$R \left(\frac{\varepsilon}{R+r} \right) = K l_2$$

...(ii)

$$\text{From (ii) / (i) we get } \frac{R}{R+r} = \frac{l_2}{l_1}$$

$$\text{or } r = R \left(\frac{l_1}{l_2} - 1 \right) = 5 \left(\frac{52}{40} - 1 \right) = 5 \left(\frac{52-40}{40} \right) = 1.5 \Omega$$

JEE Advanced

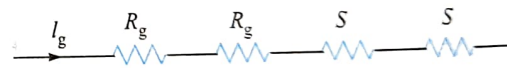
Single Correct Answer Type

1. (3) G_1 is acting as voltmeter and G_2 is acting as ammeter.

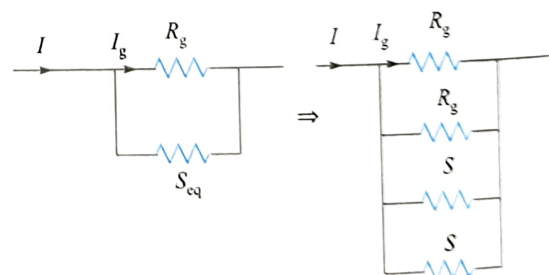
Multiple Correct Answers Type

1. (1), (3)

$$\text{For maximum voltage, } v_{\max} = I[R_g + S_{eq}]$$



For maximum current, here S_{eq} will be parallel combination of two shunt and one galvanometer.



Numerical Value Type

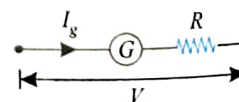
1. (5) Here, current for full scale deflection, $I_g = 0.006 \text{ A}$

Let G be resistance of the galvanometer.

To convert the given galvanometer into a voltmeter of range 0-30 V i.e., $V = 30 \text{ V}$, a resistance R is connected in series with it such that

$$V = I_g (G + R)$$

$$30 = 0.006 (G + 4990)$$



$$\frac{30}{0.006} = (G + 4990)$$

$$\frac{30 \times 1000}{6} = G + 4990$$

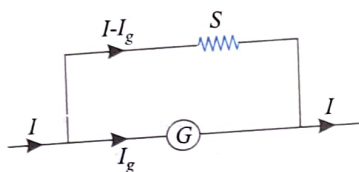
$$5000 = G + 4990$$

$$G = 10 \Omega$$

To convert the given galvanometer into ammeter of range of 0–1.5 A i.e., $I = 1.5$ A a resistance of value S is connected in parallel with it such that

$$(I - I_g)S = I_g G$$

$$(1.5 - 0.006) \times \frac{2n}{249} = 0.006 \times 10$$



$$\frac{2n}{249} = \frac{0.06}{1.494}$$

$$2n = \frac{0.06 \times 249}{1.494} = 10 \text{ or } n = 5$$

2. (5.55) Number of turns $n = 50$ turns

$$\text{Area } A = 2 \times 10^{-4} \text{ m}^2$$

Magnetic field $B = 0.02$ T

Torsion constant of wire $K = 10^{-4}$

Full scale deflection angle, $\theta = 0.2$ rad

Resistance of galvanometer $R_g = 50 \Omega$

Range of ammeter $I_A = 0 - 1.0$ A

We know torque on the coil, $\tau = MB = C\theta$

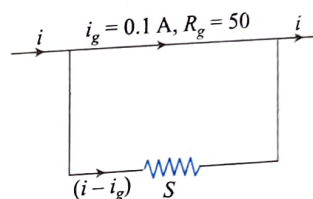
Magnetic moment of the coil $M = ni_g A$

It means $BINA = C\theta$

$$0.02 \times i_g \times 50 \times 2 \times 10^{-4} = 10^{-4} \times 0.2 \times 10$$

$$i_g = 0.1 \text{ A}$$

To convert galvanometer into an ammeter, we need to apply a shunt resistance in parallel with the galvanometer.



From figure we can write $i_g \times R_g = (i - i_g)S$

$$\text{or } 0.1 \times 50 = (1 - 0.1)S$$

$$\text{which gives } S = \frac{50}{9} = 5.55 \Omega$$

$$\text{Hence } S = 5.55 \Omega \text{ or } 5.56 \Omega$$

Concept Application Exercises

Exercise 7.1

- Resistance of a bulb is given by $R = V^2/P$. Now, 100 W bulb will have less resistance. So we are connecting less resistance in series with the heater. It means potential difference across the heater will increase. So the heater will give more heat.
- We know $R = V^2/P$. Therefore, resistance of the first bulb is $R_1 = V^2/P_1$. And resistance of the second bulb is $R_2 = V^2/P_2$. In series, same current will pass through each bulb. Therefore, power developed across the first bulb is $P_1' = I^2 (V^2/P_1)$ and that across the second bulb is $P_2' = I^2 (V^2/P_2)$.

$$\frac{\text{Power in bulb 1}}{\text{Power in bulb 2}} = \frac{P_2}{P_1} < 1 \text{ (as } P_2 < P_1 \text{)}$$

Hence, the bulb rated 220 V and 40 W will glow more.

- Here, $P = 30 \text{ W}$, $V = 6 \text{ V}$. Therefore, resistance of the bulb is

$$R_1 = V^2/P = (6)^2/30 = 1.2 \Omega$$

Current capacity of the bulb

$$I = \frac{P}{V} = \frac{30}{6} = 5 \text{ A}$$

Supply voltage, $V' = 120 \text{ V}$

Let R_2 be the resistance used in series with the bulb to have a current of 5 A in the circuit. Total resistance is

$$R_2 + R_1 = R_2 + 1.2$$

Therefore, current $I = V'/(R_2 + 1.2)$

$$\text{or } 5 = \frac{120}{R_2 + 1.2} \text{ or } R_2 = \frac{120}{5} - 1.2 = 22.8 \Omega \text{ in series}$$

Alternatively

Potential difference across the bulb should not exceed 6 V.

$$\text{For this } \frac{6}{114} = \frac{R_1}{R_2} \text{ or } R_2 = \frac{114}{6} R_1 = \frac{114}{6} \times 1.2 = 22.8 \Omega$$

- Since $m = \rho Al$, the lengths and hence the resistances of the wires are in the ratio 1:2. Also when they are connected in series, heat produced will be in the ratio 1:2, because $H = I^2 R$.

$$5. P = \frac{V^2}{R} = \frac{V^2 A}{\rho l}$$

To boil water in 10 min, power should be increased. To increase power, length should be decreased.

- Voltage drop across line is 10 V, and current is $10/2 = 5 \text{ A}$. Current drawn by one lamp is $100/230 \text{ A}$. So number of lamps is $5 \times 230/100 = 11.5$. Hence, maximum number of lamps is 11.
- Total wattage consumed (units) is

$$\frac{7 \times 40 \times 6}{1000} + \frac{2 \times 60 \times 6}{1000} + \frac{5 \times (220 \times 0.4) \times 1}{1000} + \frac{(220)^2}{48.4} \times \frac{10}{1000}$$

$$= 12.84 \text{ per day}$$

Number of units consumed in one month (Jan) is $12.84 \times 31 = 398.04$. Total bill is $2 \times 398.04 = ₹ 796.08$.

$$8. (a) R_1 = \frac{200^2}{300}, R_2 = \frac{200^2}{600}$$

The resistance of the first bulb is more. Hence, in series, the first bulb will produce more illumination.

$$(b) P = \frac{P_1 P_2}{P_1 + P_2} = \frac{300 \times 600}{300 + 600} = 200 \text{ W}$$

$$(c) P = P_1 + P_2 = 300 + 600 = 900 \text{ W}$$

- 25 W bulb will glow brighter. This is because $P = I^2 R$ where I is the current flowing and R is the resistance of the appliance. When I is same, $P \propto R$. The resistance of 25 W bulb is more and therefore P is more.

Exercise 7.2

$$1. P_{\max} = (i_{\max})^2 R \text{ or } 18 = i_{\max}^2 \times 2 \text{ or } i_{\max} = 3 \text{ A}$$

Here $R_{\text{eq}} = 3 \Omega$.

So the maximum power of circuit is $i_{\max}^2 R_{\text{eq}} = 27 \text{ W}$

- As the three bulbs are in series and identical, initially when switch S is open,

$$V_A = V_B = V_C = \frac{1}{3} V$$

$$\text{Also } P_A = P_B = P_C = \frac{(V/3)^2}{R} = \frac{V^2}{9R} = P$$

Now, the switch S is closed,

- (a) The bulb C is short-circuited and hence potential difference across it is $V'_C = 0$ and so $V'_A = V'_B = V/2$ with $V'_C = 0$. And hence,

$$P'_A = P'_B = \frac{(V/2)^2}{R} = \frac{V^2}{4R}$$

$$\text{i.e., } P'_A = P'_B = \frac{9}{4} P,$$

i.e., intensities of bulbs A and B will increase and become 2.25 of their initial values.

- (b) As $V'_C = 0$, no current will pass through bulb C , so it will give no light, i.e., $P'_C = 0$.

- (c) As R_T changes from $3R$ to $2R$, so the current in the circuit will change from

$$I = \frac{V}{3R} \text{ to } I' = \frac{V}{2R}, \text{ i.e., } I' = \frac{3}{2} I$$

Thus, current in the circuit will increase and will become 1.5 times its initial value.

- (d) As explained earlier, initially the voltage V divides equally across A , B , and C , i.e., $V_A = V_B = V_C = V/3$. Now when the switch S is closed, bulb C is short-circuited, i.e., $V'_C = 0$, so voltage V now will divide equally across A and B , i.e., $V'_A = V'_B = V/2$. So voltage across bulbs A and B will change from $V/3$ to $V/2$, while across C from $V/3$ to zero, i.e., voltage drop across A and B will increase while across C it will decrease and become zero. Further as R_T changes from $3R$ to $2R$, so power consumed in the circuit changes from

$$P_T = \frac{V^2}{3R} \text{ to } P'_T = \frac{V^2}{2R}, \text{ i.e., } P'_T = \frac{3}{2} P_T$$

Thus, power dissipated in the circuit increases and becomes 1.5 times its initial value.

3. Power drawn by motor is $P = VI = 50 \times 12 = 600 \text{ W}$

$$\text{Power loss} = \frac{70}{100} \times 600 = 420 \text{ W}$$

This power is lost in the form of heat in the resistance.

$$I^2 R = 420 \text{ or } (12)^2 R = 420 \text{ or } R = 2.9 \Omega$$

Heat produced = heat loss through surface of wire

$I^2 R = H 2\pi r l$
where H is the heat loss per unit surface area of the wire.

$$I^2 \frac{\rho l}{a} = H 2\pi r l$$

Put $a = \pi r^2$, therefore $I^2 \propto r^2$

or $\left(\frac{15}{30}\right)^2 = \left(\frac{0.15}{r}\right)^2$ or $r = 0.238 \text{ mm}$

$P_{in} = 12 \times 2 = 240 \text{ W}$, heat loss = $9 \text{ cal s}^{-1} = 9 \times 4.20 = 37.8 \text{ J s}^{-1}$

$P_{out} = 240 - 37.8 = 202.2 \text{ W}$

Efficiency $\eta = \frac{P_{out}}{P_{in}} \times 100 = 84.25\%$

Heat produced = heat loss or $\frac{V^2}{R} = \frac{KA\Delta T}{l}$

Here $V = 400 \text{ V}$, $K = 4 \times 10^{-4} \text{ cal/cm s}^\circ\text{C} = 16.8 \times 10^{-4} \text{ J/cm s}^\circ\text{C}$

$\Delta T = 100$, $l = 1 \text{ mm} = 0.1 \text{ cm}$, $A = 6(40)^2 \text{ cm}^2$

$\frac{400^2}{R} = \frac{16.8 \times 10^{-4} \times 6 \times (40)^2 \times 100}{0.1}$ or $R = 9.92 \Omega$

7. (a) When the lamps are connected in parallel, potential difference V across each lamp will be the same and will be equal to potential difference necessary for full brightness of each bulb. As $P = V^2/R$, illumination produced by the second bulb will be higher than that produced by the first bulb, i.e., the bulb having lower resistance will shine more brightly.

(b) When R_1 burns out, power is dissipated in R_2 only. Because internal resistance is quite low in lighting circuit, potential difference is still equal to V , hence power is dissipated in the second lamp, i.e.,

$$\frac{V^2}{R_2} < \left(\frac{V^2}{R_1} + \frac{V^2}{R_2} \right), \text{ net power consumed initially.}$$

In other words, net illumination will now decrease.

(c) When the two lamps are connected in series, the potential difference across each lamp will be different, but current I flowing through each lamp will be the same, as $P = I^2 R$.

Hence, illumination produced by the first lamp will be higher as compared to that produced by the second lamp, i.e., the lamp having higher resistance will glow more brightly.

(d) When lamp of resistance R_2 burns out and only lamp of resistance R_1 is connected in the circuit, then current flowing through the circuit will change. Let new current be i' . Because potential difference still remains the same (due to low internal resistance),

$$i' R_1 = i(R_1 + R_2) \text{ or } i' = [i(R_1 + R_2)] / R_1$$

If P' is the power consumed, then

$$P' = i'^2 R_1 = [i^2 (R_1 + R_2) (R_1 + R_2)] / R_1$$

When both the lamps were present, then total power consumed was given by

$$P_s = P_1 + P_2 = i^2 (R_1 + R_2), \text{ i.e., illumination gets increased when one bulb is used.}$$

8. Let the voltage supply be V . Let resistance of each bulb be R . Total resistance of circuit is nR . Power illuminated by all bulbs,

$$P_1 = \frac{V^2}{nR}$$

Power illuminated by one bulb,

$$P_2 = \frac{P_1}{n} = \frac{V^2}{n^2 R}$$

After one bulb is fused, the powers are

$$P'_1 = \frac{V^2}{(n-1)R}, P'_2 = \frac{V^2}{(n-1)^2 R}$$

(a) Fractional change in the illumination of all the bulbs is

$$f_1 = \frac{P'_1 - P_1}{P_1} = \frac{P'_1}{P_1} - 1 = \frac{n}{n-1} - 1 = \frac{1}{n-1}$$

(b) Fractional change in the illumination of one bulb is

$$f_2 = \frac{P'_2 - P_2}{P_2} = \frac{n^2}{(n-1)^2} - 1 = \frac{2n-1}{(n-1)^2}$$

9. Power input = $P_i = VI = 100 \times 6 = 600 \text{ W}$

Power output = 150 W

Efficiency: $\eta = \frac{150}{600} \times 100 = 25\%$

Heat produced is $600 - 150 = 450 \text{ W}$

or $I^2 R = 450$ where R is the resistance of windings

or $R = \frac{450}{6^2} = \frac{450}{36} = 12.5 \Omega$

10. Let stabilizer give voltage V , then power produced by the bulb is

$$P = \left(\frac{V}{220} \right)^2 100 \text{ or } \frac{dP}{P} = 2 \frac{dV}{V}$$

If $\frac{dV}{V} = \pm 1\%$, then $\frac{dP}{P} = \pm 2\%$

Hence, the maximum power produced is

$$P_{\max} = 100\% + 2\% = 102 \text{ W}$$

Minimum power produced is

$$P_{\min} = 100\% - 2\% = 98 \text{ W}$$

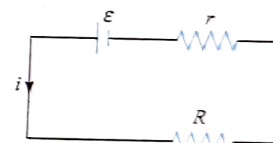
11. Efficiency = $\eta = \frac{\text{Output power}}{\text{Input power}}$

$$\eta = \frac{i^2 R}{\varepsilon i} \text{ where } i = \frac{\varepsilon}{R+r}$$

$$\eta = \frac{R}{R+r} \text{ or } 0.6 = \frac{R}{R+r} \text{ or } 2R = 3r$$

Therefore, new efficiency

$$\eta_1 = \frac{6R}{6R+r} = 0.9 = 90\%$$



Exercises

Single Correct Answer Type

1. (4) $R_{200} = \frac{200 \times 200}{100} = 400 \Omega$

So $400 = R_0 [1 + 0.005 \times 2000]$

or $R_0 = \frac{400}{11} \Omega \approx 36 \Omega$

Hence, current

$$I = \frac{200}{R_0} = 5.5 \text{ A}$$

2. (1) Power $P = I^2 R = \left(\frac{V}{R+3r} \right)^2 R$ or $P \propto \frac{1}{(R+3r)^2}$

Power ratio = $\left(\frac{V_1}{V_2} \right)^2 = \left(\frac{4.5}{1.5} \right)^2 = 3^2 = 9$

3. (1) Maximum current flows through bulb 1.

4. (2) When switch S_2 is closed, the total current shall flow through the connecting wire only, which is supposed to have zero resistance.
5. (4) Suppose V is the voltage of the supply and R is the resistance of each bulb. Now, $R_p = R/3$ and current in ammeter is $I = V/R_p = 3V/R$, provided all three bulbs are working properly. If one bulb has broken down, then

$$R_p = \frac{R}{2} \quad \text{and} \quad I = 2\frac{V}{R}$$

Therefore, current decreases and since current through each bulb is V/R , the same as before, brightness of bulbs is not affected.

6. (1) Consider two extreme cases: (i) When the resistance of the rheostat is zero, the current through Q is zero since Q is short-circuited. The circuit is then essentially a battery in series with lamp P . (ii) When the resistance of the rheostat is very large, almost no current flows through it. So the currents through P and Q are almost equal. The circuit is essentially a battery in series with lamps P and Q .

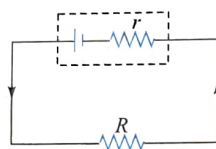
7. (4) Let current I flow through the circuit.

Energy dissipated in load is $I^2 R$.

Energy dissipated in the complete circuit is $I^2 (r + R)$.

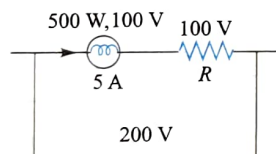
Therefore, the efficiency is

$$\frac{I^2 R}{I^2 (R + r)} = \frac{R}{R + r}$$



8. (2) Let both the batteries be connected in series. Power output is maximum when external resistance is 2Ω . Current in the circuit is $4 \text{ V}/4 \Omega = 1 \text{ A}$ and power in the external circuit is $(1)^2 \times 2 = 2 \text{ W}$.

9. (2) $P = VI$ or $I = \frac{P}{V}$ or $I = \frac{500 \text{ W}}{100 \text{ V}} = 5 \text{ A}$



Now, $5R = 100$ or $R = 20 \Omega$

10. (2) $P = \frac{V^2}{I}$ or $P \propto \frac{1}{I}$, $P' \propto \frac{1}{I - \frac{10}{100} I}$

$$\frac{P'}{P} = \frac{10}{9} \quad \text{or} \quad \left(\frac{P'}{P} - 1 \right) \times 100 = \left(\frac{10}{9} - 1 \right) \times 100$$

$$\frac{P' - P}{P} \times 100 = \frac{100}{9} \approx 11$$

11. (2) Heat developed is

$$H = \frac{V^2}{R} t$$

Heat developed will be doubled when R is halved. Further, $R = \rho l / (\pi r^2)$ and $H = V^2 \pi r^2 t / \rho l$. So heat produced will be doubled when both the length and radius of the wire are doubled.

12. (2) Heat produced $\propto I^2$, initial heat produced $= k (20)^2$, final heat produced $= 2k (20)^2$. If final current is I' , then

$$kI'^2 = 2 \times k \times (20)^2 \quad \text{or} \quad I' = 20\sqrt{2} \text{ A}$$

13. (2) Power $= \frac{V^2}{R}$

$$\text{or} \quad V = \sqrt{PR} = \sqrt{2 \times 10} = \sqrt{20} = 2\sqrt{5} \text{ V}$$

Clearly, voltage across single resistor of 10Ω cannot exceed $2\sqrt{5} \text{ V}$. Note that the resistance of parallel combination is half of 10Ω . Thus, the maximum possible voltage between A and B is $3\sqrt{5} \text{ V}$.

14. (4) Resistance of bulb is

$$\frac{1.5 \times 1.5}{4.5} \Omega = 0.5 \Omega$$

Resistance of parallel combination,

$$R = \frac{1 \times \frac{1}{2}}{1 + \frac{1}{2}} \Omega = \frac{1}{3} \Omega$$

$$\text{Now, } r = \frac{E - V}{V} R \quad \text{or} \quad \frac{8}{3} = \frac{E - 1.5}{1.5} \times \frac{1}{3} \quad \text{or} \quad E = 13.5 \text{ V}$$

15. (3) Power of the heater is $P = 1000 \text{ W}$. Potential difference is $V = 100 \text{ V}$. Therefore, resistance is

$$R_1 = \frac{V^2}{P} = \frac{100 \times 100}{1000} = 10 \Omega$$

Now resistance of the circuit is

$$R_2 = 10 + \frac{10R}{10 + R} = \frac{100 + 20R}{10 + R}$$

Therefore, power is

$$\frac{V^2}{R_2} = 625 \text{ W}$$

$$\text{or} \quad R_2 = \frac{V^2}{625} \quad \text{or} \quad \frac{100 + 20R}{10 + R} = \frac{625}{100 \times 100} \Rightarrow R = 15 \Omega$$

16. (4) $R_{60} = \frac{120 \times 120}{60} \Omega = 240 \Omega$

$$\text{Current} = \frac{120}{240 + 6} \text{ A} = \frac{120}{246} \text{ A}$$

$$\text{Voltage across bulb} = \frac{120}{246} \times 240 \text{ V} = 117.1 \text{ V}$$

$$R_{240} = \frac{120 \times 120}{240} \Omega = 60 \Omega$$

Resistance of parallel combination is

$$\frac{60 \times 240}{60 + 240} \Omega = 48 \Omega$$

Total resistance is $(48 + 6) \Omega = 54 \Omega$.

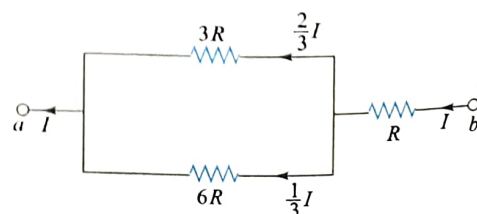
Current I is $120/54 \text{ A}$. Voltage across parallel combination is

$$\frac{120}{54} \times 48 \text{ V} = 106.7 \text{ V}$$

Change in voltage is $(117.1 - 106.7) \text{ V} = 10.4 \text{ V}$.

17. (3) Let current flow from b to a as shown (figure).

$$\text{Ratio is } \left(\frac{2}{3} I \right)^2 3R : \left(\frac{1}{3} I \right)^2 6R : I^2 R \quad \text{or} \quad \frac{4}{3} : \frac{2}{3} : 1 \quad \text{or} \quad 4 : 2 : 3$$



- (1) Let resistors P , Q , and R have resistance r . The effective resistance across the source is

$$R_{\text{eff}} = r + r \parallel r = r + \frac{(r)(r)}{r+r} = r + \frac{r}{2} = \frac{3r}{2}$$

Current drawn from source is

$$I_s^2 R_{\text{eff}} = 12$$

$$\text{or } I_s = \sqrt{\frac{12}{R_{\text{eff}}}} = \sqrt{\frac{8}{r}} \text{ A}$$

Since Q and R have equal resistance r , each draws a current of I , which is given by

$$I = \frac{1}{2} I_s = \sqrt{\frac{2}{r}} \text{ A}$$

Heat dissipation in R can now be determined and is given by

$$I^2 r = \left(\sqrt{\frac{2}{r}}\right)^2 r = 2 \text{ W}$$

19. (1) Since all the three resistors are in series, the same current will flow through them. Their resistances are given by $R = V^2/P$.

$$R_{40} = \frac{240 \times 240}{40} = 1440 \Omega$$

$$R_{60} = 960 \Omega \text{ and } R_{100} = 576 \Omega$$

Total resistance is $R = 2976 \Omega$

$$I = \frac{240}{2976} = 0.0806 \text{ A}$$

Potential difference across the 40 W bulb is

$$1440 \times 0.0806 = 116 \text{ V}$$

20. (2) Let current in 5 Ω resistor be I_1 and in 4 Ω resistors be I_2 , then $I_1/I_2 = 1/2$. The heat generated in the 5 Ω resistor is 10 cal s^{-1} . Given $I_1^2 \times 5 = 10$ or $(2I_2)^2 \times 5 = 10$ or $I_2^2 = 1/2$. Heat in the 4 Ω resistor will be $(I_2)^2 \times 4 = 2 \text{ cal } s^{-1}$.

21. (2) For the maximum power, external resistance is equal to internal resistance. Therefore, $2R = 4$ or $R = 2 \Omega$.

$$22. (1) \text{ Net resistance of circuit, } R = \frac{10 \times 5}{10 + 5} + \frac{10 \times 5}{10 + 5} = \frac{20}{3} \Omega$$

Heat generated in circuit per minute is

$$Q = I^2 R t = (10)^2 \times \frac{20}{3} \times (10 \times 60)$$

$$= 4 \times 10^5 \text{ J} = \frac{4 \times 10^5}{4.2 \times 10^3} \text{ kcal}$$

$$m = \frac{Q}{L} = \frac{4 \times 10^5}{4.2 \times 10^3 \times 80} = 1.19 \text{ kg}$$

$$23. (1) P_I = I^2 (3R), P_{II} = I^2 \left(\frac{2R}{3}\right), P_{III} = I^2 \left(\frac{R}{3}\right)$$

$$P_{IV} = I^2 \left(\frac{3R}{2}\right)$$

$$\therefore III < II < IV < I$$

24. (2) The heat supplied under these conditions is the change in internal energy $Q = \Delta U$. The heat supplied is

$$Q = I^2 R t = 1 \times 1 \times 100 \times 5 \times 60 = 30000 \text{ J} = 30 \text{ kJ}$$

25. (1) All resistances are in parallel.

So for parallel combination

$$H = \frac{V^2}{R} t \text{ or } H \propto \frac{1}{R}$$

26. (4) Let the resistance of the lamp filament be R . Then $100 = (220)^2/R$. When the voltage drops, expected power is $P = (220 \times 0.9)^2/R'$. Here, R' will be less than R , because now

the rise in temperature will be less. Therefore, P is more than $(220 \times 0.9)^2/R = 81 \text{ W}$. But it will not be 90% of the earlier value, because fall in temperature is small. Hence, option (4) is correct.

27. (2) In the first case,

$$\frac{(3E)^2}{R} t = (m) s \Delta T \quad \dots(i)$$

When the length of the wire is doubled, resistance and mass both are doubled. Therefore, in the second case,

$$\frac{(NE)^2}{2R} t = (2m) s \Delta T \quad \dots(ii)$$

Dividing Eq. (ii) by Eq. (i), we get $N = 6$.

28. (4) L is the latent heat of vaporization of water, the heat required for producing 1 g of steam. So

$$L = 540 \text{ cal} = 540 \times 4.2 = 2268 \text{ J}$$

Energy supplied is 1080 Js^{-1} . Time required to boil 100 g of water is

$$t = 540 \times 4.2 \times 100/1080 = 210 \text{ s}$$

29. (3) Given $P_A = 60 \text{ W}$, $P_B = 100 \text{ W}$. We know that current flowing through the bulb is $I = P/V$. We also know that as both the bulbs are connected in parallel, potential difference (V) across both the bulbs is the same. Thus, $I \propto P$. Since the power of bulb B is greater than that of bulb A , bulb B draws more current than bulb A .

30. (1) As both bulbs are in series, current through them will be the same. But resistance of 25 W bulb is more. Hence, from the relation $P = I^2 R$, the 25 W bulb will glow more brightly.

31. (3) Let the resistance of the two heaters be denoted by R_1 and R_2 , respectively. Then

$$R_1 = \left(\frac{V^2}{P_1}\right) \text{ and } R_2 = \left(\frac{V^2}{P_2}\right)$$

If the resistance of the series combination is denoted by R_s and the corresponding power by P_s , then $R_s = R_1 + R_2$. So

$$\frac{V^2}{P_s} = \frac{V^2}{P_1} + \frac{V^2}{P_2}$$

$$\text{or } P_s = \frac{P_1 P_2}{P_1 + P_2} = \frac{1000 \times 1000}{2000} = 500 \text{ W}$$

32. (2) Let R be the resistance of each lamp. If E is the applied emf, then the current in the circuit I_1 is given by

$$I_1 = \frac{E}{R + (R/2)} = 2E/3R$$

Current flowing through L_2 or L_3 is

$$\frac{1}{2} \left[\frac{2E}{3R} \right] = \frac{E}{3R}$$

When L_1 is fused, the whole current flows through L_2 and L_3 . Thus $I_2 = E/2R$. So current through L_1 decreases and the current through L_2 increases.

33. (3) The rate of heat developed is given by

$$\frac{V^2}{R} = \frac{(210)^2}{20} = 2205 \text{ Js}^{-1} = \frac{2205}{4.2} \text{ cal s}^{-1}$$

Let m be the amount of ice melting per sec and, then

$$m \times 80 = \frac{2205}{4.2} \text{ or } m = 6.56 \text{ gs}^{-1}$$

34. (2) Current required by each bulb is

$$I = \frac{P}{V} = \frac{100}{220} \text{ A}$$

If n bulbs are joined in parallel, then

$$nI = I_{\text{fuse}} \quad \text{or} \quad n \times \frac{100}{220} = 10 \quad \text{or} \quad n = 22$$

35. (2) If N is the initial number of turns in the coil and r is the radius of coil, then its resistance is

$$R = \rho \frac{L}{A} = \rho \frac{N 2\pi r}{A} \quad \text{or} \quad \frac{V^2 t A}{4.2 \rho N 2\pi r} = Q = msd\theta$$

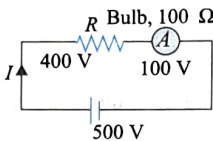
or $\frac{t}{n} = \frac{ms d\theta \times 4.2 \rho 2\pi r}{V^2 A} = \text{constant}$

or $\frac{t_1 / N_1}{t_2 / N_2} = 1$ or $t_2 = \frac{N_2}{N_1} t_1 = \left(\frac{9}{10} \frac{N_1}{N_1} \right) t_1 = \frac{9}{10} \times 16 \text{ min}$

36. (3) Resistance of bulb is $R_b = \frac{V^2}{P} = \frac{(100)^2}{100} = 100 \Omega$

If bulb consumes 100 W, then potential difference across the bulb should be 100 V. So $I = 100/100 = 1$ A. And potential difference across R is $500 - 100 = 400$ V. So

So $R = \frac{400}{I} = \frac{400}{1} = 400 \Omega$



37. (4) $P = VI$ or $50 = 5 \times I$ or $I = 10$ A

Power lost in cable $= I^2 R = 10 \times 10 \times 0.2 = 2$ W

Power supplied to the tape recorder $= 50 \text{ W} - 2 \text{ W} = 48 \text{ W}$

38. (3) In this option, emf is least and resistance is maximum, so bulbs will be dimmest in this option.

39. (3) Initially, $R_{\text{eq}} = 5R/3$. Finally, $R_{\text{eq}} = 3R/2$.

Equivalent resistance decreases, so current increases in circuit and in 1 also. Hence, brightness of 1 increases. It means pd across 1 increases, so across 2 pd decreases, hence brightness of 2 decreases.

Initially, pd across 4 is $V_{4i} = \frac{1}{2} \left[\frac{(2R/3)\epsilon}{2R/3 + R} \right] = \frac{\epsilon}{5}$

Finally, $V_{4f} = \frac{(R/2)\epsilon}{R/2 + R} = \frac{\epsilon}{3}$

Since $V_{4f} > V_{4i}$; hence, brightness of 4 increases.

40. (2) Energy stored in capacitor when it is charged up to 2 V is

$$\frac{1}{2} \times 10 \times 2^2 = 20 \mu\text{J} = u_1 \text{ (suppose)}$$

Energy stored in capacitor when it is charged up to 4 V is

$$\frac{1}{2} \times 10 \times 4^2 = 80 \mu\text{J} = u_2 \text{ (suppose)}$$

Increase in charge $= 40 - 20 = 20 \mu\text{C}$

Energy drawn from cell $= 20 \times 4 = 80 \mu\text{J} = u$ (suppose)

Heat produced $= u_1 + u - u_2$
 $= 20 + 80 - 80 = 20 \mu\text{J}$

41. (3) $i = \frac{dq}{dt}$ = slope of $q-t$ graph

$$= -5 \text{ (which is constant)}$$

Amount of heat generated in time $t = i^2 R T \propto t$

42. (2) Circuit is forming a Wheatstone bridge, so $R = 2R$. For the maximum power transfer, $2R = r$.

43. (2) $H = \int_0^4 \frac{E^2}{R} dt = \int_0^4 \frac{(6t)^2}{12} dt = 64 \text{ J}$

44. (1) Initial charge on capacitor: $Q_1 = 2 \times 100 = 200 \mu\text{C}$

Final charge: $Q_2 = 2 \times 300 = 600 \mu\text{C}$

$$W_b = 300 [Q_2 - Q_1] = 12,0000 \mu\text{J}$$

$$\Delta U = \frac{1}{2} C [300^2 - 100^2] = 80000 \mu\text{J}$$

$$\text{Heat} = W_b - \Delta U = 40000 \mu\text{J} = 0.04 \text{ J}$$

45. (1) Time period: $T_0 = 2T$, $\omega = \frac{2\pi}{T_0} = \frac{\pi}{T}$

$$I = I_0 \sin \omega t = I_0 \sin \left(\frac{\pi}{T} t \right)$$

$$Q = \int_0^T I dt$$

$$\Rightarrow I_0 \int_0^T \sin \left(\frac{\pi}{T} t \right) dt$$

$$= \frac{-I_0}{\pi/T} \left[\cos \left(\frac{\pi}{T} t \right) \right]_0^T = \frac{2TI_0}{\pi}$$

or $I_0 = Q\pi/2T$

$$\text{Heat} = \int_0^T I^2 R dt = \int_0^T I_0^2 \sin^2 \left(\frac{\pi}{T} t \right) R dt = Q^2 \pi^2 R / 8T$$

46. (1) $I = I_0 - kt$, at $t = T$, $I = 0$ or $k = I_0/T$

So $I = I_0 - I_0 t/T$, $q = \int_0^T I dt = I_0 \int_0^T \left(1 - \frac{t}{T} \right) dt$

or $q = \frac{I_0 T}{2}$ or $I_0 = \frac{2q}{T}$

$$\text{Heat} = \int_0^T I^2 R dt = I_0^2 R \int_0^T \left(1 - \frac{t}{T} \right)^2 dt = \frac{4q^2 R}{3T}$$

47. (4) $\frac{V^2}{R} t = ms\Delta T$

For first case: $\frac{(3E)^2}{(\rho L/A)} t = ms\Delta T$

For second case: $\frac{(NE)^2}{\rho 2L/A} t = 2ms\Delta T$

Dividing, we get $N = 6$.

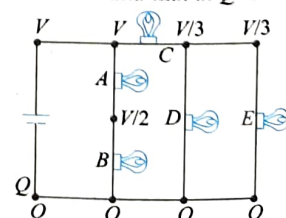
48. (4) All resistances are in parallel. The potential difference across each resistance is same.

$$\Delta H = I^2 R t = \frac{V^2 t}{R}$$

$$\Delta H \propto \frac{1}{R}$$

R_s is the smallest and generates maximum heat.

49. (2) Let potential at P is V and that at Q is 0



Potential difference across bulb C is maximum.

50. (1) For the branch containing A , B , and C .

$$R_{\text{eq}} = \frac{3R}{2}$$

Current in A: $I_1 = \frac{E}{R_{eq}} = \frac{2E}{3R}$

Current in each of B and C:

$$I_1' = \frac{I_1}{2} = \frac{E}{3R}$$

For the branch containing D, E, F, G, and H:

$$R_{eq} = \frac{7R}{3}$$

Current through D and H:

$$I_2 = \frac{E}{R_{eq}} = \frac{3E}{7R}$$

Current through each of E, F, and G:

$$I_2' = \frac{I_2}{3} = \frac{E}{7R}$$

Since $I_1 > I_2 > I_1' > I_2'$
 $A > D = H > B = C > E = F = G$

Multiple Correct Answers Type

1. (2), (3)

$$P = \frac{V^2}{R} \text{ or } R = \frac{V^2}{P} \text{ or } R \propto V^2,$$

i.e., $\frac{R_1}{R_2} = \left(\frac{200}{300}\right)^2 = \frac{4}{9}$

When connected in series, potential drop is in the ratio of their resistances. So,

$$\frac{V_1}{V_2} = \frac{R_1}{R_2} = \frac{4}{9}$$

Now, $P = I^2 R$

or $P \propto R$ (in series I is the same)

or $\frac{P_1}{P_2} = \frac{R_1}{R_2} = \frac{4}{9}$

2. (1), (2), (3)

Current through the circuit is $I = \varepsilon / (r + R)$.

Power dissipated in external resistance is

$$P = I^2 R = \frac{\varepsilon^2}{(r + R)^2} R$$

We know that power dissipated is maximum when $r = R$.

$$P_{max} = \frac{\varepsilon^2 r}{(2r)^2} = \frac{\varepsilon^2}{4r} = 9 \text{ W}, \quad i = \frac{\varepsilon}{2r} = 3 \text{ A}$$

Solve to get, $\varepsilon = 6 \text{ V}$ and $r = 1 \Omega$.

3. (1), (4)

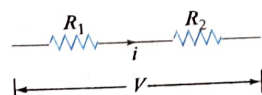
Let $V = 200 \text{ V}$ and R_1 and R_2 be the resistances of the 25 W and 100 W bulbs.

$$P_1 = 25 = V^2 / R_1 \text{ or } R_1 = V^2 / 25$$

$$\text{and } R_2 = V^2 / 100$$

When the bulbs are joined in series, the current is

$$I = \frac{V}{R_1 + R_2}$$



Power in the 25 W bulb is $R_1 I^2$ and in the 100 W bulb is $R_2 I^2$.

4. (1), (4)

Resistance of upper bulb, $R_1 = 100^2 / 25 = 400 \Omega$

Resistance of lower bulb, $R_2 = 200^2 / 100 = 400 \Omega$

Equivalent resistance of the circuit, $R_{eq} = 500 \Omega$.

Heat lost per second in the circuit is

$$e^2 / R_{eq} = 200^2 / 500 = 80 \text{ J s}^{-1}$$

Potential difference across branches AB and CD will be same. Hence, ratio of heat generated in them is

$$\frac{\text{Heat}_{AB}}{\text{Heat}_{CD}} = \frac{R_{CD}}{R_{AB}} = \frac{1000}{500} = 2:1$$

Hence, (3) is incorrect.

Since potential difference across branches AB and CD will be the same, but their resistances are different, so current in them will be different. As the resistances of the bulbs are same, so heat generated in them will be different. Hence, (2) is incorrect. Current drawn from the cell is $e / R_{eq} = 200 / 500 = 0.4 \text{ A}$.

5. (1), (2), (3), (4)

Potential difference across the terminals of the cell is given by $V = E - ir$. Hence, the graph between V and i will be a straight line having negative slope and positive intercept. Hence, (1) is correct.

Total electrical power generated is Ei and thermal power generated in the cell is $i^2 r$. Hence, thermal power generated in the external circuit is $P = Ei - i^2 r$. So, the graph between P and i is a parabola passing through the origin. Hence, choice (2) is correct.

When terminal potential difference across the cell is V , current flowing through the circuit will be $i = E - V / R$ and thermal power generated in the external circuit will be equal to

$$P = V_i = V \left(\frac{E - V}{r} \right) = \frac{EV}{r} - \frac{V^2}{r}$$

Hence, the graph between P and V will be a parabola passing through the origin and have a maximum value of P at $V = E / R$. Hence, choice (4) is correct.

At an instant, thermal power generated in the cell is equal to $i^2 r$ and total electrical power generated in the cell is equal to Ei . Hence, the fraction $\eta = i^2 r / Ei = (r / E) i$.

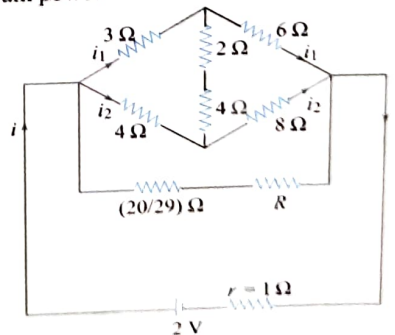
Hence, the graph between η and i will be a straight line passing through the origin and have a positive slope. Hence, (3) is correct.

6. (2), (3)

As $R \propto 1/W$ and actual power consumed $P = V^2 / R$, the lesser the resistance (i.e., higher the wattage), the greater the power consumed. If the bulbs are connected in series, actual power $P = I^2 R$ or R or $P \propto R$. Hence, in series connection, low wattage bulbs glow more.

7. (1), (3)

In the simplified circuit, the circuit is a balanced Wheatstone bridge and a branch of $20/29 \Omega$ and R is parallel with this balanced bridge for maximum power.



$$r = R_{\text{external}}$$

$$\frac{1}{r} = \frac{1}{(3+6)} + \frac{1}{(20/29+R)} + \frac{1}{(4+8)}$$

$$\text{or } R = \frac{16}{25} \Omega$$

Maximum power developed in the external circuit is

$$P_{\max} = i^2 R = \left(\frac{2}{1+1} \right)^2 \times 1 = 1 \text{ W}$$

Current through the upper branch

$$i_1 = i \left[\frac{\left(\frac{20}{29} + R \right) (4+8)}{9 + \left(\frac{20}{29} + R \right) + 12} \right]$$

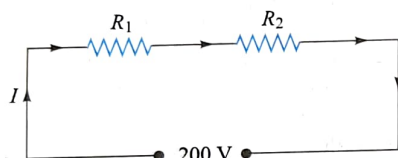
$$i_2 = i \left[\frac{\left(\frac{20}{29} + R \right) (3+6)}{9 + \left(\frac{20}{29} + R \right) + 12} \right]$$

Therefore, i_1/i_2 is independent of R .

8. (1), (4)

$$R_1 = \frac{V^2}{W_1} = \frac{(220)^2}{25} = (22)^2 \times 4 \Omega$$

$$R_2 = \frac{V^2}{W_2} = \frac{(220)^2}{100} = (22)^2 \Omega$$



Current in the circuit when the bulbs are connected in series,

$$I = \frac{220}{(22)^2 \times 4 + (22)^2} = \frac{10}{22 \times 5} = \frac{1}{11} \text{ A}$$

$$\text{Hence, } P_1 = I^2 R_1 = \left(\frac{1}{11} \right)^2 \times (22)^2 \times 4 = 16 \text{ W}$$

$$\text{And } P_2 = I^2 R_2 = \left(\frac{1}{11} \right)^2 \times (22)^2 = 4 \text{ W}$$

9. (1), (3)

Resistances 4 and 6 Ω are short-circuited. Therefore, no current will flow through these two resistances. Current passing through the battery is $I = (20/2) = 10 \text{ A}$. This is also the current passing in wire AB from B to A . Power supplied by the battery is

$$P = EI = (20)(10) = 200 \text{ W}$$

Potential difference across the 4 Ω resistance = potential difference across the 6 Ω resistance = 0

10. (1), (2), (4)

$$\text{Total charge} = \int I dt = \text{Area under the curve} = 10 \text{ C}$$

$$\text{Average current} = \frac{\int I dt}{\int dt} = 5 \text{ A}$$

$$\text{Total heat produced} = \int I^2 dt$$

$$= \int_0^2 (-5t + 10)^2 dt = \frac{200}{3} \text{ J}$$

$$\begin{aligned} \text{Maximum power} &= I^2 R \text{ when } I \text{ is maximum current} \\ &= 100 \times 1 = 100 \text{ W} \end{aligned}$$

11. (2), (3)

$$\text{For a bulb } R = \frac{V^2}{W} \quad \text{or} \quad R_B \leq R_A$$

when switch is open $I_A = I_B$

$$P_A = R_A I_A^2$$

$$P_A = R_A I_A^2$$

$$P_B < P_A \text{ and } V_A > V_B$$

$$V_A > 12 \text{ V and } V_B < 12 \text{ V}$$

After closing the switch

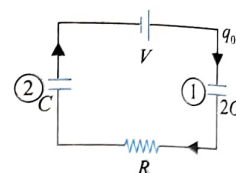
$$V = V_B = 12 \text{ V} \quad \text{or} \quad P_B = 36 \text{ W}$$

$$P_A = 24 \text{ W}$$

12. (1), (2), (4)

Before shorting the wire at steady state the charges on each capacitors will be equal to

$$q_0 = q_1 = q_2 = \left(\frac{2C}{3} \right) V$$



After shorting the capacitor marked (2) will be useless in circuit. Hence $q'_1 = 2CV = q_f$ and $q'_2 = 0$

The charge supplied by battery after shorting

$$\Delta q = q_f - q_0 = 2CV - \frac{2CV}{3} = \frac{4CV}{3}$$

$$\text{Hence work done by battery } W = \Delta q V = \frac{4CV^2}{3}$$

Initial potential energy of system

$$V_0 = \frac{1}{2} \left(\frac{2}{3} C \right) V^2 = \frac{1}{3} CV^2$$

$$\text{Final potential energy of system } U_f = \frac{1}{2} (2C) V^2 = CV^2$$

$$\text{Hence change in PE } \Delta U = U_f - U_i$$

$$\text{or } \Delta U = CV^2 - \frac{1}{3} CV^2 = \frac{2}{3} CV^2$$

Work done by battery $W = \Delta U + \text{Heat}$

$$\text{Heat developed } H = W - \Delta U$$

$$\text{or } H = \frac{4}{3} CV^2 - \frac{2}{3} CV^2 = \frac{2}{3} CV^2$$

13. (2), (4)

Resistance absorbs energy at the rate of 2 W.

Potential difference across AB is V_{AB} . $I = 50 \text{ W}$

$$V_{AB} = 50 \text{ V}$$

Drop across resistor is 2 V, therefore emf of E is 48 V.

As AB is absorbing energy at the rate of 50 W, 48 W is being absorbed by E . Thus, E is on charging. i.e., current is entering from positive terminal of E .

14. (1), (2), (4)

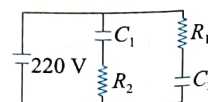
$$i = 2 \times 10^{-2} \text{ A}$$

$$P_{R_1} = i^2 R_1 = (2 \times 10^{-2})^2 \times 4 \times 10^3 = 1.6 \text{ W}$$

$$Q_{C_1} = V_{R_1} \times C_1 = 80 \times 3 \times 10^{-6} = 240 \mu\text{C}$$

$$Q_{C_2} = V_{R_2} \times C_2 = 140 \times 6 \times 10^{-6} = 840 \mu\text{C}$$

$$Q_{C_1} = E_{C_1} = 220 \times 3 \times 10^{-6} = 660 \mu\text{C}$$



Linked Comprehension Type

For Problems 1-3

1. (2), 2. (1), 3. (2)

$i_2 = i_3 = I_b$, $V_1 = (V_2 + V_3) = V$, and $P_2 = P_3$
or $R_2 = R_3$ and $V_2 = V_3 = V/2$

$P_1 = P_2 = P_3 = P_4$ (given)

$P_1 = \frac{V^2}{36}$; $P_3 = \frac{(V/2)^2}{R_3}$

As $P_1 = P_3$, so $R_3 = 9 \Omega$

Also $R_3 = R_2 = 9 \Omega$

$I_a = \frac{I}{3}$ and $I_4 = I$,

$P_1 = P_4 = 4 \text{ W}$

$\left(\frac{I}{3}\right)^2 R_1 = (I)^2 R_4 = 4 \text{ W}$ or $\frac{R_1}{9} = R_4$ or $R_4 = 4 \Omega$

Also $I = 1 \text{ A}$, $R_{eq} = 16 \Omega$ and $I = 1 \text{ A}$, $\epsilon = 16 \text{ V}$

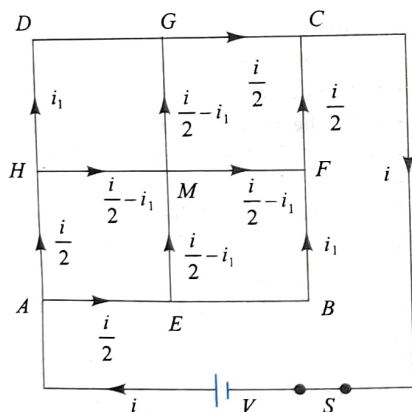
For Problems 4-6

4. (3), 5. (4), 6. (2)

In loop EBFME,

$$V_E - ri_1 - ri_1 + r\left(\frac{i}{2} - i_1\right) + r\left(\frac{i}{2} - i_1\right) = V_E$$

$$\text{or } 2i_1 = 2\left(\frac{i}{2} - i_1\right) \text{ or } i_1 = \frac{i}{4}$$



$$\text{Loop AEBCSA, } V_A - r\frac{i}{2} - ri_1 - ri_1 - r\frac{i}{2} + V = V_A$$

$$\text{or } V = ri + 2ri_1 = ri + 2r\frac{i}{4} = \frac{3ri}{2}$$

$$R_{eq} = \frac{V}{i} = \frac{3r}{2}$$

$$\frac{P_1}{P_2} = \frac{\left(\frac{i}{2}\right)^2 r}{\left(\frac{i}{2} - i_1\right)^2 r} = \frac{i^2}{(i - 2i_1)^2} = 4$$

Let l be the balancing length, then

$$\begin{aligned} kl &= V_H - V_C \\ &= \left(\frac{i}{2} - i_1\right)r + \left(\frac{i}{2} - i_1\right)r + \frac{i}{2}r \\ &= \left(\frac{3i}{2} - 2i_1\right)r = ir \end{aligned}$$

$$\text{or } l = \frac{2V}{3k}$$

For Problems 7-9

7. (4), 8. (3), 9. (1)

Initially, when all the capacitors are uncharged, they will act as conducting wires. Hence, all the resistances (except 5Ω) will be short-circuited. So no resistance occurs between A and F . Thus current through the 5Ω resistor will be $(10/5) \text{ A} = 2 \text{ A}$.

In steady state, distribution of current is shown.

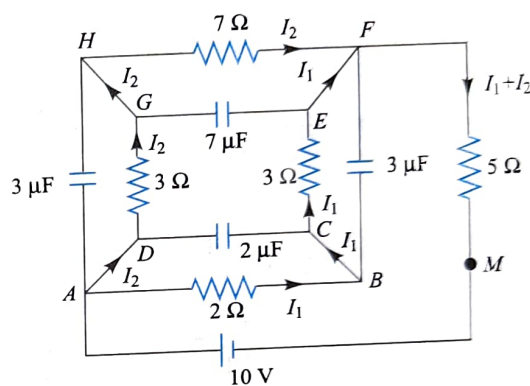
$$\text{Loop ABCEFM, } 10 = 2I_1 + 3I_1 + 5(I_1 + I_2)$$

$$\text{or } 10I_1 + 5I_2 = 10 \quad \dots(i)$$

$$\text{Loop ADGHFMA, } 10 = 3I_2 + 7I_2 + 5(I_1 + I_2)$$

$$\text{or } 5I_1 + 15I_2 = 10 \quad \dots(ii)$$

$$\text{From Eqs. (i) and (ii), } I_1 = \frac{4}{5} \text{ A, } I_2 = \frac{2}{5} \text{ A}$$



Hence, current through the 5Ω resistor is $I_2 + I_2 = (6/5) \text{ A}$. Potential difference across the $2 \mu\text{F}$ capacitor is

$$2I_1 = 2 \times \frac{4}{5} = \frac{8}{5} \text{ V}$$

Energy in it is given by

$$U_1 = \frac{1}{2} \times 2 \left(\frac{8}{5}\right)^2 = (64/25) \mu\text{J}$$

Potential difference across the $3 \mu\text{F}$ capacitor is

$$3I_1 = 3 \times \frac{4}{5} = \frac{12}{5} \text{ V}$$

Energy in it is given by

$$U_2 = \frac{1}{2} \times 3 \left(\frac{12}{5}\right)^2 = (216/25) \mu\text{J}$$

Potential difference across the $3 \mu\text{F}$ capacitor is

$$3I_2 = 3 \times \frac{2}{5} = \frac{6}{5} \text{ V}$$

Energy in it is given by

$$U_3 = \frac{1}{2} \times 3 \left(\frac{6}{5}\right)^2 = (54/25) \mu\text{J}$$

Potential difference across the $7 \mu\text{F}$ capacitor is

$$7I_2 = 7 \times \frac{2}{5} = \frac{14}{5} \text{ V}$$

Energy in it is given by

$$U_4 = \frac{1}{2} \times 7 \left(\frac{14}{5}\right)^2 = (686/25) \mu\text{J}$$

Thus, whole of the energy stored in all capacitors will be dissipated after the switch is opened. So the energy dissipated is

$$U_1 + U_2 + U_3 + U_4 = 40.8 \mu\text{J}$$

For Problems 10–14

10. (2), 11. (2), 12. (3), 13. (2), 14. (1)

$$\eta = \frac{0.4 \times 4200(\theta - \theta_1)}{420 \times 4.8(\theta - \theta_1)} \times 100 = 250/3 = 83.34\%$$

$$\eta = \frac{0.4 \times 4200(\theta - \theta_1)}{420 \times 6.4(\theta - \theta_1)} \times 100 = 62.5\%$$

$$H_A = 0.2 \times 1680(\theta - \theta_1) + 0.4 \times 4200(\theta - \theta_1) = 420 \times [0.8 + 4](\theta - \theta_1)$$

$$H_B = 0.4 \times 2450(\theta - \theta_1) + 0.4 \times 4200(\theta - \theta_1) = 420 \times (2.4 + 4)(\theta - \theta_1)$$

$$\frac{H_A}{H_B} = \frac{4.8}{6.4} = \frac{3}{4}$$

$$H_A = \frac{V^2}{R_A} \times 6 \quad \dots(i)$$

$$H_B = \frac{V^2}{R_B} \times 8 \quad \dots(ii)$$

$$\frac{H_A}{H_B} = \frac{R_B}{R_A} \times \frac{3}{4} \quad \text{but} \quad \frac{H_A}{H_B} = \frac{3}{4}$$

 Then $R_B = R_A$

$$14. (1) (i) \quad \frac{V^2}{R_A} \times 6 = \frac{V^2}{(R_A + R_B)^2} R_A \times t_A$$

$$\text{But } R_A = R_B \text{ or } \frac{V^2}{R_A} \times 6 = \frac{V^2}{4R_A^2} R_A \times t_A$$

$$t_A = 24 \text{ min}$$

$$(ii) \quad \frac{V^2}{R_B} \times 8 = \frac{V^2}{(R_A + R_B)^2} R_B \times t_B$$

$$\frac{8}{R_B} = \frac{R_B}{4R_B^2} t_B \text{ or } t_B = 32 \text{ min.}$$

For Problems 15–17

15. (1), 16. (2), 17. (2)

$$I \propto r^{3/2}$$

$$\frac{I_1}{I_2} = \frac{(4r)^{3/2}}{r^{3/2}} = \frac{8}{1}$$

$$P = \frac{V^2}{R} \text{ or } 2 \times 10^3 = \frac{V^2}{20}$$

$$\text{or } V = 200 \text{ V}$$

$$\eta = \frac{R}{R+r} \times 100, \text{ where } R = r, \text{ efficiency will be maximum.}$$

Matrix Match Type

 1. i. $\rightarrow$ c.; ii. $\rightarrow$ a., b., c., d.; iii. $\rightarrow$ d.; iv. $\rightarrow$ d.

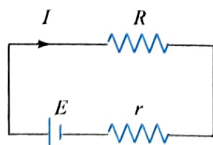
$$I = \frac{E}{R+r}$$

Terminal potential difference across the cell is $V = E - Ir$. This is maximum if $I = 0$, and this is possible if $R = \infty$.

Hence, i. $\rightarrow$ c.

Power transferred to R is maximum if $R = r$. In all other cases it will be somewhat less. Hence, ii. $\rightarrow$ a., b., c., d.

Power dissipated in cell is EI . This is maximum if I is maximum, for this $R = 0$. Hence, iii. $\rightarrow$ d.



Also if I is maximum, there will be the fastest drift of ions in electrolyte in the cell. Hence, iv. $\rightarrow$ d.

2. i. $\rightarrow$ b., d.; ii. $\rightarrow$ a., c., d.; iii. $\rightarrow$ a., c., d.; iv. $\rightarrow$ b., d.

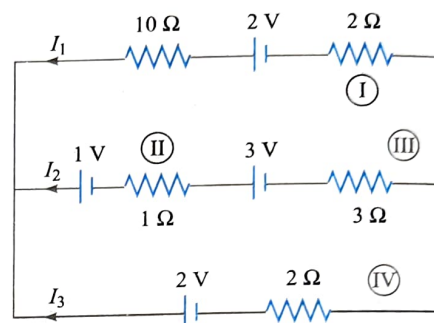
$$\text{We have } I_1 = -\frac{1}{20} \text{ A}, I_2 = \frac{7}{20} \text{ A}, I_3 = -\frac{6}{20} \text{ A.}$$

In each cell, thermal energy will be dissipated due to internal resistance whether the chemical energy of the cell is increasing or decreasing.

(i) Cell I is getting charged, hence its chemical energy increases.

(ii) Cells II and III both are getting discharged, hence their chemical energies are decreasing. So work done by both of them is positive.

(iv) Cell IV is getting charged, hence its chemical energy increases.



3. i. $\rightarrow$ a., d.; ii. $\rightarrow$ c.; iii. $\rightarrow$ c.; iv. $\rightarrow$ c.

Charging rate means current $\left(\frac{dq}{dt} = I\right)$. So if charging rate is maximum, then current is also maximum. Hence, i. $\rightarrow$ a, d.

Charge on the capacitor is maximum in steady state. In steady state, current becomes zero. Hence, ii. $\rightarrow$ c.

Power supplied by the battery is maximum initially because current is maximum initially. At this time, charge on the capacitor is zero. Hence, iii. $\rightarrow$ c.

Difference in the power supplied by battery and power consumed in R is $EI - IR = EI - I(IR) = EI - IE = 0$. Hence iv. $\rightarrow$ c.

4. i. $\rightarrow$ b.; ii. $\rightarrow$ c.; iii. $\rightarrow$ c.; iv. $\rightarrow$ c.

When the switch is closed, equivalent resistance is R . After opening the switch, the equivalent resistance becomes $2R$. Hence, equivalent resistance increases. Also current through battery decreases, hence ammeter reading decreases. Current through left R also decreases. So voltmeter reading decreases and power dissipated through left R also decreases.

5. i. $\rightarrow$ b., d.; ii. $\rightarrow$ b., c.; iii. $\rightarrow$ a., d.; iv. $\rightarrow$ a., c.

$$\text{For } P_{\max} = R = 2r$$

$$P_{\max} = \left(\frac{2E}{R+2r}\right)^2 R = \frac{E^2}{2r}$$

$$\text{For } P_{\max} : R = \frac{r}{2}$$

$$P_{\max} = \left(\frac{E}{R+r/2}\right)^2 R = \frac{E^2}{2r}$$

$$P = \left(\frac{2E}{r+2r}\right)^2 r = \frac{4E^2}{9r}$$

$$P = \left(\frac{2E}{r+r/2}\right)^2 r = \frac{4E^2}{9r}$$

6. i. $\rightarrow$ d.; ii. $\rightarrow$ c.; iii. $\rightarrow$ a.; iv. $\rightarrow$ b., d.

Rated voltage is same, $P_A > P_B > P_C$

So, $R_A < R_B < R_C$
 Currents through B and C are I_1 and I_2 , respectively, then current through A is $(I_1 + I_2)$. B and C have same applied voltage.

$\frac{V_1^2}{R_B} > \frac{V_1^2}{R_C}$. Power delivered to C is minimum.

Currents through A and B are I_1 and I_2 , respectively, then current through C is $(I_1 + I_2)$ and P_R is maximum for C .

Further, $\frac{V_1^2}{R_A} > \frac{V_1^2}{R_B}$. So, power delivered to B is minimum.

Current through A , B , and C is same. P_R is maximum for C and minimum for A .

Voltage is same. So, V^2/R is maximum for A and minimum for C .

Numerical Value Type

1. (4) Let resistance of heating coil be R , then

$$100 = \frac{220^2}{R} \text{ or } R = \frac{(220)^2}{100} \Omega$$

Resistance of each cut part is $R/2$. Now, power dissipated is

$$P = 2 \left(\frac{(220)^2}{R/2} \right) = 4 \frac{(220)^2}{R} = 4 \times 100 = 400 \text{ W}$$

2. (9) For three identical resistors in series, $P_s = V^2/3R$. If they are now in parallel over the same voltage,

$$P_p = \frac{V^2}{R_{eq}} = \frac{V^2}{R/3} = \frac{9V^2}{3R} = 9P_s = 9 \times (27 \text{ W})$$

3. (6) $150 = \frac{15 \times 15}{R_{eq}}$ or $R_{eq} = \frac{15 \times 15}{150} \Omega = \frac{3}{2} \Omega$

$$\text{Now, } \frac{2R}{2+R} = \frac{3}{2} \text{ or } R = 6 \Omega$$

4. (1) The resistance of four sections are shown in the figure.

Hence, the equivalent resistance R across AB is

$$\frac{1}{R} = \frac{1}{24} + \frac{1}{12} + \frac{1}{12} + \frac{1}{24} \text{ or } R = 4 \Omega$$

$$\text{Power} = V^2/R = 20^2/4 = 100 \text{ W}$$

5. (2) If the wire is connected as such across the battery, then current in wire,

$$i = \frac{V}{R} = \frac{200}{80} = 2.5 \text{ A}$$

and power obtained,

$$P = \frac{V^2}{R} = \frac{200 \times 200}{80} = 500 \text{ W}$$

The wire can carry maximum current of 5 A; therefore, to double the current, the resistance should be halved. Thus, if we divide the wire in two parts and the two parts are connected in parallel across 200 V mains supply, then the resistance of each part is 40 W; therefore, current in each wire is $200/40 = 5 \text{ A}$.

$$\text{Net resistance, } R' = \frac{R_1 R_2}{R_1 + R_2} = \frac{40 \times 40}{40 + 40} = 20 \Omega$$

and new power obtained is

$$P_{\max} = \frac{V^2}{R} = \frac{200 \times 200}{20} = 2000 \text{ W}$$

Thus, the maximum power is 2000 W and this is obtained when wire is cut in two halves and they are connected in parallel across the given supply.

6. (6) $i = 2 \text{ A}$, $R = 25 \text{ W}$, $t = 1 \text{ min} = 60 \text{ s}$

$$\text{Heat developed} = i^2 R t = (2 \times 2 \times 25 \times 60) = 100 \times 60 \text{ J} = 6000 \text{ J}$$

7. (2) As discussed in maximum power transfer theorem the maximum power which can be transferred is given as

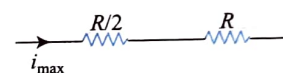
$$P_{\max} = \frac{E^2}{4r}$$

Here E and r are the equivalent EMF and internal resistance of the equivalent battery when the given batteries are connected in series as in that case EMF will be maximum which gives

$$P_{\max} = \frac{(4)^2}{4 \times 2} = 2 \text{ W}$$

8. (54) $P = i^2 R$

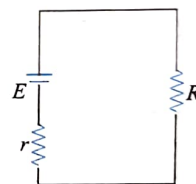
$$\therefore i_{\max} = \sqrt{\frac{P_{\max}}{R}} = \sqrt{\frac{36}{2.4}} = \sqrt{15} \text{ A}$$



$$\text{Total maximum power} = (i_{\max})^2 \left(\frac{3R}{2} \right)$$

$$= (15)(1.5)(2.4) = 54 \text{ W}$$

9. (0.3) Figure shows the situation described in question. Here r is the internal resistance of the battery and E its EMF.



Power supplied by the battery in this case is given as

$$P = \frac{E^2}{R + r}$$

Substituting the values gives

$$0.4 = \frac{(6)^2}{60 + r} \Rightarrow r = 30 \Omega$$

Maximum power is dissipated in the circuit when external resistance is equal to net internal resistance which gives

$$R = r \Rightarrow R = 30 \Omega$$

Total power supplied by the battery under this condition is given as

$$P_{\text{Total}} = \frac{E^2}{R + r} = \frac{(6)^2}{30 + 30} \Rightarrow P_{\text{Total}} = 0.6 \text{ W}$$

Out of this 0.6 W half of the power is dissipated in R and half is dissipated in r . Therefore, maximum power dissipated in R is 0.3 W.

10. (384) The resistance of heater coil is given as

$$R = \frac{V^2}{P} = \frac{(200)^2}{500} = \frac{40000}{500} = 80 \Omega$$

When connected across 160 V supply, the power consumed by heater will be given as

$$P' = \frac{V^2}{R} = \frac{(160)^2}{80} = \frac{160 \times 160}{80}$$

$$\Rightarrow P' = 320 \text{ W}$$

Total heat produced in 20 minute (1200 s) is given as

$$H = P't$$

$$\Rightarrow H = 320 \times 1200 \Rightarrow H = 384000 = 384 \text{ kJ}$$

11. (0.3) When a current is drawn from the battery, its potential difference across the terminals decreases which is given as

$$2.0 = 2.6 - Ir$$

$$\Rightarrow r = 0.6 \Omega$$

If external resistance is R , we use

$$R + r = \frac{2.6}{1} = 2.6 \Omega \Rightarrow R = 2.6 - 0.6 = 2.0 \Omega$$

Total thermal power generated in the battery is given as

$$P_{\text{Thermal}} = I^2 r = 1^2 \times 0.6 = 0.6 \text{ W}$$

Total electrical power supplied by the battery is given as

$$P_{\text{total}} = 1 \times 2 = 2 \text{ W}$$

$$\text{Hence required ratio } \frac{P_{\text{Thermal}}}{P_{\text{total}}} = \frac{0.6}{2} = 0.30$$

$$12. (20) P_{\text{initial}} (= P_2) = \frac{\varepsilon^2 R'}{(R + R')^2}$$

$$P_{\text{final}} (= P_2) = \frac{2\varepsilon^2 (R'/2)}{(R + R'/2)^2}$$

Since, $P_1 = P_2$

$$\text{Or } (R + R')^2 = 2 \left(R + \frac{R'}{2} \right)^2$$

$$\text{Or } (R + R') = \left(R + \frac{R'}{2} \right) \sqrt{2}$$

$$\text{Or } R = \frac{R'}{\sqrt{2}} = \frac{20\sqrt{2}}{\sqrt{2}} = 20 \Omega$$

13. (44.4) Initial potential energy stored in the capacitor,

$$U = \frac{1}{2} CV^2$$

$$\Rightarrow U = \frac{1}{2} \times 5 \times 10^{-6} \times (200)^2 = 0.1 \text{ J.}$$

When the switch is shifted from contact 1 to 2 capacitor starts discharging and during discharging this 0.1 J, will distribute in direct ratio of resistance thus heat dissipated in 400Ω resistance is given as

$$H_{400\Omega} = \frac{400}{400 + 500} \times 0.1$$

$$\Rightarrow H_{400\Omega} = 44.4 \times 10^{-3} \text{ J.}$$

$$\Rightarrow H_{400\Omega} = 44.4 \text{ mJ}$$

$$14. (25) \text{ The resistance } R = \frac{V^2}{P} = \frac{(250)^2}{100} = 625 \Omega$$

This is a balanced Wheatstone bridge $\Rightarrow i' = 0$

$$\text{Then, } i = \frac{\varepsilon}{R + r} = \frac{260}{625 + 25} = \frac{2}{5} \text{ A}$$

$$P = \left(\frac{i}{2} \right)^2 R = \left(\frac{1}{2} \right)^2 (625) = 25 \text{ W}$$

$$15. (7.5) \text{ Current in first case, } i = \frac{E}{5 + x} = \frac{20}{5 + x}$$

Power supplied to the resistance,

$$P = i^2 R = \frac{400}{(5 + x)^2} \times x \quad \dots(i)$$

$$\text{In second case, current in the circuit, } i' = \frac{E}{5 + \left(\frac{6x}{x + 6} \right)}$$

Power supplied by the battery in this case,

$$P' = i'^2 R' = \frac{400}{\left[5 + \left(\frac{6x}{x + 6} \right) \right]^2} \times \frac{6x}{(6 + x)} \quad \dots(ii)$$

As the power supplied in two cases are equal, from (i) and (ii)

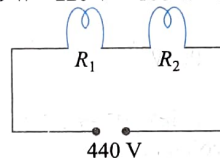
$$\frac{400x}{(5 + x)^2} = \frac{400}{\left[5 + \left(\frac{6x}{x + 6} \right) \right]^2} \times \frac{6x}{(6 + x)} \Rightarrow x = 7.5 \Omega$$

Archives

JEE Main

Single Correct Answer Type

1. (3) 25 W – 220 V 100 W – 220 V



$$\text{As } R_1 = \frac{220}{25} \times 220 \text{ and } R_2 = \frac{220}{100} \times 220$$

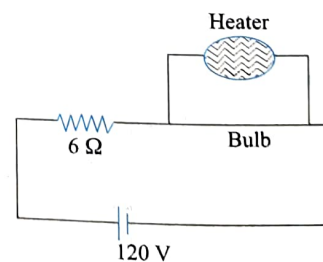
$$\begin{aligned} R &= R_1 + R_2 \\ &= 220 \times 220 \left(\frac{1}{25} + \frac{1}{100} \right) \\ &= 220 \times 220 \times \frac{1}{20} \end{aligned}$$

$$\therefore I_{\text{live}} = \frac{440}{\frac{220 \times 220}{20}} = \frac{40}{220}$$

$\therefore$ 1st bulb (25 W) will fuse only.

$$2. (3) \text{ Resistance of bulb} = \frac{120 \times 120}{60} = 240 \Omega$$

$$\text{Resistance of heater} = \frac{120 \times 120}{240} = 60 \Omega$$



Voltage across bulb before heater is switched on,

$$V_1 = \frac{120}{246} \times 240$$

Voltage across bulb after heater is switched on,

$$V_2 = \frac{120}{54} \times 48$$

Decrease in the voltage is $V_1 - V_2 = 10.04$ (approximately)

Note: Here supply voltage is taken as rated voltage.

3. (1)

Item	Number	Power
40 W bulb	15	600 watt
100 W bulb	5	500 watt
80 W fan	5	400 watt
1000 W heater	1	1000 watt

Total Wattage = 2500 Watt

$$\text{So current capacity } i = \frac{P}{V} = \frac{2500}{220} = \frac{125}{11} = 11.36 \approx 12 \text{ A}$$

JE Advanced

Single Correct Answer Type

1. (4) Power $\propto 1/R$

Multiple Correct Answers Type

1. (1), (4)

$$24 - 2 \times 10^3 I - 6 \times 10^3 (I - i) = 0$$

$$24 - 2 \times 10^3 I - 1.5 \times 10^3 i = 0$$

Hence, $I = 7.5 \text{ mA}$

$$i = 6 \text{ mA}$$

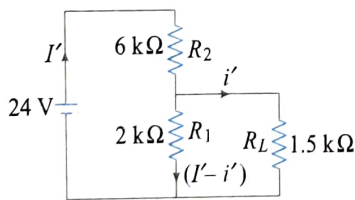
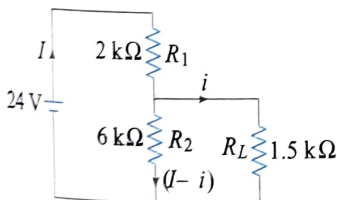
$$24 - 6 \times 10^3 I' - 2 \times 10^3 (I' - i') = 0$$

$$24 - 6 \times 10^3 I' - 1.5 \times 10^3 i' = 0$$

$$I' = 3.5 \text{ mA}$$

$$i' = 2 \text{ mA}$$

$$\frac{P_1}{P_2} = \frac{6^2}{2^2} = 9$$



2. (2), (4)

Heat required to increase the temperature of water by 40 K.

$$\Delta Q = mc\Delta T = \left(\frac{\epsilon^2}{R}\right)t$$

$$\Rightarrow \Delta Q = 0.5 \times c \times 40 = \left(\frac{\epsilon^2}{R}\right) \times 4 \text{ min}$$

$$\text{Where } R = \frac{\rho L}{\left(\frac{\pi d^2}{4}\right)}$$

The resistance of wire of length L and diameter $2d$.

$$R_1 = \frac{\rho L}{\pi(2d)^2/4} = \frac{R}{4}$$

If resistances are in series, $R_{eq} = \frac{R}{4} + \frac{R}{4} = \frac{R}{2}$

$$\text{Heat required, } \Delta Q = \left(\frac{\epsilon^2}{R}\right)4 = \frac{\epsilon^2}{(R/2)} \times t$$

Which gives $t = 2 \text{ min}$

If resistances are in parallel, $R_{eq} = \frac{\frac{R}{4} \times \frac{R}{4}}{\frac{R}{4} + \frac{R}{4}} = \frac{R}{8}$

$$\text{Heat required, } \Delta Q = \left(\frac{\epsilon^2}{R}\right)4 = \frac{\epsilon^2}{(R/8)} \times t$$

Which gives $t = 0.5 \text{ min}$

Numerical Value Type

$$1. (4) J_1 = \left(\frac{2E}{R+2}\right)^2 R$$

$$J_2 = \left(\frac{E}{R+1/2}\right)^2 R \text{ since } J_1 / J_2 = 2.25$$

$$R = 4 \Omega$$

1

APPENDIX

Chapterwise Solved January 2019 JEE Main Questions (All Sets)

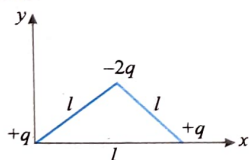
Chapter 1

Coulomb's Law and Electric Field

1. Two point charges $q_1(\sqrt{10} \mu\text{C})$ and $q_2(-25 \mu\text{C})$ are placed on the x -axis at $x = 1 \text{ m}$ and $x = 4 \text{ m}$ respectively. The electric field (in V/m) at a point $y = 3 \text{ m}$ on y -axis is

$$\left[\text{take } \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2} \right]$$

- (1) $(-63\hat{i} + 27\hat{j}) \times 10^2$ (2) $(81\hat{i} - 81\hat{j}) \times 10^2$
(3) $(63\hat{i} - 27\hat{j}) \times 10^2$ (4) $(-81\hat{i} + 81\hat{j}) \times 10^2$
2. Three charges $+Q, q, +Q$ are placed respectively, at distance, $0, d/2$ and d from the origin, on the x -axis. If the net force experienced by $+Q$, placed at $x = 0$ is zero, then value of q is
- (1) $+Q/2$ (2) $-Q/2$
(3) $-Q/4$ (4) $+Q/4$
3. Determine the electric dipole moment of the system of three charges, placed on the vertices of an equilateral triangle, as shown in the figure.



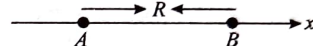
- (1) $(ql) \frac{\hat{i} + \hat{j}}{\sqrt{2}}$ (2) $\sqrt{3}ql \frac{\hat{j} - \hat{i}}{\sqrt{2}}$
(3) $-\sqrt{3}ql \hat{j}$ (4) $2ql \hat{j}$

Chapter 3 Electric Potential

4. A charge Q is distributed over three concentric spherical shells of radii a, b, c ($a < b < c$) such that their surface charge densities are equal to one another. The total potential at a point at distance r from their common center, where $r < a$, would be

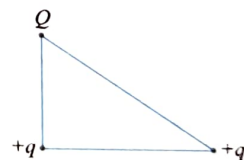
- (1) $\frac{Q}{4\pi\epsilon_0(a+b+c)}$ (2) $\frac{Q(a+b+c)}{4\pi\epsilon_0(a^2+b^2+c^2)}$
(3) $\frac{Q}{12\pi\epsilon_0} \frac{ab+bc+ca}{abc}$ (4) $\frac{Q}{4\pi\epsilon_0} \frac{(a^2+b^2+c^2)}{(a^3+b^3+c^3)}$

5. Two electric dipoles, A, B with respective dipole moments $\vec{d}_A = -4qa\hat{i}$ and $\vec{d}_B = -2qa\hat{i}$ placed on the x -axis with a separation R , as shown in the figure. The distance from A at which both of them produce the same potential is



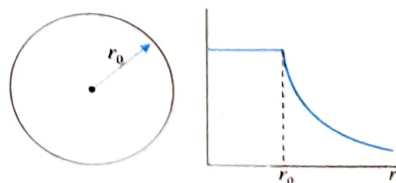
- (1) $\frac{\sqrt{2}R}{\sqrt{2}+1}$ (2) $\frac{R}{\sqrt{2}+1}$
(3) $\frac{\sqrt{2}R}{\sqrt{2}-1}$ (4) $\frac{R}{\sqrt{2}-1}$

6. An electric field of 1000 V/m is applied to an electric dipole at angle of 45° . The value of electric dipole moment is 10^{-29} cm . What is the potential energy of the electric dipole?
- (1) $-9 \times 10^{-20} \text{ J}$ (2) $-7 \times 10^{-27} \text{ J}$
(3) $-10 \times 10^{-29} \text{ J}$ (4) $-20 \times 10^{-18} \text{ J}$
7. The charges $Q, +q$ and $+q$ are placed at the vertices of a right-angle isosceles triangle as shown below. The net electrostatic energy of the configuration is zero, if the value of Q is



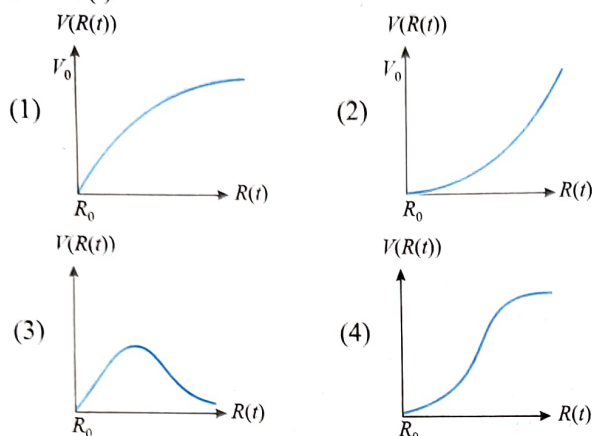
- (1) $\frac{-\sqrt{2}q}{\sqrt{2}+1}$ (2) $-2q$
(3) $\frac{-q}{1+\sqrt{2}}$ (4) $+q$

8. The given graph shows variation (with distance r from centre) of



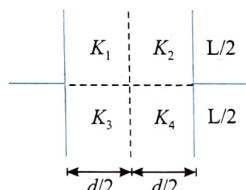
- (1) Potential of a uniformly charged sphere
(2) Potential of a uniformly charged spherical shell
(3) Electric field of uniformly charged spherical shell
(4) Electric field of uniformly charged sphere

9. There is a uniform spherically symmetric surface charge density at a distance R_0 from the origin. The charge distribution is initially at rest and starts expanding because of mutual repulsion. The figure that represents best the speed $V(R(t))$ of the distribution as a function of its instantaneous radius $R(t)$ is



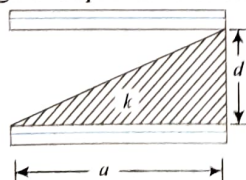
Chapter 4 Capacitor and Capacitance

10. A parallel plate capacitor with square plates is filled with four dielectrics of dielectric constants K_1, K_2, K_3, K_4 arranged as shown in the figure. The effective dielectric constant K will be



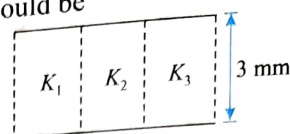
- (1) $K = \frac{(K_1 + K_2)(K_3 + K_4)}{2(K_1 + K_2 + K_3 + K_4)}$
 (2) $K = \frac{(K_1 + K_2)(K_3 + K_4)}{(K_1 + K_2 + K_3 + K_4)}$
 (3) $K = \frac{(K_1 + K_4)(K_2 + K_3)}{2(K_1 + K_2 + K_3 + K_4)}$
 (4) $K = \frac{(K_1 + K_3)(K_2 + K_4)}{K_1 + K_2 + K_3 + K_4}$

11. A parallel plate capacitor is made of two square plates of side 'a', separated by a distance d ($d \ll a$). The lower triangular portion is filled with a dielectric of dielectric constant K , as shown in the figure. Capacitance of this capacitor is

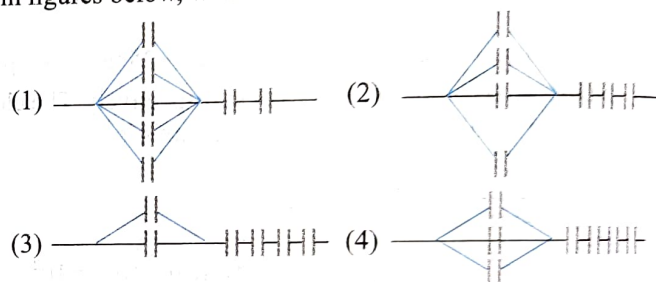


- (1) $\frac{1}{2} \frac{k\epsilon_0 a^2}{d}$ (2) $\frac{k\epsilon_0 a^2}{d} \ln K$
 (3) $\frac{k\epsilon_0 a^2}{d(K-1)} \ln K$ (4) $\frac{k\epsilon_0 a^2}{2d(K+1)}$

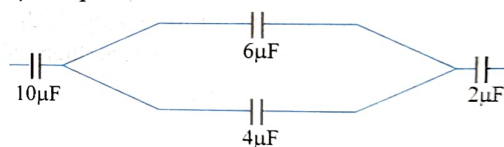
12. A parallel plate capacitor is of area 6 cm^2 and a separation of 3 mm . The gap is filled with three dielectric materials of equal thickness (see figure) with dielectric constants $K_1 = 10$, $K_2 = 12$ and $K_3 = 14$. The dielectric constant of a material which when fully inserted in above capacitor, gives same capacitance would be



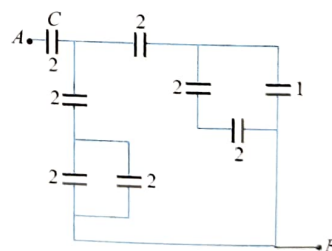
- (1) 12 (2) 4
 (3) 36 (4) 14
13. Seven capacitors, each of capacitance $2 \mu\text{F}$, are to be connected in a configuration to obtain an effective capacitance of $\left(\frac{6}{13}\right) \mu\text{F}$. Which of the combinations, shown in figures below, will achieve the desired value?



14. In the figure shown below, the charge on the left plate of the $10 \mu\text{F}$ capacitor is $-30 \mu\text{C}$. The charge on the right plate of the $6 \mu\text{F}$ capacitor is:

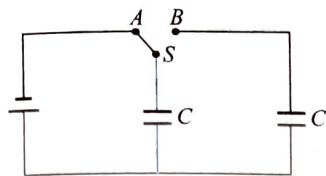


- (1) $-18 \mu\text{C}$ (2) $-12 \mu\text{C}$
 (3) $+12 \mu\text{C}$ (4) $+18 \mu\text{C}$
15. In the circuit shown, find C if the effective capacitance of the whole circuit is to be $0.5 \mu\text{F}$. All values in the circuit are in μF .



- (1) $\frac{7}{10} \mu\text{F}$ (2) $\frac{7}{11} \mu\text{F}$
 (3) $\frac{6}{5} \mu\text{F}$ (4) $4 \mu\text{F}$
16. A parallel plate capacitor with plates of area 1 m^2 each, are at a separation of 0.1 m . If the electric field between the plates is 100 N/C , the magnitude of charge each plate is:-
 (Take $\epsilon_0 = 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N-m}^2}$)
- (1) $7.85 \times 10^{-10} \text{ C}$ (2) $6.85 \times 10^{-10} \text{ C}$
 (3) $9.85 \times 10^{-10} \text{ C}$ (4) $8.85 \times 10^{-10} \text{ C}$

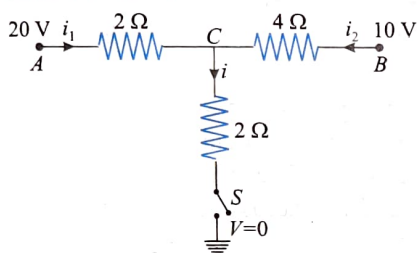
17. In the figure shown, after the switch 'S' is turned from position 'A' to position 'B', the energy dissipated in the circuit in terms of capacitance 'C' and total charge 'Q' is



- (1) $\frac{3Q^2}{8C}$ (2) $\frac{3Q^2}{4C}$
 (3) $\frac{1Q^2}{8C}$ (4) $\frac{5Q^2}{8C}$

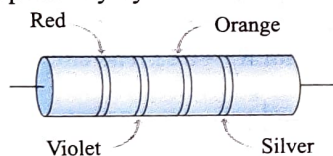
Chapter 5 Electric Current and Circuits

18. When the switch S, in the circuit shown, is closed, then the value of current i will be



- (1) 3 A (2) 5 A
 (3) 4 A (4) 2 A

19. A resistance is shown in the figure. Its value and tolerance are given respectively by



- (1) 27 kΩ, 20% (2) 270 kΩ, 5%
 (3) 270 kΩ, 10% (4) 27 kΩ, 10%

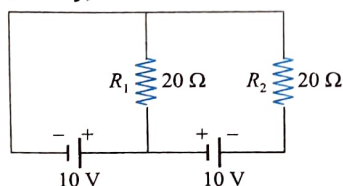
20. A copper wire is stretched to make it 0.5% longer. The percentage change in its electrical resistance if its volume remains unchanged is

- (1) 2.5% (2) 0.5%
 (3) 1.0% (4) 2.0%

21. A uniform metallic wire has a resistance of 18 Ω and is bent into an equilateral triangle. Then, the resistance between any two vertices of the triangle is

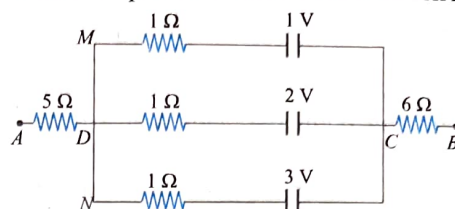
- (1) 8 Ω (2) 12 Ω
 (3) 4 Ω (4) 2 Ω

22. In the given circuit the cells have zero internal resistance. The currents (in amperes) passing through resistance R_1 , and R_2 respectively, are



- (1) 2, 2 (2) 0, 1
 (3) 1, 2 (4) 0.5, 0

23. In the circuit, the potential difference between A and B is

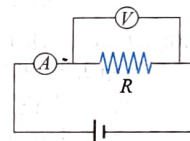


- (1) 6 V (2) 1 V
 (3) 3 V (4) 2 V

Chapter 6 Electrical Measuring Instruments

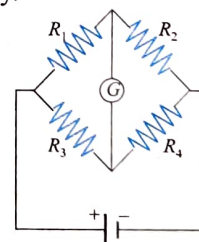
24. The actual value of resistance R , shown in the figure is 30 Ω. This is measured in an experiment as shown using the standard formula $R = \frac{V}{I}$, where V and I are the readings

of the voltmeter and ammeter, respectively. If the measured value of R is 5% less, then the internal resistance of the voltmeter is



- (1) 350 Ω (2) 570 Ω
 (3) 35 Ω (4) 600 Ω

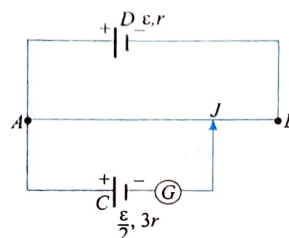
25. The Wheatstone bridge shown in Fig. here, gets balanced when the carbon resistor used as R_1 has the colour code (Orange, Red, Brown). The resistors R_2 and R_4 are 80 Ω and 40 Ω, respectively.



Assuming that the colour code for the carbon resistors gives their accurate values, the colour code for the carbon resistor, used as R_3 , would be

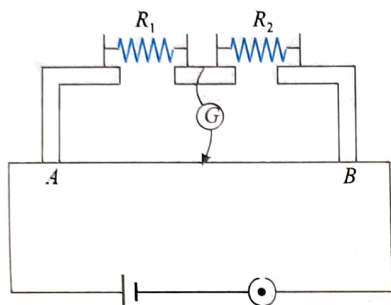
- (1) Red, Green, Brown (2) Brown, Blue, Brown
 (3) Grey, Black, Brown (4) Brown, Blue, Black

26. A potentiometer wire AB having length L and resistance $12r$ is joined to a cell D of emfs and internal resistance r . A cell C having emf $\epsilon/2$ and internal resistance $3r$ is connected. The length AJ at which the galvanometer as shown in fig. shows no deflection is



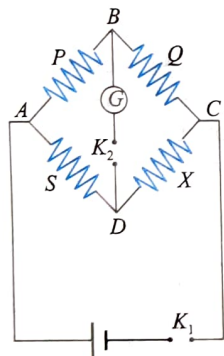
- (1) $\frac{5}{12}L$ (2) $\frac{11}{24}L$
 (3) $\frac{11}{12}L$ (4) $\frac{13}{24}L$

27. In the experimental set up of metre bridge shown in the figure, the null point is obtained at a distance of 40 cm from A. If a $10\ \Omega$ resistor is connected in series with R_1 , the null point shifts by 10 cm. The resistance that should be connected in parallel with $(R_1 + 10)\ \Omega$ such that the null point shifts back to its initial position is



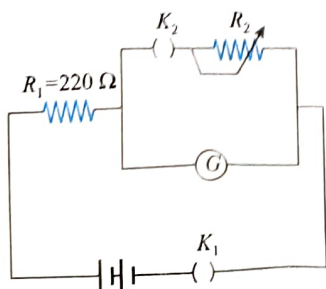
- (1) $40\ \Omega$ (2) $60\ \Omega$
 (3) $20\ \Omega$ (4) $30\ \Omega$

28. In a Wheatstone bridge (see fig.), resistances P and Q are approximately equal. When $R = 400\ \Omega$, the bridge is equal. When $R = 400\ \Omega$, the bridge is balanced. On inter-changing P and Q , the value of R , for balance, is $405\ \Omega$. The value of X is close to



- (1) $403.5\ \text{ohm}$ (2) $404.5\ \text{ohm}$
 (3) $401.5\ \text{ohm}$ (4) $402.5\ \text{ohm}$

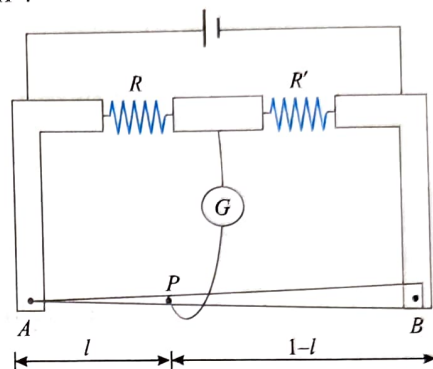
29. The galvanometer deflection, when key K_1 is closed but K_2 is open, equals θ_0 (see figure). On closing K_2 also and adjusting R_2 to $5\ \Omega$, the deflection in galvanometer becomes $\frac{\theta_0}{5}$. The resistance of the galvanometer is, then, given by



- (1) $12\ \Omega$ (2) $25\ \Omega$
 (3) $5\ \Omega$ (4) $22\ \Omega$

30. In a meter bridge, the wire of length 1 m has a non-uniform cross-section such that, the variation $\frac{dR}{dl}$ of its resistance R with length l is $\frac{dR}{dl} \propto \frac{1}{\sqrt{l}}$. Two equal resistances are

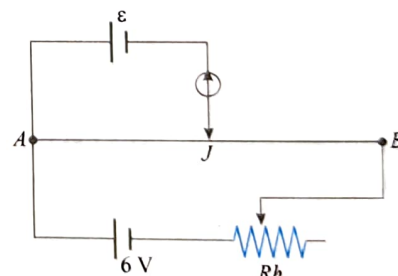
connected as shown in the figure. The galvanometer has zero deflection when the jockey is at point P . What is the length AP ?



- (1) $0.25\ \text{m}$ (2) $0.3\ \text{m}$
 (3) $0.35\ \text{m}$ (4) $0.2\ \text{m}$

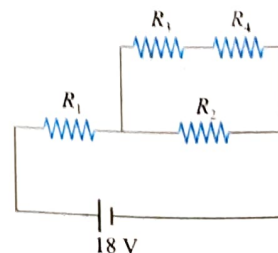
31. An ideal battery of 4 V and resistance R are connected in series in the primary circuit of a potentiometer of length 1 m and resistance $5\ \Omega$. The value of R , to give a potential difference of 5 mV across 10 cm of potentiometer wire, is
 (1) $490\ \Omega$ (2) $480\ \Omega$
 (3) $395\ \Omega$ (4) $495\ \Omega$

32. The resistance of the meter bridge AB in the given figure is $4\ \Omega$. With a cell of emf $\varepsilon = 0.5\ \text{V}$ and rheostat resistance $R_h = 2\ \Omega$ the null point is obtained at some point J . When the cell is replaced by another one of emf $\varepsilon = \varepsilon_2$ the same null point J is found for $R_h = 6\ \Omega$. The emf ε_2 is



- (1) $0.6\ \text{V}$ (3) $0.5\ \text{V}$
 (3) $0.3\ \text{V}$ (4) $0.4\ \text{V}$

33. In the given circuit the internal resistance of the 18 V cell is negligible. If $R_1 = 400\ \Omega$, $R_3 = 100\ \Omega$ and $R_4 = 500\ \Omega$ and the reading of an ideal voltmeter across R_4 is 5 V, then the value R_2 will be



- (1) $300\ \Omega$ (3) $230\ \Omega$
 (3) $450\ \Omega$ (4) $550\ \Omega$

Chapter 7

Heating Effects of Current

34. A current of 2 mA was passed through an unknown resistor which dissipated a power of 4.4 W. Dissipated power when an ideal power supply of 11 V is connected across it is
 (1) 11×10^{-5} W (2) 11×10^{-4} W
 (3) 11×10^5 W (4) 11×10^{-3} W
35. A 2Ω carbon resistor is color coded with green, black, red and brown respectively. The maximum current which can be passed through this resistor is
 (1) 63 mA (2) 0.4 mA
 (3) 100 mA (4) 20 mA

36. Two equal resistance when connected in series to a battery, consume electric power of 60 W. If these resistances are now connected in parallel combination to the same battery, the electric power consumed will be
 (1) 60 W (2) 240 W
 (3) 30 W (4) 120 W
37. Two electric bulbs, rated at (25 W, 220 V) and (100 W, 220 V), are connected in series across a 220 V voltage source. If the 25 W and 100 W bulbs draw powers P_1 and P_2 respectively, then
 (1) $P_1 = 9$ W, $P_2 = 16$ W (2) $P_1 = 4$ W, $P_2 = 16$ W
 (3) $P_1 = 16$ W, $P_2 = 4$ W (4) $P_1 = 16$ W, $P_2 = 9$ W

Answers Key

- | | | | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1. (3) | 2. (3) | 3. (3) | 4. (2) | 5. (3) | 21. (3) | 22. (4) | 23. (4) | 24. (2) | 25. (2) |
| 6. (2) | 7. (1) | 8. (2) | 9. (1) | 10. (4) | 26. (4) | 27. (2) | 28. (4) | 29. (4) | 30. (1) |
| 11. (3) | 12. (1) | 13. (4) | 14. (4) | 15. (2) | 31. (3) | 32. (3) | 33. (1) | 34. (1) | 35. (4) |
| 16. (4) | 17. (1) | 18. (2) | 19. (4) | 20. (3) | 36. (2) | 37. (3) | | | |